Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 4, pp. 913–921
DOI : 10.47974/JIM-1911

Stability of Caputa–Katugampola fractional differential nonlinear control system with delay Riemann–Katugampola

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Abstract

This paper presents the existence, uniqueness and stability of nonlinear fractional order systems of the Caputa–Katugampola Fractional Differential with delay Riemann –Katugampola in finite time, which takes advantage of the "generalized Grünewald inequality". Therefore, we propose necessary and sufficient conditions to guarantee the finite time stability of a system of order $0 < \alpha < 1$ and we provide a basic criterion for the stability of systems. We also provide examples and illustrations that show the relevance of the proposed theory. Moreover, our results indicate that our conclusions are more accurate than the existing literature on stability requirements.

Subject Classification: *34A08.*

Keywords: *Caputo–Katugampola, Nonlinear control system, Stability, Riemann–Katugampola.*

1. Introduction

In science and engineering, "fractional" calculus is often overlooked . Recently, several important results have been obtained in fractional differential systems [l,2,3,4]. Stability analysis is a major difficulty in fractional differential systems. While many scientists focus on asymptotic stability, it is equally important in actual engineering applications to ensure that the state trajectories

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of the system remain within a given constraint over a specified period of time. Various systems, such as rockets, satellites, flight control systems, and robotic manipulation systems [5], show the need for short-range operation and adherence to trajectory constraints. Lazarevic [6] was the first researcher to study finite-time stability in fractional-delay differential systems. Following [8], many papers [7-11] applied a similar technique to examine fractional delay differential systems. These publications, on the other hand, include the development of their fundamental hypotheses, $f(t) = \sup_{-\tau\le\theta\le 0}\|x(t+\theta)\|$ is not necessarily monotonically growing with respect to t, resulting in $F_1(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f(s)ds$,in this instance, the Gronwall inequality cannot be used, necessitating the creation of additional methods that do not rely on the function. $f(t) = \sup_{-\tau\le\theta\le 0}\|x(t+\theta)\|$.

In this paper to develop a sufficient condition for the finite-time stability of nonlinear fractional-order delay systems.

$$\begin{cases} {}^{CK}_{a}D_t^{\alpha,\mu}x(t) = A_0x(t) + \sum_{I=1}^{n} A_i x(t-\tau) + f\left(t, x(t), {}^{RK}_{a}D_t^{\alpha,\mu}x(t-\tau)\right) + Bu(t), \ \ t \in [0.T] \\ x(t) = \int_0^t f(s)ds, \qquad t \in [-\tau, 0] \end{cases} \tag{1}$$

Where ${}^{CK}_{a}D_t^{\alpha,\mu}$ denotes the Caputo −Katugampola Fractional Derivatives of order $\alpha, \mu > 0$ ${}^{RK}_{a}D_t^{\alpha,\mu}$ denotes the Riemann– Katugampola derivative of order $\alpha > 0$ $,\mu > 0$ and $x(.) \in R^n$ for t∈ [−∞,T] , where $0 < a < b < \infty$, ${}^{RK}_{a}D_t^{\alpha,\mu}$ denotes the Riemann −Katugampola Fractional derivative of order $0 < \alpha < 1, \mu$ be a positive value . $f(.,.,.): [0.T] \times R^n \times R^n \to R^n$, $B_{n\times m}$ is a control matrix and $u(t): [0,T] \to R^m$ is a control function. Finely the function $x(t)$ is a nonlocal function defined on $[-\tau, 0]$, $\tau \in (-\infty, 0)$.

Preliminaries

This section talks through certain definitions and lemmas relating to fractional calculus.

