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Anti-neutrosophic fuzzy ideals of near-rings

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Abstract

In this article, we characterize the anti-neutrosophic fuzzy ideals of near-rings with an example. Here we introduce notion of anti-neutrosophic fuzzy subnear-rings and how they related to ideals within the near-rings. Finally, we provide an example illustrating the application of anti -neutrosophic fuzzy ideals within the context of a specific near-ring.

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Keywords: *Near-ring, Anti-fuzzy subnear-ring, Neutrosophic set, Anti-neutrosophic fuzzy ideal.*

1. Introduction

L. A. Zadeh [7] introduced the concept of fuzzy sets with membership function μ this values represent the degree of truth or falsity of each element with in the given universe of discourse, a number from the unit interval whole and nothing these indication represent the degree of membership function. The concept of fuzzy sets was established in 1965, generalizing classical set theory by allowing between everything and nothing. L.A.Zadeh [8] established the concept of interval valued (in short i-v) fuzzy logic membership functions typically assign values between 0 and 1 to represent the degree of membership of an element in a fuzzy set. These values can indeed be closed interval numbers within that range. In 1975, abbreviations were used according to the single valued. Abou-zaid

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developed concepts of fuzzy subnear-ring and ideals [1]. Davvaz [3] proposed the concept of a fuzzy ideals of near-rings with interval valued membership values in 2001. Biwaz [2] pioneered the concept of anti-fuzzy groups of subgroups K.H.Kim et al. [5] pioneered the notion of anti fuzzy ideals of near-ring. K.Hemabala and B.Srinivasa Kumar [4] introduced anti-neutrosophic multi fuzzy ideals of γ near rings and determined some algebraic properties. As a new branch of thought A.A.Salama and Florentin Smarandache [6] pioneered the concept of characteristic function of a neutrosophic set. Neutrosophy logic extends fuzzy logic by introducing a third component: the interminate element. In addition to "true" and "false" values, neutrosophic logic considers "indeterminate" values, which represent uncertain or vague information. This extension allows for a more nuanced representation of uncertainty, making it particularly useful in decision-making processes where uncertainty is prevalent. Our article is organized as follows: In section 2, preliminary in that some of the definitions and useful notations are provided. Finally, we examine the anti-neutrosophic fuzzy subnear-rings(in short ANFSN) and anti-neutrosophic fuzzy left(right) ideals of a near-rings (in short ANFIN). We use the examples in section 3 to examine some of its properties.

2. Anti-Neutrosophic fuzzy ideals of near-rings

In this section we defines concepts of ANFSN and ideals of left near-ring. In this article $\mathfrak{N}$ represents the left near-ring, $\mathcal{P}$ denotes membership function, $\mathcal{Q}$ denotes indeterminancy membership function, $\mathcal{R}$ represents non-membership function.

Definition 2.1: A neutrosophic fuzzy subset *S* of near-ring $\mathfrak{N}$ is called an ANFSN of $\mathfrak{N}$ if

(1) $\mathcal{P}_S(\varrho-\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$

$\mathcal{Q}_S(\varrho-\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$

$\mathcal{R}_S(\varrho-\varsigma) \ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\}$.

(2) $\mathcal{P}_S(\varrho\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$

$\mathcal{Q}_S(\varrho\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$

$\mathcal{R}_S(\varrho\varsigma) \ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\}$.

The neutrosophic fuzzy subset S of near-ring $\mathfrak{N}$ is called the ANFIN of $\mathfrak{N}$ if S is the ANFSN of $\mathfrak{N}$ and

(3) $\mathcal{P}_S(\varsigma+\varrho-\varsigma) \le \mathcal{P}_S(\varrho)$

$\mathcal{Q}_S(\varsigma+\varrho-\varsigma) \ge \mathcal{Q}_S(\varrho)$

$\mathcal{R}_S(\varsigma+\varrho-\varsigma) \ge \mathcal{R}_S(\varrho).$

(4) $\mathcal{P}_S(\varrho\varsigma) \le \mathcal{P}_S(\varsigma)$

$\mathcal{Q}_S(\varrho\varsigma) \ge \mathcal{Q}_S(\varsigma)$

$\mathcal{R}_S(\varrho\varsigma) \ge \mathcal{R}_S(\varsigma).$

(5) $\mathcal{P}_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \le \mathcal{P}_S(\vartheta)$

$\mathcal{Q}_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \ge \mathcal{Q}_S(\vartheta)$

$\mathcal{R}_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \ge \mathcal{R}_S(\vartheta).$ for all $\varrho,\varsigma,\vartheta \in \mathfrak{N}.$

Note that S is the anti-neutrosophic fuzzy left ideal of $\mathfrak{N}$ if it satisfies (1), (2), (3) and (4) and S is the anti-neutrosophic fuzzy right ideal of $\mathfrak{N}$ if it satisfies (1), (2), (3) and (5).

