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Supra *v*-open set and some of its properties

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Abstract

The idea of ***supra v-open*** set is introduced, and several different properties of ***supra v-open*** set are proven. Furthermore, the notion of ***supra v-space*** has been presented and which is satisfying the conditions of supra topology. In addition, the relations between ***v-open*** set and ***supra v-open*** set have been shown. Finally, the concept ***supra v-base*** and some of its properties were studied.

Subject Classification: *00A05, 00A06.*

Keywords: *Algebra, Topology, δ-field, α-σ-field.*

1. Introduction

The notion of $\sigma\text{-}\sigma$ −algebra is one of the important topics in real analysis and probability. Ibrahim S. Ahmed [1], [2], [3], [4], and [5] introduced some kinds of classes of sets, which are generalizations to σ−algebra such as δ −*field* , $\alpha - field\ and\ \alpha - \sigma -$ field. Topology is offshoot of the mathematics which deal with the study of those properties of sure stuff that remain invariant under certain type of transformations as bending or stretching. In simple terms, topology is the discussion of continuity, compactly and connectivity [6]. Njastad in 1965 [7] was first introduced $\alpha -$ open set concept as a generalization of open set, where $D \subseteq \mathcal{X}$ is a $\alpha -$ open iff $D \subseteq (int(cl(int(D))))$. Many authors are interested studies $\alpha -$ open , see [8] and [9]. The concept of λ−algebra is introduced by Ebrahim [10].

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The notion of $\beta - the$ open set was studied in [11], where $D \subseteq \mathcal{X}$ is a β $-$ open iff $D \subseteq (cl\left(int\left(cl(D)\right)\right))$. A subset $D \subseteq \mathcal{X}$ is a θ - open if $\forall d \epsilon D$ there is open E such that $d \epsilon E \subseteq cl(E) \subseteq D$ [12-14].

2. The main results

Definition 2.1: Let $\{\mathfrak{F}_\kappa\}_{\kappa \in J}$, $\kappa \geq 2$, be a collection of supra topologies on $\mathcal{X}$ and $\mathcal{V} \subseteq \mathcal{X}$. Then $\mathcal{V}$ is called $supra\ \mathcal{v} - open$ set in $\mathcal{X}$ if there is $T \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ such that $\phi \neq T \subseteq \mathcal{V}$ where $\mathcal{V} \neq \phi$ and $T = \phi$ where $\mathcal{V} = \phi$.

Definition 2.2: The set of all $supra\ \mathcal{v} - open$ in $\mathcal{X}$ is denoted by $\mathcal{S}\mathcal{v}O_\mathcal{X}$ and defined as:

$$\mathcal{S}\mathcal{v}O_\mathcal{X} = \{\mathcal{V}: \mathcal{V} \text{ is } supra\ \mathcal{v} - open\ in\ \mathcal{X}\}.$$

Definition 2.3: The complement of a $supra\ \mathcal{v} - open$ set is said $supra$ $\mathcal{v} - closed$ set $\mathcal{S}\mathcal{v}C_\mathcal{X}$ is denote to set of every $supra\ \mathcal{v} - closed$.

Definition 2.4: Assume $\mathcal{X}$ be a universal set, $\{\mathfrak{F}_\kappa\}_{\kappa \in J}$, $\kappa \geq 2$, be a collection of supra topologies on $\mathcal{X}$ and let $\mathcal{S}\mathcal{v}O_\mathcal{X}$ be set of all $supra\ \mathcal{v}$ $-$open in $\mathcal{X}$. Then an ordered pair $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is called $supra\ \mathcal{v}$ $-$space concerning the given supra topologies.

Proposition 2.5: Let $\mathcal{X}$ be a universal set and $\{\mathfrak{F}_\kappa\}_{\kappa \in J}$, $\kappa \geq 2$, be a collection of supra topologies on $\mathcal{X}$. Then $\mathcal{S}\mathcal{v}O_\mathcal{X}$ satisfies the conditions of supra topology on $\mathcal{X}$.

Proof: Since $\{\mathfrak{F}_\kappa\}_{\kappa \in J}$, $\kappa \geq 2$, be a collection of supra topologies on $\mathcal{X}$, Then $\bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ is a supra topology on $\mathcal{X}$, hence $\phi, \mathcal{X} \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$. Consequentially, $\phi, \mathcal{X} \in \mathcal{S}\mathcal{v}O_\mathcal{X}$.