Definition 1 [12]: Let $\alpha > 0, \mu > 0,\ x \in L^1([a,b]), R),\ 0 < a < b < \infty$. The left and right Riemann−Katugampola fractional derivatives (R-K FDs) of order α is defined by ${}^{RK}_{a}D_t^{\alpha,\mu}x(t) = \frac{\mu^\alpha}{\Gamma(1-\alpha)}(t^{1-\mu}\frac{d}{dt})\int_a^t \frac{\tau^{\mu-1}}{(t^\mu-\tau^\mu)^\alpha}\ x(\tau)\ d\tau$

$${}^{RK}_{b}D_t^{\alpha,\mu}x(t) = \frac{-\mu^\alpha}{\Gamma(1-\alpha)}(t^{1-\mu}\frac{d}{dt})\int_t^b \frac{\tau^{\mu-1}}{(\tau^\mu-t^\mu)^\alpha}\ x(\tau)\ d\tau.$$

Definition 2 [13,14]: Let $\alpha > 0, \mu > 0,\ x \in L^1([a,b]), R),\ 0 < a < b < \infty$. The left and right Riemann−Katugampola fractional integrals (R-KFIs) of order α is defined by: ${}^{RK}_{a}D_t^{-\alpha,\mu}x(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \left(\frac{t^\mu-\tau^\mu}{\mu}\right)^{\alpha-1} x(\tau)\ \frac{d\tau}{\tau^{1-\mu}}$

$$ {}^{RK}_{\ b}D_t^{-\alpha,\mu}x(t) = \frac{1}{\Gamma(\alpha)}\int_t^b \left(\frac{\tau^\mu - t^\mu}{\mu}\right)^{\alpha-1} x(\tau)\ \frac{d\tau}{\tau^{1-\mu}}. $$

Definition 3 [15, 16]: Let $\alpha \in (0,1), \mu > 0, [a,b] \in R, 0 < a < b < \infty$, The left and right Caputo −Katugampola fractional derivatives (C–KFDs) are defined respectively by: ${}^{CK}_{\ a}D_t^{\alpha,\mu}x(t) = \frac{\mu^\alpha}{\Gamma(1-\alpha)}(t^{1-\mu}\frac{d}{dt})\int_a^t \frac{\tau^{\mu-1}}{(t^\mu-\tau^\mu)^\alpha}\ (x(\tau) - x(b))\ d\tau$,

$$ {}^{CK}_{\ b}D_t^{\alpha,\mu}x(t) = \frac{-\mu^\alpha}{\Gamma(1-\alpha)}(t^{1-\mu}\frac{d}{dt})\int_t^b \frac{\tau^{\mu-1}}{(\tau^\mu-t^\mu)^\alpha}\ (x(\tau) - x(b))\ d\tau. $$

Lemma 1 [6,17]: *Let $\alpha > 0, g_1(t)$ and $g_2(t)$ are nonnegative, nondecreasing function locally integrable on [0, T) (some $T \le +\infty$), $g(t) \le M$, and suppose that $x(t)$ is nonnegative and locally integrable on [0, T) with*

$$ x(t) \le g_1(t) + g_2(t)\,{}^{R}_{a}D_t^{-\alpha}x(t) $$

on this interval. Then $x(t) \le g_1(t)E_\alpha(g_2(t)t^\alpha), \quad t \in [0,T)$,
where E_α is the Mittag-Leffler function defined by $\boldsymbol{E_\alpha(v)} = \sum_{\boldsymbol{\mathcal{L}=0}}^{\infty}\frac{\boldsymbol{v^{\mathcal{L}}}}{\Gamma(\boldsymbol{\mathcal{L}\alpha}+1)}$

Lemma 2 [6]: *Assume* $0 < g_2 \le g_1, \alpha \in (0,1]. Then\ g_1{}^\alpha - g_2{}^\alpha \le (f - g)^\alpha.$

Problem Formulation for Proposal system

In this section, we look at the existence, uniqueness and stability theorems for -order nonlinear differential equations.

Theorem 1: *Let $\alpha > 0$. Then the equality ${}^{RK}_{\ a}D_t^{\alpha,\mu}({}^{RK}_{\ a}D_t^{-\alpha,\mu}x) = x(t)$ is $x(t)$ while ${}^{RK}_{\ a}D_t^{-\alpha,\mu}({}^{RK}_{\ a}D_t^{\alpha,\mu}x) = x(t)$ is satisfied for $x(t) \in L_1(a,b)$,$0 < \alpha < 1$.*

Lemma 3 [18]: *Let f be a continuous function on a rectangle $R = [a,b]\ x\ [c,d]$,then $\int\int f(x,y)dA$ have the following $\int_a^b(\int_c^d f(x,y)dy)dx = \int_c^d(\int_a^b f(x,y)dx)dy$.*

