Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 4, pp. 619–623
DOI : 10.47974/JIM-1504

Some properties of intuitionistic fuzzy n-binormal operator on IFH-space

Marwah Abdulridha Shareef *
Hadeel Ali Shubber †
Department of Mathematics
College of Education for Pure Science
University of Thi-Qar
Thi-Qar
Iraq

Abstract

The Intuitionistic Fuzzy n-binormal operator on Hilbert space H (*IFn – BN*) is introduced in this paper. We demonstrate that the addition and multiplication of two *IFn – BN* operators is the *IFn – BN* operator. We research some of its characteristics.

Subject Classification: *03E72, 05C72.*
Keywords: *Fuzzy set, Fuzzy theory, Binormal analysis, Fuzzy operator.*

1. Introduction

The notion of a fuzzy was first introduced by Zadeh [6]; Atanassov [8] has defined the intuitionistic fuzzy and studied its properties applied to Fuzzy metric space. Krishna and Saram [9] studied the separation and non-separation properties of fuzzy normed linear space. In 2010[1], Alzaraiqi introduced the definition of an n-normal operator. Rasoual Eskandari, Farzollah Mirzapour & Ali Morassaei introduced in 2012[7] More on (α, β)–Normal operators in Hilbert space. Panayappan & Sivamani [10] defined n-binormal operator(n-BN). Several research studies have been devoted to this direction, here, we will refer to [2,4,5,12,13].

2. Basic rules

Definition 2.1 [4]: Let μ & ϑ $: \mathcal{V}2 \times (0, +\infty) \to I$ be Fuzzy, $\mu(e, r, \eta) + \vartheta(e, r, \eta) \le 0, \forall e, r \in \mathcal{V}$ & $\eta > 0$ ($IFIP$-Sapce)

This article has been corrected with minor changes. These changes do not impact the academic content of the article.

* *E-mail:* **mrwhalmry661@gmail.com (Corresponding Author)**
† *E-mail:* **hadeelali2007@utq.edu.iq**

$(IFI.1\ )F_{\mu,v}(r,e,t)\ =)\ F\mu,v(e,r,t)$.
$(IFI.2)F_{\mu,v}(r,e,0)=0$ and $F_{\mu,v}(r,e,t)>0$, for every $t>0$.
$(IFI.3)$ $\forall\alpha\epsilon R$,

$$F_{\mu,v}(\alpha x,y,t)=\begin{cases} F_{\mu,v}\left(r,e,\frac{t}{\alpha}\right), & \alpha>0 \\ \mathrm{H(t)} & \alpha=\mathrm{i}0 \\ \mathcal{N}_s\left(F_{\mu,v}\left(r,e,\frac{t}{\alpha}\right)\right), & \alpha\leq 0 \end{cases}$$

$(IFI.4)$ $F_{\mu,v}\ (e,r,t)\neq H(t)\ ,t\in R\ iff\ r\neq 0$,
Where

$$H(\ t)=\{\begin{matrix}1\ , & if\ \mathrm{it}>0 \\ 0\ , & if\ t\ \leq\ 0\end{matrix}$$

$(IFI.5)$ sup $\{\tau(F_{\mu,\mathrm{i}v}(r,z\ ,s),F_{\mu,v}(e,z,r))\}=\ F_{\mu,v}(r+e,z,t)$.
$(IFI.6)$ $\lim_{t\to 0}\ F_{\mu,v}(r,e,t)=1$.
$(IFI.7)F_{\mu,v}(r,e,.)\colon R\to[0,1]$ is continuously on $R\backslash\{0\}$.

Definition 2.2 [6]: Let i$X\neq\{\ 0\}$ and let $I=[0,1]$. fuzzy subset μ from X is a function from X into I $if\,x\epsilon X$ to real number $\mu(x)$ in the interval $I.\mu(x)$ is the grade of membership function to x in μ.

Definition 2.3 [3]: If i$X\neq\{\ 0\}$. An intuitionistic fuzzy set (IFS) $A\colon\Upsilon=\{\ (x,\ddot{\mu}_\Upsilon(x),v_\Upsilon(x)\colon x\ \in X)\}$, mappings $\ddot{\mu}_\Upsilon\colon X\to I$ is said to be the extent of being a member, $v_\Upsilon\colon X\to I$ the extent of not being a member, & $0\leq\ddot{\mu}_\Upsilon(x)+v_\Upsilon(x)\leq 1\ \forall x\in\ X$.