Let $\{\mathcal{V}_{\beta_i}\}_{\beta_i \in I} \in \mathcal{S}\mathcal{v}O_\mathcal{X}$. Then $\mathcal{V}_{\beta_i}$ is a $upra\ \mathcal{v} - open$ $\forall \beta_i \in I$. Hence, there is $\{T_{\beta_i}\}_{\beta_i \in I} \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ such that $T_{\beta_i} \subseteq \mathcal{V}_{\beta_i}$. Thus $\bigcup_{\beta_i \in I} T_{\beta_i} \subseteq \bigcup_{\beta_i \in I} \mathcal{V}_{\beta_i}$, but $\{T_{\beta_i}\}_{\beta_i \in I} \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ and $\bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ is a supra topology, which implies that $\bigcup_{\beta_i \in I} T_{\beta_i} \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$. So, we have $\bigcup_{\beta_i \in I} \mathcal{V}_{\beta_i}$is a $supra\ \mathcal{v} - open$, that is $\bigcup_{\beta_i \in I} \mathcal{V}_{\beta_i} \in \mathcal{S}\mathcal{v}O_\mathcal{X}$.

Therefore, $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is a supra $-$ topological space.

Definition 2.6: Suppose $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ is a $supra\ \mathcal{v} - space$ & $\mathfrak{b} \in \mathcal{X}$, then $\mathcal{V} \subseteq \mathcal{X}$ is said $supra\ \mathcal{v} - neighbourhood$ of $\mathfrak{b}$ if there is $\mathcal{D} \in \mathcal{S}\mathcal{v}O_\mathcal{X}$ containing $\mathfrak{b}$ such that $\mathcal{D} \subseteq \mathcal{V}$.

Theorem 2.7: *Let* $(\mathcal{X}, \mathcal{S}\mathcal{v}O_\mathcal{X})$ *be a* $supra\ \mathcal{v} - space$ *and* $\mathcal{V} \subseteq \mathcal{X}$. *Then* $\mathcal{V} \in \mathcal{S}\mathcal{v}O_\mathcal{X}$ *iff* $\mathcal{V}$ *includes* $supra\ \mathcal{v} - neighbourhood$ *of each of its points.*

Proposition 2.8: suppose $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$ are a family of supra topologies on $\mathcal{X}$. If $T \in \mathfrak{F}_\kappa$ for all $\kappa \in J$, then $T \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$.

Definition 2.8: Suppose $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ $supra\ \mathcal{v} - space$ & $\phi \neq \mathcal{Y} \subseteq \mathcal{X}$, the restriction of $\mathcal{S}\mathcal{v}O_{\mathcal{X}}$ relative to $\mathcal{Y}$ is denoted by $(\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}}$ and defined as: $(\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}} = \{R: R = \mathcal{V}\cap\mathcal{Y}, for\ some\ \mathcal{V} \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}\}$.

Proposition 2.9: Let $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ be a $supra\ \mathcal{v} - space$ and $\phi \neq \mathcal{Y} \subseteq \mathcal{X}$. Then $(\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}}$ is satisfies the conditions of supra topology on a set $\mathcal{Y}$.

Proof: Since $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is satisfies the condition of supra topology on $\mathcal{X}$, then $\phi, \mathcal{X} \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$. Now, $\phi = \phi\cap\mathcal{Y}$ and $\mathcal{Y} = \mathcal{X}\cap\mathcal{Y}$, then $\phi, \mathcal{Y} \in (\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}}$.

Let $\{R_{\beta_i}\}_{\beta_i\in I} \in (\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}}$, then $R_{\beta_i}=V_{\beta_i}\cap\mathcal{Y}$ where $\{V_{\beta_i}\}_{\beta_i\in I} \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$. Hence $\bigcup_{\beta_i\in I} R_{\beta_i}=\bigcup_{\beta_i\in I}(V_{\beta_i} \cap \mathcal{Y}) = (\bigcup_{\beta_i\in I} V_{\beta_i}) \cap \mathcal{Y}$. But $\bigcup_{\beta_i\in I} V_{\beta_i} \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$, so we have $\bigcup_{\beta_i\in I} R_{\beta_i} \in (\mathcal{S}\mathcal{v}O_{\mathcal{X}})|_{\mathcal{Y}}$. This completes the proof.

Proposition 2.10: Let $\mathcal{X}$ be a universal set, and $\mathfrak{F}$ be a nonempty collection of subsets of $\mathcal{X}$. If $(\mathcal{X}, \mathfrak{F})$ is a measurable space over $\delta -$field, then $(\mathcal{X}, \mathfrak{F})$ is a supra $-$ topological space.