Theorem 2: *Let $\alpha > 0, \beta > 0, 1 \le p \le \infty, 0 < a < b < \infty$ and let $\mu \in R$ and $c \in R$ be such that $\mu \ge c$. Then for $x \in X_c^p(a,b)$ .Then ${}^{RK}_{\ a}D_t^{-\alpha,\mu}\left({}^{RK}_{\ a}D_t^{-\beta,\mu}x\right) = {}^{RK}_{\ a}D_t^{-\alpha-\beta,\mu}x(t)$*

Lemma 4 [19, 20]: If $f(t) \in C^n[0,+\infty]$ and $n-1 < \alpha < n \in z^+$,

1. ${}^{RK}_{\ a}D_t^{\alpha,\mu}({}^{RK}_{\ a}D_t^{-\alpha,\mu}f(t)) = f(t)$
2. ${}^{CK}_{\ a}D_t^{\alpha,\mu}f(t) = {}^{RK}_{\ a}D_t^{\alpha,\mu}f(t) - \frac{\mu^\alpha f(a)}{\Gamma(1-\alpha)}(t^\mu - a^\mu)^{\boldsymbol{-\alpha}}$

Lemma 5: *The relation between Riemann−Katugampola fractional integrals and Caputo −Katugampola fractional derivatives have the following formulation* ${}^{RK}_{a}D_t^{-\alpha,\mu}({}^{CK}_{a}D_t^{\alpha,\mu} f(t)) = f(t)$

Lemma 6: *Let us consider a nonnegative function a(t) that is locally integrable on 0 ≤ t ≤ T (with some T≤+∞) and g(t) that is a nonnegative, nondecreasing continuous function defined on 0≤t≤T g(t)≤ M (constant), where α is greater than zero. Furthermore, let us assume that u(t) is nonnegative and locally integrable on 0≤t≤T, with a constant.*

$u(t) = a(t) + g(t)\int_0^t \left(\frac{(t^\mu - s^\mu)}{\mu}\right)^{\alpha-1} u(s)\,ds$ Then $u(t) \le a(t)E_\alpha\left(g(t)\left(\frac{t^\mu - a^\mu}{\mu}\right)^\alpha\right)$

Proof: Let $B\varphi(t) = \frac{g(t)}{\Gamma(\alpha)}\int_a^t \left(\frac{(t^\mu - s^\mu)}{\mu}\right)^{\alpha-1} \frac{x(\tau)}{\tau}d\tau \ ,t \ge 0$. Then $u(t) \le a(t) + Bu(t)$ imply $u(t) \le \sum_{k=0}^{n-1} B^k a(t) + B^n u(t)$

Now to prove

$$B^n u(t) \le \int_a^t \frac{(g(t)\Gamma(\alpha))^n}{\Gamma(n\alpha)}\left(\frac{t^\mu - \tau^\mu}{\mu}\right)^{n\alpha-1} a(\tau)\frac{d\tau}{\tau^{1-\mu}} \tag{2}$$

$$B^n u(t) \to 0 \ as \ n \longrightarrow +\infty \quad \forall t \in [0,T)$$

We are aware that this relation (2) is true when n equals 1. Make the assumption that it is correct for some n = k. If n is equal to k plus one, then the induction hypothesis suggests that

$$B^{k+1}u(t) = B(B^k) \le$$
$$\frac{g(t)}{\Gamma(\alpha)}\int_a^t \left(\frac{t^\mu - s^\mu}{\mu}\right)^{\alpha-1}\left[\int_a^s \frac{(g(s)\Gamma(\alpha))^k}{\Gamma(k\alpha)}\left(\frac{s^\mu - \tau^\mu}{\mu}\right)^{k\alpha-1} u(\tau)d\tau\right]\frac{ds}{s^{1-\mu}}.$$

