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Analytical solution of unsteady MHD immiscible fluid with thermal radiation and chemical reaction in a porous medium under heat source

P. R. Sakthivel †
L. Sivakami *
Department of Mathematics and Statistics
Faculty of Science and Humanities
SRM Institute of Science and Technology
Kattankulathur 603203
Chengalpattu
Tamil Nadu
India

Abstract

Immiscible fluids find diverse applications across production and engineering sectors. In this study, we are presenting an analytical solution for the dynamics of unsteady immiscible fluid systems incorporating thermal radiation and chemical reactions within the porous medium subjected to a heat source. Initially, we address this complex system by formulating partial differential equations governing momentum, energy, and concentration. By employing nondimensional variables, we convert these equations into forms that have no dimensions, making it straightforward to evaluate them.

Following that, we implement the Laplace methods for obtaining accurate solutions for concentration, temperature, and velocity, ensuring that both initial and boundary conditions are maintained. Then by employing MATLAB, we create visual aids to clarify some phenomena existing in the system.

Subject Classification: *44A10, 76D05, 76S05, 80A99.*

Keywords: *Casson fluid, Chemical reaction, Heat source, Laplace transform, MHD, Porous medium, Thermal radiation.*

1. Introduction

Newtonian and non-Newtonian fluids differ greatly from one another. Wu et al. [1] studied the Recent progress in the working mechanism

† *E-mail:* **sr8485@srmist.edu.in**
* *E-mail:* **sivakaml@srmist.edu.in (Corresponding Author)**

of non-Newtonian fluids. There are several distinctions between the Newtonian and the non-Newtonian fluids, such as Kleppe, J., and W. J. Marner [2] explained Bingham plastic in a flat vertical plate. Olajuwon, B.I [3] investigate the flow and convection heat transfer of a pseudoplastic power-law fluid past the vertical plate with heat generation. Ilyas Khan et al. [4] explored the unsteady free convection flow utilizing a Walter-B fluid, Hassan et al. [5] investigated the natural convection that occurs in viscoplastic, and Swati Mukhopadhyay et al. [6] explored the fluid explicit movements over unstable stretching medium.

Cason [7] initially presented the Casson fluid flow model to forecast pigment-oil suspension patterns. Anwar et al. [8] explored the unstable MHD circulation within the Casson fluid on a boundless vertical platform. K. Ramesh and M. Devakar [9] derived the mathematical results corresponding to the Casson fluid flow problem with slip boundary conditions. C.S.K. Raju et al. [10] strengthened it by bringing out the analysis method for heat along with the mass transmission in the MHD Cason fluid through an increasingly permeable stretched surface. Li Li [11] presented a precise methodology for swiftly processing two-dimensional problems at a high velocity. The mathematical expression of partial differential equations can be analyzed using MATLAB programming to grasp the link between the graph, function value, and independent variable. Through the use of numerical approaches, the results can be visualized. Lakshmi Priya et al. [12] delved into the influence of transfer of heat and mass on MHD convective circulation of an immiscible medium in a horizontal channel comprising a reaction between chemicals and a heat source. Kashif Ali Khan et al. [13] reported on the behavior of heat along with the mass transfer in an unpredictable two-dimensional in-nature boundary layer flow system incorporating Cason fluid.

Hari R. Kataria and Harshad R. Patel [14] looked into the impact of heat generation/absorption as well as chemical interactions on the MHD Casson fluid's movement via a linearly propelled vertical panel. M. Hamid et al. [15] disclosed the phenomenon of natural convection movement in a substantially warm trapezoidal chamber harboring a non-Newtonian Casson fluid. The Caputo fractional model was constructed by Nadeem Ahmad Sheikh et al. [16] through the refinement of Fick's and Fourier's laws. Akolade et al. [17] assessed its effects by manipulating the combination of mass diffusion and viscous on the unconstrained convective unstable motion of the Casson fluid. Zainab Bukhari et al. [18] put out the findings of numerical modeling of the pulsatile motion of the non-Newtonian Cason fluid in a channel that is rectangular and asymmetrically localized constriction. N.F.M. Omar et al. [19] carried investigations on the implications of radiation and porous convection from different Casson fluids. Mohana Ramana R. et al. [20] set a strategy for identifying the consequence of the temperature of the source/sink and chemical responses upon the MHD stagnant state movement of Casson fluid. Ahmed Muhammed Juma'a [21] stated his findings that an approximate method for solving the boundary value problem generated by the convection heat transfer of a Newtonian incompressible fluid on a flat plate.