Definition 2.2: If S_ϖ be the family of neutrosophic fuzzy subset of $\mathfrak{N}$. Then for all $\varrho \in \mathfrak{N}$, $\bigcup_{\varpi\in\mathfrak{N}} S_\varpi$ and $\bigcap_{\varpi\in\mathfrak{N}} S_\varpi$ are defined as follows,

(1) $\bigcup_{\varpi\in\mathfrak{N}} \mathcal{P}_{S_\varpi}(\varrho) = \sup\{\mathcal{P}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}$

$\bigcup_{\varpi\in\mathfrak{N}} \mathcal{Q}_{S_\varpi}(\varrho) = \inf\{\mathcal{Q}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}$

$\bigcup_{\varpi\in\mathfrak{N}} \mathcal{R}_{S_\varpi}(\varrho) = \inf\{\mathcal{R}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}.$

(2) $\bigcap_{\varpi\in\mathfrak{N}} \mathcal{P}_{S_\varpi}(\varrho) = \inf\{\mathcal{P}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}$

$\bigcap_{\varpi\in\mathfrak{N}} \mathcal{Q}_{S_\varpi}(\varrho) = \sup\{\mathcal{Q}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}$

$\bigcap_{\varpi\in\mathfrak{N}} \mathcal{R}_{S_\varpi}(\varrho) = \sup\{\mathcal{R}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}.$

Example 2.3: Let $\mathfrak{N} = \{\omega_a, \omega_b, \omega_c, \omega_d\}$ be a set with two binary operations defined as follows:

$+$	ω_a	ω_b	ω_c	ω_d
ω_a	ω_a	ω_b	ω_c	ω_d
ω_b	ω_b	ω_a	ω_d	ω_c
ω_c	ω_c	ω_d	ω_b	ω_a
ω_d	ω_d	ω_c	ω_a	ω_b

$\cdot$	ω_a	ω_b	ω_c	ω_d
ω_a	ω_a	ω_a	ω_a	ω_a
ω_b	ω_a	ω_a	ω_a	ω_a
ω_c	ω_a	ω_a	ω_a	ω_a
ω_d	ω_a	ω_b	ω_c	ω_d

Clearly $(\mathfrak{N},+,\cdot)$ is left near-ring. Determine $\mathcal{P}_S$, $\mathcal{Q}_S$, $\mathcal{R}_S : \mathfrak{N} \to [0,1]$ by $\frac{\omega_a}{\{0.02,0.05,0.07\}}$, $\frac{\omega_b}{\{0.04,0.03,0.05\}}$, $\frac{\omega_c}{\{0.06,0.01,0.03\}}$, $\frac{\omega_d}{\{0.06,0.01,0.03\}}$. It is verifiable $\{< \varrho, \mathcal{P}_S, \mathcal{Q}_S, \mathcal{R}_S > \mid \varrho \in \mathfrak{N}\}$ is the ANFIN of $\mathfrak{N}$.

Proposition 2.4: If S is a neutrosophic fuzzy subset S in N satisfies the condition $S(\varsigma + \varrho - \varsigma) = S(\varrho)$ then $S(\varrho + \varsigma) = S(\varsigma + \varrho)$

Proof: Consider S is ANFIN $\mathfrak{N}$. In any case $\varrho, \varsigma \in \mathfrak{N}$, we now have $S(\varrho + \varsigma) = S(\varsigma + (\varrho + \varsigma) - \varsigma) = S(\varsigma + (\varsigma + \varrho) - \varsigma) = S(\varsigma + \varrho)$.

Theorem 2.5: *If* $\{S_\varpi \mid \varpi \in \mathfrak{N}\}$ *is family of ANFIN* $\mathfrak{N}$, *then* $\bigcup_{\varpi \in \Theta} S_\varpi$ *is also ANFIN* $\mathfrak{N}$, *where* Θ *is any index set.*

Proof: Let $\varrho, \varsigma, \vartheta \in \mathfrak{N}$. Then,

$$\begin{aligned}
\bigcup_{\varpi \in \mathfrak{N}} \mathcal{P}_{S_\varpi}(\varrho - \varsigma) &= \sup\{\mathcal{P}_{S_\varpi}(\varrho - \varsigma) \mid \varpi \in \mathfrak{N}\} \\
&\le \sup\{\max\{\mathcal{P}_{S_\varpi}(\varrho), \mathcal{P}_{S_\varpi}(\varsigma)\} \mid \varpi \in \mathfrak{N}\} \\
&= \max\{\sup\{\mathcal{P}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}, \sup\{\mathcal{P}_{S_\varpi}(\varsigma) \mid \varpi \in \mathfrak{N}\}\} \\
&= \max\{\bigcup_{\varpi \in \mathfrak{N}} \mathcal{P}_{S_\varpi}(\varrho), \bigcup_{\varpi \in \Theta} \mathcal{P}_{S_\varpi}(\varsigma)\}
\end{aligned}$$