Definition 2.4 [11]: It is claimed that the 5-tuple $(X,\mathcal{V},*)$ is an intuitionistic fuzzy normed space (for short $IFNS$) if X is a linear space over field $F,*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and μ,v are fuzzy sets in $X\times\ (0,\infty)\ (i.e.\mu\ ,v\ :\ X\ \times(0,\infty)\ \to\ [0,1]\ )$ holding the condition: for all $\alpha,\beta\ \in X$ and $t,s>0$,

(IFN.1) $\mu(\alpha,t)\ +\ v(\alpha,t)\leq 1$,
(IFN.2) $\mu(\alpha,t)\ >\ 0$,
(IFN.3) $\mu(\alpha,t)\ =\ 1$ iff $\alpha=\ 0$,
(IFN. 4) $\mu(\propto\alpha,t)\ =\ \mu\ (\alpha.\frac{t}{|\propto|})$ for all each $F\backslash\{0\}$,
(IFN.5) $\mu(\alpha\ +\beta,t\ +\ s)\ \geq\ \mu\ (x,t)\ *\mu(y,s)$,
(IFN.6) $M(\alpha,.)\colon(0,\infty)\ \to[0,1]$ is continuous,
(IFN.7) $lim_{t\to\infty}\,\mu\ (\alpha,t)\ =\ 1$ and $lim_{t\to 0}\,\mu(x,t)\ =\ 0$,
(FN.8) $v(\alpha,t)\ <\ 1$,
(IFN.9) $v(\alpha,t)\ =\ 0$ iff $\alpha\ =\ 0$,
(IFN.10) $v(a\alpha\ t)\ =\ 0$ iff $\alpha\ =\ 0$,
(IFN.11) $v(\alpha+\beta,t+s)\leq v(\alpha,t)\diamond v(\beta,s)$,

(IFN. I2) $v(\alpha, .): (0, \infty) \rightarrow [0, 1]$ is continuous,
(IFN.13) $lim_{t\rightarrow\infty}\ v(\alpha, t) = 0$ and $lim_{t\rightarrow\infty} v(\alpha, t) = 1$,
Furthermore, assume that $(X, \mu, v, *, \diamond)$ holds the conditions:
(IFN.14) $\alpha * \alpha = \alpha$ and $\alpha \diamond \alpha = \alpha, \forall \alpha \in [0,1]$,
(FN.15) $\mu(\alpha, t) > 0$ and $v(\alpha, t) < 1,, \forall t > 0 \Rightarrow \alpha = 0$.

3. Intuitionistic Fuzzy Binormal and n-Binormal Operator

Definition 3.1: $(\mathcal{V}, F_{\mu,v}, \mathcal{T})$ is IFH space with $\langle \delta, \gamma \rangle = sup\ \{ r \in \mathbb{R}: F_{\mu,v} (\delta, \gamma, r) < 1\}, \ \forall \delta, \gamma \in \mathcal{V}$. $\mathfrak{T}$ is $[IFBN]$ operator if $\mathfrak{T}^*\mathfrak{T}\mathfrak{T}\,\mathfrak{T}^* = \mathfrak{T}\,\mathfrak{T}^*\,\mathfrak{T}^*\,\mathfrak{T}$.

Theorem 3.1: *Suppose* $\mathfrak{T} \in IFB(\mathcal{V})$ *is normal, Then* $\mathfrak{T} \in [IFBN]$ *operatar.*

Theorem 3.2: *Let* $\mathfrak{T} \in [IFBN]$ *and* $S \in [IFBN]$. *If* $\mathfrak{T}$ *and* S *are doubly commuting, then* $\mathfrak{T}S \in [IFBN]$.

Theorem 3.3: *Let* $\mathfrak{T} \in [IFBN]$ *and* $S \in [IFBN]$. *If* $\mathfrak{T}$ *and* S *are doubly commuting, then* $(S + \mathfrak{T}) \in [IFBN]$.

Definition 3.2: $(\mathcal{V}, F_{\mu,v}, \mathcal{T})$ is IFH space with $\langle \delta, \gamma \rangle = \sup \{ r \in \mathbb{R}: F_{\mu,v}(\delta, \gamma, r) < 1\}, \ \forall \delta, \gamma \in \mathcal{V}$. $\mathfrak{T}$ is $[IFBN]$ operator if $\mathfrak{T}^*\mathfrak{T}^n\mathfrak{T}^n\,\mathfrak{T}^* = \mathfrak{T}^n\,\mathfrak{T}^*\mathfrak{T}^*\mathfrak{T}^n$.