N.F.M.Omar et al [22] established the analytical solution for the erratic motion of a Casson fluid in a porous substrate, considering the effects of heat radiation and chemical reaction.

Inspired by the above research, this study will cover the effects of porosity with the heat source in the Casson fluids. Analytical solutions for all governing equations will be solved using the Laplace transform and the Graphical results will be compiled using MATLAB limiting cases have been done to ensure the results are parallel with the previous finding.

2. Mathematical Formulation

We have now effectively dealt with the Casson flow of fluids with chemical processes and thermal propagation with a source of heat. In the present research, the flow at x > 0, where x serves as the reference value orthogonal to the surface, is looked at by considering an unstable Casson fluid that passes by an accelerating plate. The fluid along with the plate is motionless when time t = 0, were the fluid's temperature and concentration stays constant. After that, at t > 0, the plate races about a velocity of $u' = At$ at t > 0. As shown in Figure 1, the concentration C' and temperature T' of the plate is increased simultaneously to C'_W and T'_W respectively.

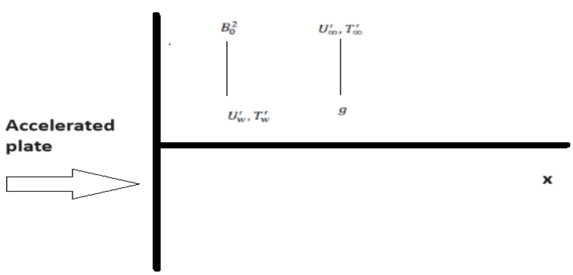

Figure 1

Flow geometry

The basic three governing equations are dimensional momentum, energy, and concentration

$$\rho\frac{\partial u'}{\partial t'}=\mu\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u'}{\partial x'^2}+\rho g\beta(T'-T'_\infty)-\sigma B_0^2 u'-\frac{v}{K}u'+\rho g\beta(C'-C'_\infty) \quad (1)$$

$$\rho C_P\frac{\partial T'}{\partial t'}=k\frac{\partial^2 T'}{\partial x'^2}-\frac{\partial q'_r}{\partial x'}+Q_s(T'-T_\infty) \quad (2)$$

$$\rho C_P\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial x'^2}-Kr'(C'-C'_\infty) \quad (3)$$

Here, B is the external magnetic field, u ′ represents fluid in the x-direction, Specific heat at constant pressure is symbolized by C_P; the Casson parameter is written by γ; Coefficient of thermal expansion is stated by β; the temperature of the fluid as it nears the surface of the plate is denoted by T′; the temperature along the plate is T'_∞; the density is portrayed by ρ; the fluid density is referred to by k; the porosity is showed by K; the radiative heat flux is implied by q_r; Q_S is the heat source with initial and boundary conditions; Ddenotes chemical reaction; t represents the time variable:

$$\begin{gathered} u'(x',0)=0;u'(0,t')=At;u'(\infty,t')=0;\\ T'(x',0)=T'_\infty;T'(0,t')=T'_w;T'(\infty,t')=T'_\infty;\\ C'(x',0)=C'_\infty;C'(0,t')=C'_w;C'(\infty,t')=C'_\infty; \end{gathered} \quad (4)$$

and by introducing dimensionless variable

$$u=\frac{u'}{(vA)^{\frac{1}{3}}};t=\frac{t'A^{\frac{2}{3}}}{v^{\frac{1}{3}}};x=\frac{x'A^{\frac{1}{3}}}{v^{\frac{2}{3}}};T=\frac{T'-T'_\infty}{T'_W-T'_\infty};C=\frac{C'-C'_\infty}{C'_W-C'_\infty} \quad (5)$$

with parameters

$$N=\frac{16\sigma T_\infty}{3k},M=\frac{\sigma B_0^2 v^{\frac{1}{3}}}{\rho A^{\frac{2}{3}}},\ \Pr=\frac{\mu C_\rho}{k},Gr=\frac{g\beta(T_w-T_\infty)}{A},\ Gc=\frac{g\beta(C_w-C_\infty)}{A}.$$