$$\begin{aligned}
\bigcup_{\varpi \in \mathfrak{N}} \mathcal{Q}_{S_\varpi}(\varrho - \varsigma) &= \inf\{\mathcal{Q}_{S_\varpi}(\varrho - \varsigma) \mid \varpi \in \mathfrak{N}\} \\
&\ge \inf\{\min\{\mathcal{Q}_{S_\varpi}(\varrho), \mathcal{Q}_{S_\varpi}(\varsigma)\} \mid \varpi \in \mathfrak{N}\} \\
&= \min\{\inf\{\mathcal{Q}_{S_\varpi}(\varrho) \mid \varpi \in \mathfrak{N}\}, \inf\{\mathcal{Q}_{S_\varpi}(\varsigma) \mid \varpi \in \mathfrak{N}\}\} \\
&= \min\{\bigcup_{\varpi \in \mathfrak{N}} \mathcal{Q}_{S_\varpi}(\varrho), \bigcup_{\varpi \in \mathfrak{N}} \mathcal{Q}_{S_\varpi}(\varsigma)\}
\end{aligned}$$

Similarly we can prove

$$\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho-\varsigma)\geq\min\{\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho),\bigcup_{\eta\in\Re}\mathcal{R}_{S_\varpi}(\varsigma)\}$$

$$\begin{aligned}\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(l-m)&=\inf\{\mathcal{R}_{S_i}(l-m)\mid i\in\Omega\}\\&\geq\inf\{\min\{\mathcal{R}_{S_i}(l),\mathcal{R}_{S_i}(m)\}\mid i\in\Omega\}\\&=\min\{\inf\{\mathcal{R}_{S_i}(l)\mid i\in\Omega\},\inf\{\mathcal{R}_{S_i}(m)\mid i\in\Omega\}\}\\&=\min\{\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(l),\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(m)\}\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varrho\varsigma)&=\sup\{\mathcal{P}_{S_\varpi}(\varrho\varsigma)\mid\varpi\in\Re\}\\&\leq\sup\{\max\{\mathcal{P}_{S_\varpi}(\varrho),\mathcal{P}_{S_\varpi}(\varsigma)\}\mid\varpi\in\Re\}\\&=\max\{\sup\{\mathcal{P}_{S_\varpi}(\varrho)\mid\varpi\in\Re\},\sup\{\mathcal{P}_{S_\varpi}(\varsigma)\mid\varpi\in\Re\}\}\\&=\max\{\{\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varrho),\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varsigma)\}\mid\varpi\in\Re\}\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varrho\varsigma)&=\inf\{\mathcal{Q}_{S_\varpi}(\varrho\varsigma)\mid\varpi\in\Re\}\\&\geq\inf\{\min\{\mathcal{Q}_{S_\varpi}(\varrho),\mathcal{Q}_{S_\varpi}(\varsigma)\}\mid\varpi\in\Re\}\\&=\min\{\inf\{\mathcal{Q}_{S_\varpi}(\varrho)\mid\varpi\in\Re\},\inf\{\mathcal{Q}_{S_\varpi}(\varsigma)\mid\varpi\in\Re\}\}\\&=\min\{\{\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varrho),\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varsigma)\}\mid\varpi\in\Re\}\end{aligned}$$

Similarly we can prove

$$\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho\varsigma)\geq\min\{\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho),\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varsigma)\}$$

$$\begin{aligned}\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(lm)&=\inf\{\mathcal{R}_{S_i}(lm)\mid i\in\Omega\}\\&\geq\inf\{\min\{\mathcal{R}_{S_i}(l),\mathcal{R}_{S_i}(m)\}\mid i\in\Omega\}\\&=\min\{\inf\{\mathcal{R}_{S_i}(l)\mid i\in\Omega\},\inf\{\mathcal{R}_{S_i}(m)\mid i\in\Omega\}\}\\&=\min\{\{\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(l),\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(m)\}\mid i\in\Omega\}\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varsigma+\varrho-\varsigma)&=\sup\{\mathcal{P}_{S_\varpi}(\varsigma+\varrho-\varsigma)\mid\varpi\in\Re\}\\&\leq\sup\{\mathcal{P}_{S_\varpi}(\varrho)\mid\varpi\in\Re\}\\&=\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varrho)\$\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varsigma+\varrho-\varsigma) &= \inf\{\mathcal{Q}_{S_\varpi}(\varsigma+\varrho-\varsigma)\,|\,\varpi\in\Re\}\\ &\geq \inf\{\mathcal{Q}_{S_\varpi}(\varrho)\,|\,\varpi\in\Re\}\\ &= \bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varrho)\end{aligned}$$