Since *g(t)* is nondecreasing, it follows that

$$B^{k+1}u(t) \le \frac{(g(t))^{k+1}}{\Gamma(\alpha)}\int_a^t \left(\frac{t^\mu - s^\mu}{\mu}\right)^{\alpha-1}\left[\int_a^s \frac{(\Gamma(\alpha))^k}{\Gamma(k\alpha)}\left(\frac{s^\mu - \tau^\mu}{\mu}\right)^{k\alpha-1} u(\tau)d\tau\right]\frac{ds}{s^{1-\mu}}$$

$$B^{k+1}u(t) \le$$
$$\frac{(g(t))^{k+1}}{\Gamma(\alpha)}\int_a^t\left[\int_\tau^t \frac{(\Gamma(\alpha))^n}{\Gamma(n\alpha)}\left(\left(\frac{t^\mu - \tau^\mu}{\mu}\right)(1-z)\right)^{\alpha-1}\left(\frac{z(t^\mu - \tau^\mu)}{\mu}\right)^{k\alpha-1} ds\right]u(\tau)\frac{d\tau}{\tau^{1-\mu}}$$

Since

$$B^n u(t) \le \int_a^t\left[\frac{(g(t)\Gamma(\alpha))^n}{\Gamma(\alpha)\Gamma((k+1)\alpha)}\left(\frac{(t^\mu - s^\mu)}{\mu}\right)^{n\alpha-1}\right]u(s)ds \to 0, n \to \infty \ \forall t \in [0,T)$$

$$u(t) \le a(t)\left(\frac{1}{\Gamma(\alpha)}\sum_{n=0}^{\infty}\frac{\left(g(t)\Gamma(\alpha)\left(\frac{t^\mu - a^\mu}{\mu}\right)^\alpha\right)^n}{\Gamma(n\alpha+1)}\right), u(t) \le a(t)E_\alpha\left(g(t)\left(\frac{t^\mu - a^\mu}{\mu}\right)^\alpha\right).$$

Lemma 7: *Let* $\alpha > 0, \mu > 0$,*and* $x(t) \in L^1([a,b]), R)$, $0 < a < b < \infty$ *then* $(\left\| \left({}^{RK}_{a}D^{\alpha,\mu}_{t}\right)x(t) - \left({}^{RK}_{a}D^{\alpha,\mu}_{t}\right)y(t)) \right\| \leq C_1(t)\|x(\tau) - y(\tau)\|$ *where*

$$C_1(t) = \frac{\mu^\alpha}{\Gamma(1-\alpha)}\left((t^\mu - a^\mu)\right) \tag{3}$$

Lemma 8: *Let* $\alpha > 0, \mu > 0$,*and* $x(t) \in L^1([a,b]), R)$, $0 < a < b < \infty$ $\left\| {}^{RK}_{a}D^{-\alpha,\mu}_{t} x(t) - {}^{RK}_{a}D^{-\alpha,\mu}_{t} y(t) \right\| \leq C_2(t)\|x(t) - y(t)\|$ *where*

$$C_2(t) = \frac{1}{\mu\alpha\Gamma(\alpha)}\left(\frac{t^\mu - a^\mu}{\mu}\right) \tag{4}$$

Theorem 3: *Let* $x: [-\tau, T] \to R^m$ *be continuous differential function* , $\tau > 0$ *then* $x(t)$ *is a solution of the Caputo −Katugampola fractional order nonlinear differential control nonlocal system (1), if and only if*

$$x(t) = \begin{cases} A_0x(t) + \sum_{I=1}^{n} A_i x(t-\tau) + f\left(t, x(t), {}^{RK}_{a}D^{\alpha,\mu}_{t} x(t-\tau)\right) + Bu(t) \\ \\ \int_0^t f(s)ds \qquad\qquad for -\tau \leq t \leq 0 \end{cases} \tag{5}$$

Theorem 4: *Consider the following Caputo −Katugampola fractional order nonlinear differential control nonlocal system and* ${}^{CK}_{a}D^{\alpha,\mu}_{t} x(t) = A_0 x(t) + \sum_{I=1}^{n} A_i x(t-\tau) + f\left(t, x(t), {}^{RK}_{a}D^{\alpha,\mu}_{t} x(t-\tau)\right) + Bu(t)$ *with* $(f(t, 0,0) = [0, \ldots, 0]^T$*) has a unique continuous solution*

3.1 *Stability of the Caputa–Katugampola and Riemann−Katugampola Fractional order nonlinear differential control nonlocal system with maximal interval (0,T]*