Now the dimensionless form of the Eqs. (1), (2), and (3) are

$$\frac{\partial u}{\partial t}=A\frac{\partial^2 u}{\partial x^2}+GrT-Bu+GcC \quad (6)$$

$$\lambda\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+QT \quad (7)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial x^2}-KrC \quad (8)$$

Now the initial and boundary condition:

$$\text{“}u(x,0)=0; u(0,t)=at; u(\infty,t)=0;$$
$$T(x,0)=0; T(0,t)=1; T(\infty,t)=0; \tag{9}$$
$$C(x,0)=0; c(0,t)=1; C(\infty,t)=0;\text{”}$$

The L.T. model is then used to solve for the result.

$$\frac{\partial^2\bar{U}}{\partial x^2}+\frac{(s+B)}{A}\bar{U}=-R_1\bar{T}-R_2\bar{C} \tag{10}$$

$$\frac{\partial^2 T}{\partial x^2}-\lambda s\bar{T}-Q\bar{T}=0 \tag{11}$$

$$\frac{\partial^2\bar{C}}{\partial x^2}-\bar{C}(ScKr+Scs)=0 \tag{12}$$

Where the parameters used in this research are:

$$A=1+\frac{1}{\gamma},\ B=M+\frac{1}{k},\ \lambda=\frac{\Pr}{1+N},\ R_1=\frac{Gr}{A},\ R_2=\frac{Gc}{A}$$

Then by using the inverse Laplace transforms equations (10), (11), and (12) are solved:

$$T(x,t)=\frac{e^{\frac{Q}{\lambda}t}}{2}\left\{e^{-x\sqrt{\frac{Q}{\lambda}}}\ .erfc\left(\frac{x\sqrt{\lambda}}{2\sqrt{t}}-\sqrt{\frac{Q}{\lambda}t}\right)+e^{x\sqrt{\frac{Q}{\lambda}}}\ .erfc\left(\frac{x\sqrt{\lambda}}{2\sqrt{t}}+\sqrt{\frac{Q}{\lambda}t}\right)\right\} \tag{13}$$

$$C(x,t)=\frac{e^{krt}}{2}\left\{e^{-x\sqrt{krSc}}\ .erfc\left(\frac{x\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Krt}\right)+e^{x\sqrt{KrSc}}\ .erfc\left(\frac{x\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Krt}\right)\right\} \tag{14}$$

$$\text{“}U(x,t)=u0(x,t)+u1(x,t)+u2(x,t)+u3(x,t)+u4(x,t)\text{"} \tag{13}$$

3. Results and Discussion

With Gr = 1, Gc = 1, Pr = 20, Sc = 3, kr = 1, k = 0.2, M = 5, N = 3, t = 0.33, gamma = 0.2, and Q = 4, the velocity, concentration, and temperature profile values are compiled using MATLAB.

Figure 2(a) highlights that the radiation constraint improves along with the temperature gradient. We know that the overall thickness of the thermal barrier decreases as one increases the radiation variable, and figure 2(b) reveals an impact of time on the temperature sector. Time as well as temperature increase synchronously.

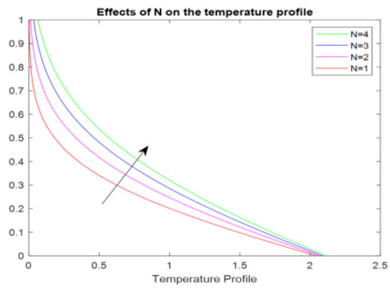

Figure 2(a)

Effects of N on the temperature profile

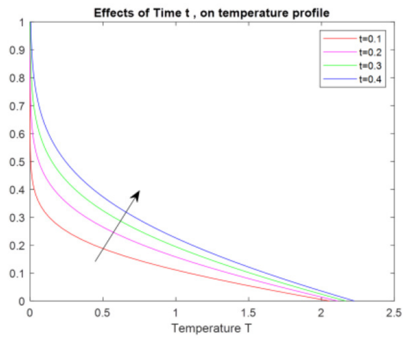

Figure 2(b)

Effect of time on the temperature profile.