Similarly we can prove

$$\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varsigma+\varrho-\varsigma)\geq\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho)$$

$$\begin{aligned}\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(m+l-m) &= \inf\{\mathcal{R}_{S_i}(m+l-m)\,|\,i\in\Omega\}\\ &\geq \inf\{\mathcal{R}_{S_i}(l)\,|\,i\in\Omega\}\\ &= \bigcup_{i\in\Omega}\mathcal{R}_{S_i}(l)\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varrho\varsigma) &= \sup\{\mathcal{P}_{S_\varpi}(\varrho\varsigma)\,|\,\varpi\in\Theta\}\\ &\leq \sup\{\mathcal{P}_{S_\varpi}(\varsigma)\,|\,\varpi\in\Re\}\\ &= \bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\varsigma)\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varrho\varsigma) &= \inf\{\mathcal{Q}_{S_\varpi}(\varrho\varsigma)\,|\,\varpi\in\Re\}\\ &\geq \inf\{\mathcal{Q}_{S_\varpi}(\varsigma)\,|\,\varpi\in\Re\}\\ &= \bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\varsigma)\end{aligned}$$

Similarly we can prove

$$\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varrho\varsigma)\geq\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\varsigma)$$

$$\begin{aligned}\bigcup_{i\in\Omega}\mathcal{R}_{S_i}(lm) &= \inf\{\mathcal{R}_{S_i}(lm)\,|\,i\in\Omega\}\\ &\geq \inf\{\mathcal{R}_{S_i}(m)\,|\,i\in\Omega\}\\ &= \bigcup_{i\in\Omega}\mathcal{R}_{S_i}(m)\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}((\varrho+\vartheta)\varsigma-\varrho\varsigma) &= \sup\{\mathcal{P}_{S_\varpi}((\varrho+\vartheta)\varsigma-\varrho\varsigma)\,|\,\varpi\in\Re\}\\ &\leq \sup\{\mathcal{P}_{S_\varpi}(\vartheta)\,|\,i\in\Re\}\\ &= \bigcup_{\varpi\in\Re}\mathcal{P}_{S_\varpi}(\vartheta)\end{aligned}$$

$$\begin{aligned}\bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}((\varrho+\vartheta)\varsigma-\varrho\varsigma) &= \inf\{\mathcal{Q}_{S_\varpi}((\varrho+\vartheta)\varsigma-\varrho\varsigma)\mid\varpi\in\Re\}\\ &\geq \inf\{\mathcal{Q}_{S_\varpi}(\vartheta)\mid\varpi\in\Re\}\\ &= \bigcup_{\varpi\in\Re}\mathcal{Q}_{S_\varpi}(\vartheta)\end{aligned}$$

Similarly we can prove

$$\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}((\varrho+\vartheta)\varsigma-\varrho\varsigma)\geq\bigcup_{\varpi\in\Re}\mathcal{R}_{S_\varpi}(\vartheta)$$

$$\begin{aligned}\bigcup_{i\in\Omega}\mathcal{R}_{S_i}((l+n)m-lm) &= \inf\{\mathcal{R}_{S_i}((l+n)m-lm)\mid i\in\Omega\}\\ &\geq \inf\{\mathcal{R}_{S_i}(n)\mid i\in\Omega\}\\ &= \bigcup_{i\in\Omega}\mathcal{R}_{S_i}(n)\end{aligned}$$

Therefore $\bigcup_{\varpi\in\Re}S_\varpi$ is an ANFIN $\Re$.

Definition 2.6: If $\beth$ be subset of a near-ring $\Re$. Let S be neutrosophic fuzzy subset of $\Re$. Define $\Xi_\beth(S) = \{<\varrho, \Xi_\beth(\mathcal{P})_S, \Xi_\beth(\mathcal{Q})_S, \Xi_\beth(\mathcal{R})_S> \mid \varrho\in\Re\}$, where $\Xi_\beth(\mathcal{P})_S:\Re\to[0,1]$, $\Xi_\beth(\mathcal{Q})_S:\Re\to[0,1]$ and $\Xi_\beth(\mathcal{R})_S:\Re\to[0,1]$ as follows:

$$\Xi_\beth(\mathcal{P})_S(\varrho)=\begin{cases}0 & \text{if } \varrho\in\beth\\ 1 & \text{otherwise}\end{cases}$$

$$\Xi_\beth(\mathcal{Q})_S(\varrho)=\begin{cases}1 & \text{if } \varrho\in\beth\\ 0 & \text{otherwise}\end{cases}$$

$$\Xi_\beth(\mathcal{R})_S(\varrho)=\begin{cases}1 & \text{if } \varrho\in\beth\\ 0 & \text{otherwise}\end{cases}$$

Clearly $\Xi_\beth(S)$ is a neutrosophic fuzzy subset of $\Re$. $\Xi_\beth(S)$ is called an anti-neutrosophic fuzzy characteristic function of $\beth$.

Theorem 2.7: *Let $\beth$ be subset of near-ring $\Re$. The characteristic function $\Xi_\beth(S):\Re\to[0,1]$ is an ANFIN $\Re$ iff $\beth$ is left(right) ideal of $\Re$.*

Proof: If $\beth$ be left(right) ideal of $\Re$. Let

$$\Xi_{\beth}(\mathcal{P})_S(\varrho)=\begin{cases}0 & \text{if} \quad \varrho\in\beth\\ 1 & \text{otherwise}\end{cases}$$

$$\Xi_{\beth}(\mathcal{Q})_S(\varrho)=\begin{cases}1 & \text{if} \quad \varrho\in\beth\\ 0 & \text{otherwise}\end{cases}$$

$$\Xi_{I}(\mathcal{R})_S(\varrho)=\begin{cases}1 & \text{if} \quad \varrho\in\beth\\ 0 & \text{otherwise}\end{cases}$$

Thus $\Xi_{\beth}(S)$ is an ANFIN $\mathfrak{N}$.

