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An exploration of the nth order fuzzy differential equations through numerical analysis, employing He's variational iteration method

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Abstract

Using a broader concept of higher-order fuzzy differentiability, the HVI Method (HVIM) was developed to solve nth order FDE(FDE) (Sekar and Sakthivel, [13]; Sekar and Prabhavathi, [14]). A different approach detailed in the works of Sekar and Prabhavathi [14] and Sekar and Sakthivel [13] was contrasted with the discrete solutions that were derived. Reduced computational cost means less time spent using the new method. It is presumed that the fuzzy function and its derivative possess Hukuhara differentiability. The effectiveness of this method was illustrated by the numerical example.

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1. Introduction

Prior work by Abbasbandy et al. has shown convergence outcomes and numerical approaches for solving fuzzy differential equations. S. Abbasbandy and Alavic [11], Abbasbandy and Viranloo [1, 2], S. Abbasbandy and Nieto [12], and Duraisamy Duraisamy and Usha [5] are among the acknowledged sources. However, these approaches have limitations beyond their 1st order accuracy, which can be maintained with a few more assumptions. Unless the solution are different, these approaches will converge to a subset of the numerical solutions. As mentioned by Lakshmikantham and Liu [9], one frequently encountered type of FDE.

The problems with a high degree of uncertainty can be represented using FDE(FDEs). It was written by M. Othadi and Mosleh in [10]. Researching FDEs can be done in various ways. B. Hashemi and M. Dehghan Fuzzy linear systems were addressed by Dehghan and Hashemi [4] through the use of an iterative technique. O. S. Fard [6] suggested a technique that iteratively solves a system of extended linear fuzzy differential equations. Applying their results to fuzzy differential equations, Bede and Gal [3] obtained generalizations about the differentiability of value functions for fuzzy integers.

In order to resolve FDEof nth order, R. G. Sharmila and E. C. H. Amirtharaj [15] used the Runge-Kutta method, more especially the Centroidal Mean (RKCeM) approach. S. Sekar and others, notably Prabhavathi [13] and Sakthivel [14], demonstrated effective methods for solving nth-order FDEby combining the Leapfrog and He's Perturbation approaches. Previous research on the subject has been carried out by scholars such as T. Jayakumar and Indrakumar [16], and Amirtharaj [15], and S. Sekar and his colleagues in Sekar and Prabhavathi [13] and Sekar and Sakthivel [14].

2. HVI Method

This section offers a brief summary of the key elements of the technique known as the HVI approach. It was first introduced by He in 1999 and has since been further developed in 2007. This method is based on a universal Lagrange multiplier method developed by He [7] - He [8]. The differential equation is solved using a method called the variational iteration approach.

The concept involves the evaluation of both linear and nonlinear operators, represented by L and N, respectively. Furthermore, there is an additional term denoted as g(t) that is not uniform. The corrective function is implemented using the specified technique.

$$u_{n+1}(t) = u_n(t) + \int \lambda [L[u_n(s)] + N[\tilde{u}_n(s)] - g(s)] ds$$

Using this approach, the Lagrange multiplier λ is first determined via variational theory, i.e., it should cause the correction function to be stationary, i.e., $\delta u_{n+1}\ (u_n(t), t) = 0.$ Next, using any selective initial function and the determined $u_n, n \geq | 0$ of the solution u_0 derived. Consequently $u = \lim_{n \to \infty} u_n$. Through the utilization of the correction functional, a multitude of approximations are derived, ultimately culminating in the emergence of the precise solution as the limit of these successive approximations.

3. Numerical Examples for the nth – order FDE

We have selected a problem presented to showcase the effectiveness of the HVI Method. T. Jayakumar and Indrakumar [16], R. Gethsi The study was conducted by Sharmila and E. C. Henry Amirtharaj. Several studies conducted by different researchers utilized a step size of r = 0.1. These studies were conducted by Sharmila and Amirtharaj, S. Sekar and their colleagues, and Sekar and Prabhavathi. In their research, they provided the precise solutions as well. In a similar study, Sekar and Sakthivel also conducted research on this topic. The solutions were obtained using two different approaches: He's Homotopy Perturbation Method and HVI Method. Figures 1-4 showcase the absolute errors between them.

3.1 *Example 1*

Based on previous discussions in the works of Prabhavathi, and Sekar and Sakthivel, this section takes into consideration the following FDE

$$z''_{(t)\,4} z'(t) + 4z(t) = 0\,(t \geq 0)$$

$$z(0) = (2 + \beta, 4 - \beta)$$

$$z'(0) = (5 + \beta, 7 - \beta)$$

solution is

$$\underline{z}\ (t,m) = (2+m)e^{2t} + (1-m)t\,e^{2t}$$

$$\overline{z}(t,m) = (4-m)e^{2t} + (m-1)t\,e^{2t}$$

Example 2

Within this part, we examine a fuzzy differential equation that includes a fuzzy initial value. The authors of this equation are, T. Jayakumar and Indrakumar.

$$z''_{(t)=2} z''(t) + 3z'(t)\ \ (0 \le t \le 1)$$
$$z(0) = (3+\beta, 5-\beta)$$
$$z'(0) = (-3+\beta, -1-\beta)$$
$$z''(0) = (8+\beta, 10-\beta)$$

Figures 1-4 show graphical representations of chosen r = 0:1 values to show how effective HVI Method is in bringing attention to the effect of errors on the exact answers. These depictions include effects in three dimensions.

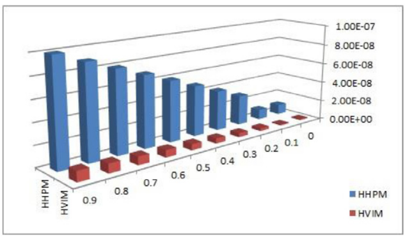

Figure 1

Error estimation of $\underline{z}(t,m)$

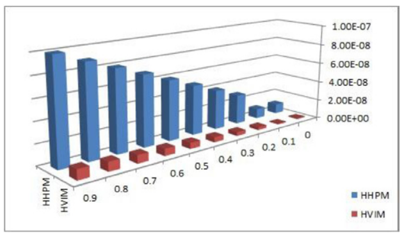

Figure 2

Error estimation of $\overline{z}(t,m)$

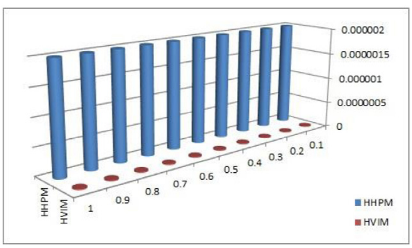

Figure 3

Error estimation of $Z_1(t, m)$

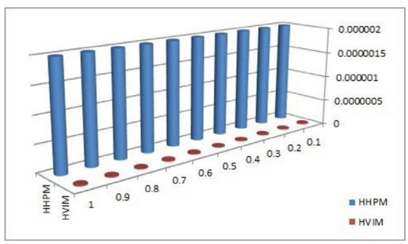

Figure 4

Error estimation of $Z_1(t, m)$

4. Conclusions

Results from solving n-th-order FDEwith the Seikkala derivative and IVP show that HVI Method works. Figures 1-4 suggest that, in comparison to He's Homotopy Perturbation Method, HVI Method yields a substantially smaller inaccuracy. When dealing with nth-order FDEthat use the Seikkala derivative and an IVP, the method that is most recommended is HVI.

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