Theorem 3.4: *Let* $\mathfrak{T}$ *is* $[IF\ n - BN]$ *operator if* $\mathfrak{T}^{-1}$ *exists then* $(\mathfrak{T}^{-1})$ *is* $[IF\ n - BN]$ *operator.*

Proof: Clearly by using properties of $\mathfrak{T}^{-1}$.

Theorem 3.5: *Let* $\mathfrak{T}_1, \ldots\ldots, \mathfrak{T}_m$ *be* $IFn - BN$ *operators in* $B(\mathcal{V})$. *Then* $(\mathfrak{T}_1 \oplus \ldots\ldots \oplus \mathfrak{T}_m)$ *and* $(\mathfrak{T}_1 \otimes \ldots\ldots \otimes \mathfrak{T}_m)$ *are [* $IFn - BN]$ *operators.*

Proof: Easily to prove this by definition of direct sum and direct product.

Theorem 3.6: *Let* w *and* g *be commuting* $[IFn - BN]$ *operators such that* $(w + g)^*$ *commutes with* $\sum_{k=0}^{n-1} \binom{n}{k} w^{n-k} g^k$.*Then* $(w + g)$*is* $[IFn - BN]$.

Proof: Consider $\mathcal{P}_{\mu,v}(((w + g)^* (w + g)^n (w + g)^n (w + g)^* - (w + g)^n (w + g)^* (w + g)^* (w + g)^n)u, t)$.

$$\mathcal{P}_{\mu,v}((((w + g)^* \textstyle\sum_{k=0}^{n}\binom{n}{k} w^{n-k}g^k)(\sum_{k=0}^{n}\binom{n}{k} w^{n-k}g^k (w + g)^*) - (\sum_{k=0}^{n}\binom{n}{k} w^{n-k}g^k (w + g)^*)((w + g)^* \sum_{k=0}^{n}\binom{n}{k} w^{n-k}g^k))u, t)).$$

$$= \mathcal{P}_{\mu,v}((((w^* w^n + g^*w^n + (w + g)^* \textstyle\sum_{k=1}^{n-1} \binom{n}{k} w^{n-k}g^k + w^* g^n + g^*g^n)(w^n w^* + w^n g^* + \sum_{k=1}^{n-1} \binom{n}{k} w^{n-k} g^k (w + g)^* + g^n w^* + g^n g^*) - \sum_{k=0}^{n}\binom{n}{k} w^{n-k} g^k (w + g)^* (w + g)^* \sum_{k=0}^{n}\binom{n}{k} w^{n-k} g^k))u, t).$$

Since w and g are commuting $[IFn-BN]$ operators such that $(w+g)^*$commutes with $\sum_{k=0}^{n-1} \binom{n}{k} w^{n-k} g^k$.

$= \mathcal{P}_{\mu,v}((w^n w^* + w^n g^* + \sum_{k=1}^{n-1} \binom{n}{k} w^{n-k} g^k (w+g)^* + g^n w^* + g^n g^*)(w^* w^n + g^* w^n + (w+g)^* \sum_{k=1}^{n-1} \binom{n}{k} w^{n-k} g^k + w^* g^n + g^* g^n \sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k (w+g)^* (w+g)^* \sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k))u,t)$.

$= \mathcal{P}_{\mu,v}(((\sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k (w+g)^* (w+g)^* \sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k) - (\sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k (w+g)^* (w+g)^* \sum_{k=0}^{n} \binom{n}{k} w^{n-k} g^k))u,t) = 0$.

Hence $(w+g)$is $[IFn-BN]$.

Proposition 3.1: *Let* $\mathfrak{T}$ *is* $[IFn-BN]$ *operator then Any* $S \in IFB(\mathcal{V})$ *that is unitarily equivalent to* $\mathfrak{T}$ *is* $[IFn-BN]$.

Proof: Let $S = U\mathfrak{T}U^*, S^n = U\mathfrak{T}^n U^*$

Then

$$\mathcal{P}_{\mu,v}((U\,\mathfrak{T}^*\,U^*\,U\mathfrak{T}^n U^*\,U\mathfrak{T}^n U^*\,U\mathfrak{T}^* U^* - U\mathfrak{T}^n\,U^*\,U\,\mathfrak{T}^*\,U^*\,U\mathfrak{T}^* U^*\,U\mathfrak{T}^n U^*)u,t) = \mathcal{P}_{\mu,v}\big((U\mathfrak{T}^n\,\mathfrak{T}^*\,\mathfrak{T}^*\,\mathfrak{T}^n\,U^* - U\mathfrak{T}^n\,\mathfrak{T}^*\,\mathfrak{T}^*\,\mathfrak{T}^n\,U^*)u,t\big) = 0$$

since $\mathfrak{T}$ is $[IFn-BN]$.