Conversely,

Consider $\Xi_{\beth}(S)$ is an ANFIN $\mathfrak{N}$.

Let $\varrho,\varsigma,\vartheta\in\beth$. Then $\Xi_{\beth}(\mathcal{P})_S(\varrho)=0=\Xi_{\beth}(\mathcal{P})_S(\varsigma)$, $\Xi_{\beth}(\mathcal{Q})_S(\varrho)=1=\Xi_{\beth}(\mathcal{Q})_S(\varsigma)$ and $\Xi_{\beth}(\mathcal{R})_S(\varrho)=1=\Xi_{\beth}(\mathcal{R})_S(\varsigma)$.

$$\Xi_{\beth}(\mathcal{P})_S(\varrho-\varsigma)\le\max\{\Xi_{\beth}(\mathcal{P})_S(\varrho),\Xi_{\beth}(\mathcal{P})_S(\varsigma)\}=\max\{0,0\}=0$$

$$\Xi_{\beth}(\mathcal{Q})_S(\varrho-\varsigma)\ge\min\{\Xi_{\beth}(\mathcal{Q})_S(\varrho),\Xi_{\beth}(\mathcal{Q})_S(\varsigma)\}=\min\{1,1\}=1$$

$$\Xi_{\beth}(\mathcal{R})_S(\varrho-\varsigma)\ge\min\{\Xi_{\beth}(\mathcal{R})_S(\varrho),\Xi_{\beth}(\mathcal{R})_S(\varsigma)\}=\min\{1,1\}=1.$$

So $\Xi_{\beth}(\mathcal{P})_S(\varrho-\varsigma)=0,\Xi_{\beth}(\mathcal{Q})_S(\varrho-\varsigma)=1,\Xi_{\beth}(\mathcal{R})_S(\varrho-\varsigma)=1.$

Therefore $\varrho-\varsigma\in\beth$. Hence $\beth$ is an additive subgroup of $\mathfrak{N}$.

$$\Xi_{\beth}(\mathcal{P})_S(\varrho\varsigma)\le\max\{\Xi_{\beth}(\mathcal{P})_S(\varrho),\Xi_{\beth}(\mathcal{P})_S(\varsigma)\}=\max\{0,0\}=0$$

$$\Xi_{\beth}(\mathcal{Q})_S(\varrho\varsigma)\ge\min\{\Xi_{\beth}(\mathcal{Q})_S(\varrho),\Xi_{\beth}(\mathcal{Q})_S(\varsigma)\}=\min\{1,1\}=1$$

$$\Xi_{\beth}(\mathcal{R})_S(\varrho\varsigma)\ge\min\{\Xi_{\beth}(\mathcal{R})_S(\varrho),\Xi_{\beth}(\mathcal{R})_S(\varsigma)\}=\min\{1,1\}=1.$$

So $\Xi_{\beth}(\mathcal{P})_S(\varrho\varsigma)=0$, $\Xi_{\beth}(\mathcal{Q})_S(\varrho\varsigma)=1$, $\Xi_{\beth}(\mathcal{R})_S(\varrho\varsigma)=1$.

Therefore $\varrho\varsigma\in\beth$. Hence $\beth$ is a multiplicative semigroup of $\mathfrak{N}$.

Let $\varrho\in\beth$ and $\varsigma\in\mathfrak{N}$. Then

$$\Xi_{\beth}(\mathcal{P})_S(\varsigma+\varrho-\varsigma)\le\Xi_{\beth}(\mathcal{P})_S(\varrho)=0$$
$$\Xi_{\beth}(\mathcal{Q})_S(\varsigma+\varrho-\varsigma)\ge\Xi_{\beth}(\mathcal{Q})_S(\varrho)=1$$
$$\Xi_{\beth}(\mathcal{R})_S(\varsigma+\varrho-\varsigma)\ge\Xi_{\beth}(\mathcal{R})_S(\varrho)=1.$$

Hence, $\Xi_{\beth}(\mathcal{P})_S\ (\varsigma+\varrho-\varsigma) = 0$, $\Xi_{\beth}(\mathcal{Q})_S\ (\varsigma+\varrho-\varsigma) = 1$, $\Xi_{\beth}\ (\mathcal{R})_S\ (\varsigma+\varrho-\varsigma) = 1$ which is implies that $\varsigma+\varrho-\varsigma \in \beth$. Thus $\beth$ is normal subgroup of $\mathfrak{N}$.