Proof: Since $(\mathcal{X}, \mathfrak{F})$ is a measurable space over $\delta -$field, then $\phi \in \mathfrak{F}$.

Since $\mathfrak{F}$ is a nonempty collection of subsets of $\mathcal{X}$, then there is a nonempty set $\mathcal{V}$ in $\mathfrak{F}$, but $\mathcal{V} \subseteq \mathcal{X}$, by the definition of $\delta -$field, we have $\mathcal{X} \in \mathfrak{F}$. Let $\{\mathcal{V}_{\beta_i}\}_{\beta_i\in I} \in \mathfrak{F}$. Then $\mathcal{V}_{\beta_i} \subseteq \bigcup_{\beta_i\in I} \mathcal{V}_{\beta_i} \subseteq \mathcal{X}$ for all $\beta_i \in I$. Hence, $\bigcup_{\beta_i\in I} \mathcal{V}_{\beta_i} \in \mathfrak{F}$. Therefore, $(\mathcal{X}, \mathfrak{F})$ is a supra $-$ topological space.

The opposite of a previous proposition need not be true.

Example 2.11: Let $\mathcal{X} = \{s_1, s_2, s_3, s_4\}$ and $\mathfrak{F} = \begin{Bmatrix} \{s_3, s_4\}, \{s_2\}, \{s_1, s_3\}, \{s_2, s_3, s_4\}, \\ \{s_1, s_2, s_3\}, \{s_1, s_3, s_4\}, \phi, \mathcal{X} \end{Bmatrix}$. Then $(\mathcal{X}, \mathfrak{F})$ is a supra topological space. In the other hand, $(\mathcal{X}, \mathfrak{F})$ is not a measurable space over $\delta -$field, since $\{s_2\} \in \mathfrak{F}$, but $\{s_2\} \subset \{s_1, s_2\} \notin \mathfrak{F}$.

Proposition 2.12: Let $\mathcal{X}$ be a universal set and $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, be a collection of $\delta -$field on $\mathcal{X}$. If $\mathcal{V}$ is a measurable set in $\mathfrak{F}_\kappa$ $\forall\kappa \in J$. Then $\mathcal{V}$ is a supra $\mathcal{v} -$ open set.

Proof: Assume that $\mathcal{V} \in \mathfrak{F}_\kappa$ for all $\kappa \in J$. If $\mathcal{V} = \phi$ or $\mathcal{V} = \mathcal{X}$. We are done. So, we assume $\phi \neq \mathcal{V} \neq \mathcal{X}$ and we get $\mathcal{V} \in \bigcap_{\kappa\in J} \mathfrak{F}_\kappa$.

Since $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, is a collection of $\delta -$field on $\mathcal{X}$, then $\bigcap_{\kappa\in J} \mathfrak{F}_\kappa$ is $\delta -$field on $\mathcal{X}$, that is $(\mathcal{X}, \bigcap_{\kappa\in J} \mathfrak{F}_\kappa)$ is a measurable space over $\delta -$field, hence by Proposition (2.10), we have $\bigcap_{\kappa\in J} \mathfrak{F}_\kappa$ is a supra topology on $\mathcal{X}$.

Now, $\phi \neq \mathcal{V} \subseteq \mathcal{V}$, so $\mathcal{V}$ is supra $\mathcal{v} -$ open set.

Remark 2.13: The following example indicates that the supra $\mathcal{v}-$ space $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is not a measurable space over $\delta-$field

Example 2.14: Let $\mathcal{X}=\{x_1, x_2, x_3, x_4\}$. Define $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$, $\mathfrak{F}_4$ as follows:

$$\mathfrak{F}_1=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_1,x_2,x_3\},\mathcal{X}\},$$

$$\mathfrak{F}_2=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_2\},\{x_1,x_2,x_3\},\mathcal{X}\},$$

$$\mathfrak{F}_3=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_2,x_3\},\{x_1,x_2,x_3\},\mathcal{X}\} \text{ and}$$

$$\mathfrak{F}_4=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_3\},\{x_1,x_2,x_3\},\mathcal{X}\}.$$

Then $(\mathcal{X},\mathfrak{F}_1)$, $(\mathcal{X},\mathfrak{F}_2)$, $(\mathcal{X},\mathfrak{F}_3)$ and $(\mathcal{X},\mathfrak{F}_4)$ are supra topologies spaces. Now, $\bigcap_{\kappa=1}^{4}\mathfrak{F}_\kappa=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_1,x_2,x_3\},\mathcal{X}\}$.