Theorem 5: *The solution to the Caputo −Katugampola fractional derivatives Riemann−Katugampola fractional order nonlinear differential control nonlocal system (1) is* $x(t) \in R^n$ *Then the following inequalities holds:*

$$\| x(t) \| \leq \delta(t) E_\alpha\left(\omega(t) \left(\frac{t^\mu - a^\mu}{\mu}\right)^\alpha \right) \tag{6}$$

Proof: Since $x(t)$ has the following formulation,

$$x(t) = \begin{cases} {}^{RK}_{a}D^{-\alpha,\mu}_{t} \left(A_0 x(t) + \sum_{I=1}^{n} A_i x(t-\tau) + f\left(t, x(t), {}^{RK}_{a}D^{\alpha,\mu}_{t} x(t-\tau)\right) + Bu(t) \right) \\ \int_0^t f(s)ds \qquad\qquad t \in [-\tau, 0] \end{cases}.$$

Thus, for 0 ≤ t ≤ T, we have

$x(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \left(\frac{t^\mu-\tau^\mu}{\mu}\right)^{\alpha-1} \left(A_0 x(t) + \sum_{I=1}^n A_i x(t-\tau) + f\left(t, x(t), {}_a^{RK}D_t^{\alpha,\mu} x(t-\tau)\right) + Bu(t)\right) \frac{d\tau}{\tau^{1-\mu}}$

$\|x(t)\| \leq \|B\|\|u(\tau)\| \frac{\mu^{1-\alpha}}{\alpha\Gamma(\alpha)}(t^\mu - a^\mu)^\alpha + \left(\|A_0\| + \|\sum_{I=1}^n A_i\| + L + LC_1(t)\right) \frac{1}{\Gamma(\alpha)}\int_a^t \left(\frac{t^\mu-\tau^\mu}{\mu}\right)^{\alpha-1} \left\|\sup_{\vartheta\in[-\tau,0]} x(t+\vartheta)\right\| \frac{d\tau}{\tau^{1-\mu}}$

Set $\delta(t) = \|B\|\|u(\tau)\| \frac{\mu^{1-\alpha}}{\alpha\Gamma(\alpha)}(t^\mu - a^\mu)^\alpha$, $\omega(t) = (\|A_0\| + \|\sum_{I=1}^n A_i\| + L + LC_1(t))$, implies that

$\|x(t)\| \leq \delta(t) + \left(\omega(t)\left({}_a^{RK}D_t^{-\alpha,\mu}\left\|\sup_{\vartheta\in[-\tau,0]} x(t+\vartheta)\right\|\right)\right).$

$\| x(t) \| \leq \delta(t)E_\alpha\left(\omega(t)\left(\frac{t^\mu-a^\mu}{\mu}\right)^\alpha\right).$

3.2 *Stability of the Caputa–Katugampola and Riemann−Katugampola Fractional order nonlinear differential control nonlocal system with by using step method.*

Theorem 6: *Assume that Caputo −Katugampola fractional order nonlinear differential control nonlocal system (1) satisfy the following conditions lemma (4) Then the solution of (1) is finite-time stable, if the following conditions is satisfied:* $\delta_T(\tau)E_\alpha\sigma_0(T)\left(\frac{T^\mu}{\mu}\right)^\alpha \leq \varepsilon$ *where*

$$\delta_T(\tau) = \delta_1(T) + \left(\sum_{I=1}^n \|A_i\| + LC_1(t)\right)\frac{1}{\Gamma(1+\alpha)}\left(\frac{(j\tau^\mu)}{\mu}\right)^\alpha$$
$$[\left(\sum_{j=1}^n \delta_j(\tau)E_\alpha\sigma_0(j\tau)\left(\frac{(j\tau^\mu)}{\mu}\right)^\alpha\right)$$
$$+\frac{1}{\Gamma(1+\alpha)}\left(\frac{(T^\mu)-((n+1)\tau^\mu)}{\mu}\right)^\alpha \delta_{n+1}(\tau)E_\alpha\sigma_0(n+1)\left(\frac{((n+1)\tau^\mu)}{\mu}\right)^\alpha]$$
$$\delta_1(t) = (\|B\|\|u(\tau)\|)\frac{\left(\frac{t^\mu}{\mu}\right)^\alpha}{\Gamma(1+\alpha)}.$$