If $\varrho \in \mathfrak{N}$, $\varsigma \in \beth$.

$$\Xi_{\beth}(\mathcal{P})_S(\varrho\varsigma) \le \Xi_{\beth}(\mathcal{P})_S(\varsigma) = 0$$
$$\Xi_{\beth}(\mathcal{Q})_S(\varrho\varsigma) \ge \Xi_{\beth}(\mathcal{Q})_S(\varsigma) = 1$$
$$\Xi_{\beth}(\mathcal{R})_S(\varrho\varsigma) \ge \Xi_{\beth}(\mathcal{R})_S(\varsigma) = 1.$$

Hence, $\Xi_{\beth}(\mathcal{P})_S(\varrho\varsigma) = 0$, $\Xi_{\beth}(\mathcal{Q})_S(\varrho\varsigma) = 1$, $\Xi_{\beth}(\mathcal{R})_S(\varrho\varsigma) = 1$ which is implies that $\varrho\varsigma \in \beth$. Thus $\beth$ is left ideal of $\mathfrak{N}$.

If $\varrho,\ \varsigma \in \mathfrak{N},\ \vartheta \in \beth$. Then

$$\Xi_{\beth}(\mathcal{P})_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \le \Xi_{\beth}(\mathcal{P})_S(\vartheta) = 0$$
$$\Xi_{\beth}(\mathcal{P})_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \ge \Xi_{\beth}(\mathcal{P})_S(\vartheta) = 1$$
$$\Xi_{\beth}(\mathcal{P})_S((\varrho+\vartheta)\varsigma-\varrho\varsigma) \ge \Xi_{\beth}(\mathcal{P})_S(\vartheta) = 1.$$

Hence, $\Xi_{\beth}(\mathcal{P})_S\ ((\varrho+\vartheta)\ \varsigma-\varrho\varsigma) = 0$, $\Xi_{\beth}(\mathcal{P})_S\ ((\varrho+\vartheta)\ \varsigma-\varrho\varsigma) = 1$, $\Xi_I(\mathcal{P})_S\ ((\varrho+\vartheta)\ \varsigma-\varrho\varsigma) = 1$ which is implies that $(\varrho+\vartheta)\varsigma - \varrho\varsigma \in \beth$. Thus $\beth$ is right ideal of $\mathfrak{N}$.

Therefore $\beth$ is left(right) ideal of $\mathfrak{N}$.

Definition 2.8: Let S be neutrosophic fuzzy subset $\mathfrak{N}$. Then for any $\varepsilon \in [0,1]$, then ε - lower level sets of S denoted by $L(S:\varepsilon)$ is defined as follows: $L(S:\varepsilon) = \{\varrho \in W \mid \mathcal{P}_S(\varrho) \le \varepsilon,\ \mathcal{Q}_S(\varrho) \ge \varepsilon,\ \mathcal{R}_S(\varrho) \ge \varepsilon\}$.

Theorem 2.9: If S be neutrosophic fuzzy subset of $\mathfrak{N}$. S is anti-neutrosophic fuzzy left(right) ideal of $\mathfrak{N}$ *iff* $L(S:\varepsilon)$ is a left(right) ideal of $\mathfrak{N}$ *for all* $\varepsilon \in [0,1]$.

Proof: Consider S is an ANFIN $\mathfrak{N}$. Let $\varepsilon \in [0,1]$ such that $\varrho,\varsigma,\vartheta \in L(S:\varepsilon)$

Then,

$$\mathcal{P}_S(\varrho-\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\} \le \max\{\varepsilon,\varepsilon\} = \varepsilon$$
$$\mathcal{Q}_S(\varrho-\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\} \ge \min\{\varepsilon,\varepsilon\} = \varepsilon$$
$$\mathcal{R}_S(\varrho-\varsigma) \ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\} \ge \min\{\varepsilon,\varepsilon\} = \varepsilon.$$

Thus, $\varrho-\varsigma \in L(S:\varepsilon)$. Hence $L(S:\varepsilon)$ is additive subgroup of $\mathfrak{N}$.

Let $\varrho \in L(S:\varepsilon)$ and $\varsigma \in \mathfrak{N}$

$$\mathcal{P}_S(\varrho\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\} \le \max\{\varepsilon, \varepsilon\} = \varepsilon$$
$$\mathcal{Q}_S(\varrho\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\} \ge \min\{\varepsilon, \varepsilon\} = \varepsilon$$
$$\mathcal{R}_S(\varrho\varsigma) =\ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\} \ge \min\{\varepsilon, \varepsilon\} = \varepsilon.$$

Thus, $\varrho\varsigma \in L(S:\varepsilon)$. Hence $L(S:\varepsilon)$ is an multiplicative semigroup of $\mathfrak{N}$.

Let $\varrho \in L(S:\varepsilon)$ and $\varsigma \in \mathfrak{N}$

$$\mathcal{P}_S(\varsigma + \varrho - \varsigma) \le \mathcal{P}_S(\varrho) = \varepsilon$$
$$\mathcal{Q}_S(\varsigma + \varrho - \varsigma) \ge \mathcal{Q}_S(\varrho) = \varepsilon$$
$$\mathcal{R}_S(\varsigma + \varrho - \varsigma) \ge \mathcal{R}_S(\varrho) = \varepsilon.$$

Therefore, $\varsigma + \varrho - \varsigma \in L(S:\varepsilon)$.