So, we have

$\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}=\{\phi,\{x_1,x_2\},\{x_1,x_3\},\{x_1,x_2,x_3\},\{x_1,x_2,x_4\},\{x_1,x_3,x_4\},\mathcal{X}\}$

Therefore, $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a $supra\ \mathcal{v}-space$.

We note that $\{x_1,x_2\},\{x_1,x_3\}\in\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$.

But $\{x_1,x_2\}\cap\{x_1,x_3\}=\{x_1\}\notin\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$. So, we conclude that $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is not a measurable space over the $\delta-$field.

Proposition 2.15: If $\mathcal{X}$ is a finite set and $\mathfrak{F}$ is a collection of subsets of $\mathcal{X}$ such that $\{\phi\}\neq\mathfrak{F}\neq\{\mathcal{X}\}$. If $\mathfrak{F}$ is a $\lambda-$algebra on $\mathcal{X}$, then $\mathfrak{F}$ is supra$-$topology on $\mathcal{X}$.

Proposition 2.16: Assume $\mathcal{X}$ is a finite set & $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa\geq 2$, be a collection of $\lambda-$algebra on $\mathcal{X}$ such that $\{\phi\}\neq\mathfrak{F}_\kappa\neq\{\mathcal{X}\}$ $\forall\kappa\in J$. If $\mathcal{V}\in\mathfrak{F}_\kappa$ $\forall\kappa\in J$. Then $\mathcal{V}\in\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$.

Proposition 2.17: The supra $\mathcal{v}-$ space $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a measurable space over $\alpha-\sigma-$ field.

Proof: Assume that $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a supra $\mathcal{v}-$ space, then by Proposition (2.5), we have $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a supra$-$topological space. Hence $\phi,\mathcal{X}\in\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$. Now, if $\mathcal{V}_1,\mathcal{V}_2,\ldots\in\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$, then $\bigcup_{n=1}^{\infty}\mathcal{V}_n\in\mathfrak{F}$. Thus, $\mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}}$ is $\alpha-\sigma-$field on $\mathcal{X}$. Therefore, $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a measurable space over $\alpha-\sigma-$ field.

Proposition 2.18: The supra $\mathcal{v}-$ space $(\mathcal{X}, \mathcal{S}\mathcal{v}\mathcal{O}_{\mathcal{X}})$ is a measurable space over the $\alpha-$ field.

The converse of the Proposition (2.18) need not be true.

Example 2.19: Let $\mathcal{X}=\mathbb{R}$ and $\mathfrak{F}$ consist of all finite disjoint unions of right – semi-closed intervals. Then $(\mathcal{X},\mathfrak{F})$ is a measurable space over $\alpha-$ field. But $(\mathcal{X},\mathfrak{F})$ is not supra $\mathcal{v}-$ space.

Since if $(\mathcal{X},\mathfrak{F})$ is supra $\mathcal{v}-$ space, then $(\mathcal{X},\mathfrak{F})$ is a supra topological space, this is a contradiction.

Because if we take $\mathcal{V}_n$ = (0, 1- (1/n)], n=1,2,…, then $\mathcal{V}_n \in \mathfrak{F}$ $\forall$n, but $\bigcup_{n=1}^{\infty} \mathcal{V}_n$ = (0,1) $\notin$ $\mathfrak{F}$.

Conclusion.

The main results obtained are:

1. Let $\mathcal{X}$ is a universal set, and $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, be a collection of supra topologies on $\mathcal{X}$. Then $\mathcal{S}\mathcal{v}O_{\mathcal{X}}$ satisfies the conditions of supra topology on $\mathcal{X}$.
2. Let $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ be a $supra\ \mathcal{v} - space$ and $\mathcal{V} \subseteq \mathcal{X}$. Then $\mathcal{V} \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$ iff $\mathcal{V}$ includes $supra\ \mathcal{v} - neighbourhood$ of each of its points.
3. Let $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$ be a family of supra topologies on $\mathcal{X}$. If $T \in \mathfrak{F}_\kappa$ for all $\kappa \in$ J, then $T \in \mathcal{S}\mathcal{v}O_{\mathcal{X}}$.
4. Let $\mathcal{X}$ be a universal set and $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, be a collection of δ −field on $\mathcal{X}$. If $\mathcal{V}$ is a measurable set in $\mathfrak{F}_\kappa$ $\forall\kappa \in$ J. Then $\mathcal{V}$ is a supra $\mathcal{v}$ − open set.
5. The supra $\mathcal{v}$ − space $(\mathcal{X}, \mathcal{S}\mathcal{v}O_{\mathcal{X}})$ is a measurable space over $\alpha - \sigma -$ field.

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