Remark 1: The Caputo −Katugampola fractional order nonlinear differential control nonlocal system 1.1 is finite-time stable if satisfy the following condition $\delta_T(\tau)\, E_\alpha\sigma_0(T)\left(\frac{T^\mu-a^\mu}{\mu}\right)^\alpha \leq \varepsilon$,where $\delta_T(\tau) = (\|B\|\|u(T)\|)\frac{1}{\Gamma(1+\alpha)}\left(\frac{T^\mu-a^\mu}{\mu}\right)^\alpha$ If $T \in (0,\tau]$.

Illustrative example

This section presents example that demonstrate the finite time stability of the Caputo −Katugampola fractional order nonlinear differential control nonlocal system 1.1 as shown in Table 1.

Example 1: Consider the following the Caputo −Katugampola fractional order nonlinear differential control nonlocal system 1.1,

$$ {}_{a}^{CK}D_t^{\alpha,\mu}x(t) = \sum_{I=1}^{n} A_i \begin{pmatrix} \sin(t-0.1) \\ cos(t-0.1) \end{pmatrix} + \begin{pmatrix} t\sin(t)\, {}_{a}^{RK}D_t^{\alpha,\mu}(\sin(t)) \\ tcos(t)\, {}_{a}^{RK}D_t^{\alpha,\mu}(cos(t)) \end{pmatrix} + $$

$\begin{pmatrix} \sin(t-0.1)\, {}_{a}^{RK}D_t^{-\alpha,\mu}(0.03) \\ \cos(t-0.1)\, {}_{a}^{RK}D_t^{-\alpha,\mu}(0.04) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t).$ Where $u(t) = [u_1(t), u_2(t)]^T$,is a vector control functions. We have that ,$A_0 = \begin{pmatrix} 1 & -3 \\ -2 & 0 \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 1 \\ -3 & 0 \end{pmatrix}, A_3 = \begin{pmatrix} 7 & 1 \\ -2 & -4 \end{pmatrix}$, and. By using inequality (3.6) in Theorem (3.5) with $\alpha \in (0,1), L = 1, \|B\| = 1, u(t) = 1$

Solution: $\|x(t)\| \leq \|B\|\|u(\tau)\| \frac{\mu^{1-\alpha}}{\alpha\Gamma(\alpha)}(t^{\mu} - a^{\mu})^{\alpha} E_{\alpha}(\|A_0\| + \|\sum_{I=1}^{n} A_i\| + L + LC_1(t)) \left(\frac{t^{\mu}}{\mu} - \frac{a^{\mu}}{\mu}\right)^{\alpha}$

Table 1
The value of ε, for $a = 0.1, \alpha = 0.9$, initial value of $t = 0.2 > a$

T	$\mu = 0.1$	$\mu = 0.2$	$\mu = 0.3$	$\mu = 0.4$	$\mu = \alpha$	$\mu = 0.6$	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$
0.2	6.8483	4.8298	3.4119	2.4141	1.7105	1.2137	0.8623	0.6134	0.4369
0.4	25.4114	19.1405	14.4740	10.9869	8.3708	6.4005	4.9111	3.7810	2.9205
0.6	41.8710	32.8082	25.8579	20.4977	16.3403	13.0976	10.5545	8.5492	6.9596
0.8	56.2449	45.3473	36.8296	30.1335	24.8350	20.6136	17.2277	14.4937	12.2718
1	69.0499	56.9507	47.3603	39.7275	33.6158	28.6870	24.6827	21.4057	18.7047

Conclusion

The Caputa and Riemann −Katugampola Fractional order nonlinear differential control nonlocal system 1.1 was very difficult for studding since the fractional derivative types has expression are difficult constriction.The uniqueness and existence depended on Generalized Gronwall Inequality of The Riemann – Caputa fractional derivative was presented first time and make good role in stability of the system .The stability of finite time for our presented system was depended on maximal interval or on step size of maximal interval to obtain the guarantee estimation of epsilon.

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Received February, 2024