Hence, $L(S:\varepsilon)$ is normal subgroup of $\mathfrak{N}$. Let $\varsigma \in L(S:\varepsilon)$, $\varrho \in \mathfrak{N}$.

$$\mathcal{P}_S(\varrho\varsigma) \le \mathcal{P}_S(\varsigma) = \varepsilon, \mathcal{Q}_S(\varrho\varsigma) \ge \mathcal{Q}_S(\varsigma) = \varepsilon, \mathcal{R}_S(\varrho\varsigma) \ge \mathcal{R}_S(\varsigma) = \varepsilon.$$

Therefore, $\varrho\varsigma \in L(S:\varepsilon)$.

Let $\vartheta \in L(S:\varepsilon)$ and $\varrho, \varsigma \in \mathfrak{N}$.

$$\mathcal{P}_S((\varrho + \vartheta)\varsigma - \varrho\varsigma) \le \mathcal{P}_S(\vartheta) = \varepsilon$$

$$\mathcal{Q}_S((\varrho + \vartheta)\varsigma - \varrho\varsigma) \ge \mathcal{Q}_S(\vartheta) = \varepsilon$$

$$\mathcal{R}_S((\varrho + \vartheta)\varsigma - \varrho\varsigma) \ge \mathcal{R}_S(\vartheta) = \varepsilon.$$

Therefore, $((\varrho + \vartheta)\varsigma - \varrho\varsigma) \in L(S:\varepsilon)$. Hence $L(S:\varepsilon)$ is left(right) ideal of the near ring $\mathfrak{N}$.

Conversely, assume $L(S:\varepsilon)$ is left(right) ideal of $\mathfrak{N}$, for all $\varepsilon \in [0,1]$.

Let $\varrho, \varsigma \in \mathfrak{N}$. Suppose, $\mathcal{P}_S(\varrho - \varsigma) > \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{P}_S(\varrho - \varsigma) > \kappa > \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$. It implies that $\mathcal{P}_S(\varrho) < \kappa$ and $\mathcal{P}_S(\varsigma) < \kappa$. Suppose, $\mathcal{Q}_S(\varrho - \varsigma) < \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{Q}_S(\varrho - \varsigma) < \kappa < \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$.

It implies that $\mathcal{Q}_S(\varrho) > \kappa$ and $\mathcal{Q}_S(\varsigma) > \kappa$.

Similar way we can derive for $\mathcal{R}_S$.

Then we have $\varrho, \varsigma \in L(S:\kappa)$ and since $L(S:\kappa)$ is a left(right) ideal of $\mathfrak{N}$, $\varrho - \varsigma \in L(S:\kappa)$.

Hence, $\mathcal{P}_S(\varrho - \varsigma) \le \kappa, \mathcal{Q}_S(\varrho - \varsigma) \ge \kappa, \mathcal{R}_S(\varrho - \varsigma) \ge \kappa$ which is a contradiction.

Thus,

$$\mathcal{P}_S(\varrho-\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$$
$$\mathcal{Q}_S(\varrho-\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$$
$$\mathcal{R}_S(\varrho-\varsigma) \ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\}.$$

Suppose, $\mathcal{P}_S(\varrho\varsigma) > \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$. Choose $\kappa \in [0,1]$, such that, $\mathcal{P}_S(\varrho\varsigma) > \kappa > \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$.

This implies that $\mathcal{P}_S(\varrho) < \kappa$ and $\mathcal{P}_S(\varsigma) < \kappa$.

Suppose, $\mathcal{Q}_S(\varrho\varsigma) < \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{Q}_S(\varrho\varsigma) < \kappa < \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$.

This implies that $\mathcal{Q}_S(\varrho) > \kappa$ and $\mathcal{Q}_S(\varsigma) > \kappa$.

Similar way we can derive for $\mathcal{R}_S$.

Then we have $\varrho, \varsigma \in L(S:\kappa)$ and since $L(S:\kappa)$ is a left(right) ideal of $\mathfrak{N}$, $\varrho\varsigma \in L(S:\kappa)$.

Hence, $\mathcal{P}_S(\varrho\varsigma) \le \kappa, \mathcal{Q}_S(\varrho\varsigma) \ge \kappa, \mathcal{R}_S(\varrho\varsigma) \ge \kappa$ which is a contradiction.

Thus,

$$\mathcal{P}_S(\varrho\varsigma) \le \max\{\mathcal{P}_S(\varrho), \mathcal{P}_S(\varsigma)\}$$
$$\mathcal{Q}_S(\varrho\varsigma) \ge \min\{\mathcal{Q}_S(\varrho), \mathcal{Q}_S(\varsigma)\}$$
$$\mathcal{R}_S(\varrho\varsigma) \ge \min\{\mathcal{R}_S(\varrho), \mathcal{R}_S(\varsigma)\}.$$

Suppose, $\mathcal{P}_S(\varsigma+\varrho-\varsigma) > \mathcal{P}_S(\varrho)$. Choose $\kappa \in [0,1]$, such that, $\mathcal{P}_S(\varsigma+\varrho-\varsigma) > \kappa > \mathcal{P}_S(\varrho)$.