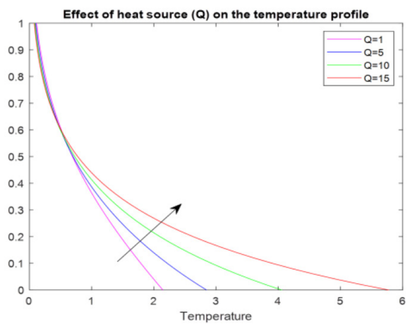

Figure 2(c)

Effect of heat source(Q) on the temperature profile.

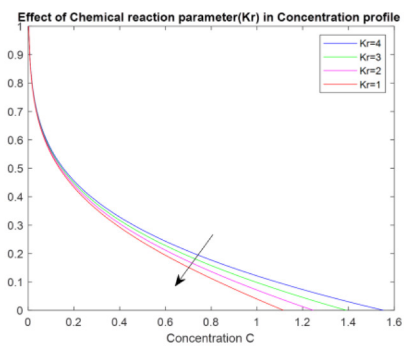

Figure 3(a)

Effects of chemical reaction parameter (Kr) in Concentration profile

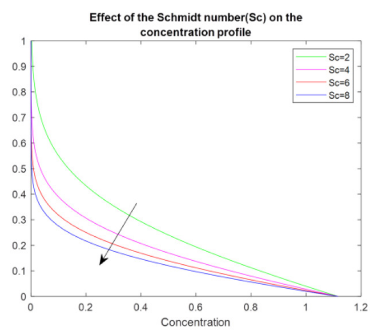

Figure 3(b)

Effect of the Schmidt number (Sc) on the concentration profile

Figure 2(c) demonstrates how the heat input (Q) alters the temperature curve. Both the temperature and source of heat expand instantaneously owing to their immediate correlation. The concentration decreases as the chemical response characteristic goes up, which is apparent in figure 3(a).

Schmidt level climbs with declining concentration, which is apparent in figure 3(b). Figure 4(a) indicates the way the Schmidt number modulates the velocity field. The velocity curve gets steeper in tandem with the rise in the Schmidt factor.

From Figure 4(b) the chemical responses parameter count impacts the velocity field, with velocity ascending as the kr count grows. This demonstrates how the velocity distribution is significantly influenced by the chemical reaction coefficient or kr. As we can see in figure 4(c), there is a drop in velocity as K decreases.

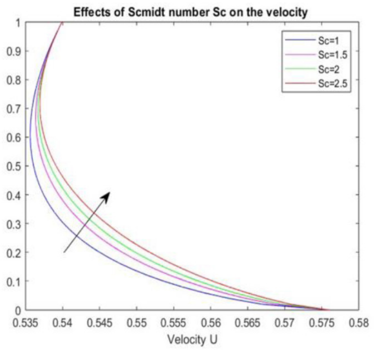

Figure 4 (a)

Effect of Sc in Velocity profile.

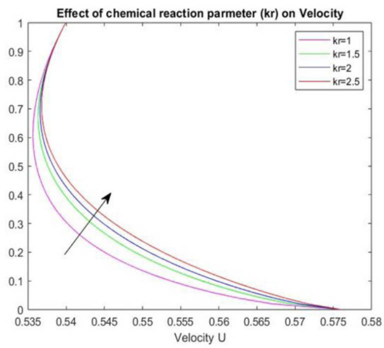

Figure 4 (b)

Effect of Kr in Velocity profile.