This implies that $\mathcal{P}_S(\varrho) < \kappa$.

Suppose, $\mathcal{Q}_S(\varsigma+\varrho-\varsigma) < \mathcal{Q}_S(\varrho)$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{Q}_S(\varsigma+\varrho-\varsigma) < \kappa < \mathcal{Q}_S(\varrho)$.

This implies that $\mathcal{Q}_S(\varrho) > \kappa$.

Similar way we can derive for $\mathcal{R}_S$.

Then $\varrho \in L(S:\kappa)$. Since $L(S:\kappa)$ is normal subgroup of $(\mathfrak{N},+)$, $\varsigma+\varrho-\varsigma \in L(S:\kappa)$. As a results, $\mathcal{P}_S(\varsigma+\varrho-\varsigma) \le \kappa$, $\mathcal{Q}_S(\varsigma+\varrho-\varsigma) \ge \kappa$, $\mathcal{R}_S(\varsigma+\varrho-\varsigma) \ge \kappa$ which is a contradiction.

Thus,

$$\mathcal{P}_S(\varsigma+\varrho-\varsigma) \le \mathcal{P}_S(\varrho)$$
$$\mathcal{Q}_S(\varsigma+\varrho-\varsigma) \ge \mathcal{Q}_S(\varrho)$$
$$\mathcal{R}_S(\varsigma+\varrho-\varsigma) \ge \mathcal{R}_S(\varrho).$$

Suppose, $\mathcal{P}_S(\varrho\varsigma) > \mathcal{P}_S(\varsigma)$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{P}_S(\varrho\varsigma) > \kappa > \mathcal{P}_S(\varsigma)$. This implies that $\mathcal{P}_S(m\varsigma) < \kappa$.

Suppose, $\mathcal{Q}_S(\varrho\varsigma) < \mathcal{Q}_S(\varsigma)$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{Q}_S(\varrho\varsigma) < \kappa < \mathcal{Q}_S(\varsigma)$.

This implies that $\mathcal{Q}_S(\varsigma) > \kappa$.

Similar way we can derive for $\mathcal{R}_S$.

Then $\varsigma \in L(S:\kappa)$. Since $L(S:\kappa)$ is left(right) ideal of $\mathfrak{N}$, $\varrho\varsigma \in L\ (S:\kappa)$.

Hence, $\mathcal{P}_S(\varrho\varsigma) \le \kappa, \mathcal{Q}_S(\varrho\varsigma) \ge \kappa, \mathcal{R}_S(\varrho\varsigma) \ge \kappa$. which is a contradiction.

Thus,

$$\mathcal{P}_S(\varrho\varsigma) \le \mathcal{P}_S(\varsigma)$$
$$\mathcal{Q}_S(\varrho\varsigma) \ge \mathcal{Q}_S(\varsigma)$$
$$\mathcal{R}_S(\varrho\varsigma) \ge \mathcal{R}_S(\varsigma).$$

Suppose, $\mathcal{P}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) > \mathcal{P}_S(\vartheta)$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{P}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) > \kappa > \mathcal{P}_S(\vartheta)$.

This implies that $\mathcal{P}_S(\vartheta) < \kappa$.

Suppose, $\mathcal{Q}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) < \mathcal{Q}_S(\vartheta)$.

Choose $\kappa \in [0,1]$, such that, $\mathcal{Q}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) < \kappa < \mathcal{Q}_S(\vartheta)$.

This implies that $\mathcal{Q}_S(\vartheta) > \kappa$.

Similar way we can derive for $\mathcal{R}_S$.

Then $\varsigma \in L(S:\kappa)$. Since is $L(S:\kappa)$ left(right) ideals of $\mathfrak{N}$,

$$(\varrho+\vartheta)\varsigma - \varrho\varsigma \in L(S:\kappa)$$

Hence,

$$\mathcal{P}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \le \kappa$$
$$\mathcal{Q}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \ge \kappa$$
$$\mathcal{R}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \ge \kappa.$$

which is a contradiction.

Thus,

$$\mathcal{P}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \le \mathcal{P}_S(\vartheta)$$
$$\mathcal{Q}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \ge \mathcal{Q}_S(\vartheta)$$
$$\mathcal{R}_S((\varrho+\vartheta)\varsigma - \varrho\varsigma) \ge \mathcal{R}_S(\vartheta).$$

Therefore, S is an ANFIN $\mathfrak{N}$.

Conclusion

This article we studied the concept of union of ANFIN is ANFIN. Further we proved characteristic property and level set property. This paper can be extended by using various type of ideals, namely bi-ideals, quasi ideals, prime ideals etc.,

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