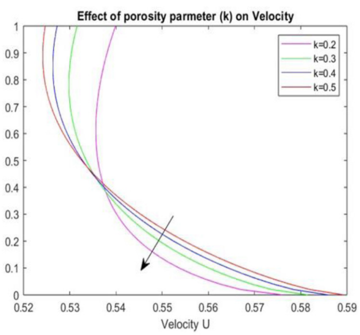

Figure 4 (c)

Effects of porosity parameter,k on velocity profile

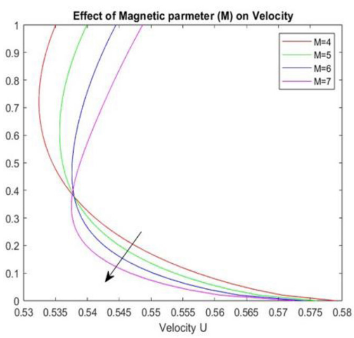

Figure 4 (d)

Effects of Magnetic parameter, M with velocity

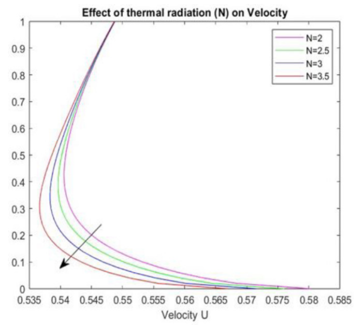

Figure 4 (e)

Effects Of thermal parameter N, with velocity

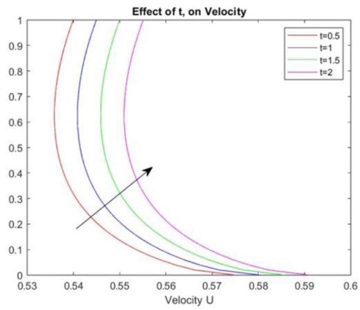

Figure 4 (f)

Effect of t, in velocity profile u

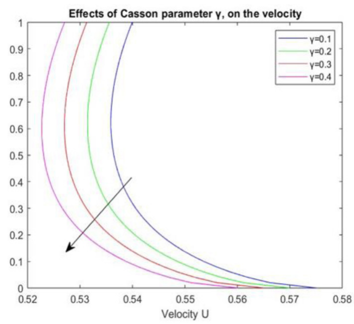

Figure 4 (g)

Effect of the Casson parameter in Velocity profile u

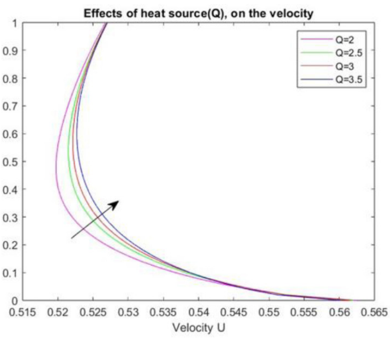

Figure 4 (h)

Effect of the heat source Q in velocity profile U.

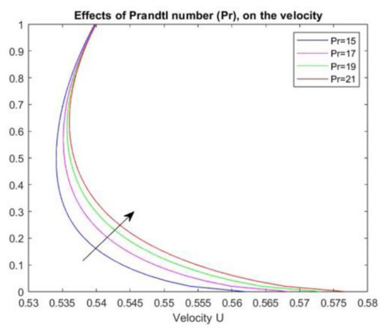

Figure 4 (i)

Effect of Pr in Velocity profile

In Figure 4(d) we see the velocity falls as M grows.; Figure 4(e) illustrates this relationship between the velocity and the increase in heat radiation, N.

The implications of t upon velocity u are displayed in Figure 4(f). Corresponding with t, the velocity u climbs. As seen in Figure 4(g), the Casson indicator declines across the velocity gradient.

Figure 4(h) demonstrates how the upward trajectory of the heat source alters the velocity profile. Heat drives the velocity to move up. Figure 4(i) shows the Pr influences on the velocity distribution. It is demonstrated that the Prandtl number affects the velocity discipline. The flow's velocity exceeds with a rise in the Prandtl number as momentum diffusivity progressively increases thermal diffusivity.

4. Conclusions

Solutions have been obtained through the utilization of the Laplace transform method. Through our analysis, we've discerned that temperature is positively influenced by thermal radiation, time, and the intensity of the heat source. On the other hand, a substantial drop in the fluid's concentration is associated with higher values for the Schmidt factor and chemical reaction value.

Furthermore, our results reveal that augmenting the magnetic parameter, thermal radiation, porous medium characteristics, and Casson parameters results in a diminished velocity field. Conversely, elevating the Schmidt number, chemical reaction rate, time, pressure, heat source intensity, and Prandtl number leads to an opposite effect on the velocity field.

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