Hence S is $[IFn-BN]$.

Proposition 3.2: Let $\mathfrak{T}$ is $[IFn-BN]$ operator then the restriction $\mathfrak{T}/M$ of $\mathfrak{T}$ all closed subspaces M of H that decrease $\mathfrak{T}$ is $[IFn-BN]$.

Proof: Since $\mathfrak{T}$ is $[IFn-BN]$

$$\mathcal{P}_{\mu,v}((\mathfrak{T}/M)^*\,(\mathfrak{T}/M)^n\,(\mathfrak{T}/M)^n\,(\mathfrak{T}/M)^* - (\mathfrak{T}/M)^n\,(\mathfrak{T}/M)^*\,(\mathfrak{T}/M)^*\,(\mathfrak{T}/M)^n)u,t)$$

$$= \mathcal{P}_{\mu,v}\left(\left(\left(\mathfrak{T}^*\,\mathfrak{T}^n\mathfrak{T}^n\mathfrak{T}^*/_M\right) - \left(\mathfrak{T}^n\mathfrak{T}^*\,\mathfrak{T}^*\mathfrak{T}^n/_M\right)\right)u,t\right) = 0$$

since $\mathfrak{T}$ is $[IFn-BN]$.

Hence $\mathfrak{T}\,/\,M \in [IFn-BN]$.

4. Conclusion

With the help of our findings, we have established the concept of $[IFn-BN]$ operator on IFH-space.

- Let $\mathfrak{T}_1, \dots, \mathfrak{T}_m$ be $IFn-BN$ operators in B$(\mathcal{V})$. Then $(\mathfrak{T}_1 \oplus \dots \oplus \mathfrak{T}_m)$ and $(\mathfrak{T}_1 \otimes \dots \otimes \mathfrak{T}_m)$ are$[IFn-BN]$ operators.
- Let w and g be commuting $[IFn-BN]$ operators such that $(w+g)^*$ commutes with $\sum_{k=0}^{n-1} \binom{n}{k} w^{n-k} g^k$. Then $(w+g)$is $[IFn-BN]$.

References

[1] Alzuraiqi, S. A., & Patel, A. B. On n-normal operators. *General Mathematics Notes*, 1(2), 61-73 (2010).

[2] Radharamani, A., & Maheswari, S. "Intuitionistic Fuzzy adjoint & Intuitionistic fuzzy self-adjoint operators in Intuitionistic fuzzy Hilbert space". *Inter. J. Research and Analytical Reviews* (IJRAR), 5(4), 248-251. (2018).

[3] Dubois, D. J. "Fuzzy sets and systems: theory and applications" (Vol. 144). Academic press. (1980).

[4] Goudarzi, M., S. M. Vaezpour, and Reza Saadati. "On the intuitionistic fuzzy inner product spaces." Chaos, Solitons & Fractals 41.3 : 1105-1112. (2009).

[5] Salim, T. N. " On Fixed Point theory in Intuitionistic Fuzzy Soft and Metric Space University Of Thi-Qar. (2018).

[6] Zadeh, L.. " Fuzzy sets", *Information And Control,* 338-353.4. (1965).

[7] Eskandari, R., Mirzapour, F., & Morassaei, A. More on (α, β)-normal operators in Hilbert spaces. In Abstract and Applied Analysis (Vol. 2012, pp. 1-11). Hindawi. (2012).

[8] Atanassov, Krassimir T. "Intuitionistic fuzzy sets." Physica, Heidelberg, 1-137. (1999).

[9] Krishna, S. V., & Sarma, K. K. M. Separation of fuzzy normed linear spaces. *Fuzzy sets and systems*, 63(2), 207-217. (1994).

[10] Panayappan , S., & Sivamani ,N. " On n-binormal operators, General Mathematics Notec", 10 (2) 1-8. (2012).

[11] Saadati , R., & Park, J. H. On the intuitionistic fuzzy topological spaces. Chaos, Solitons & Fractals, 27(2), 331-344. (2006).

[12] Shubham Sharma & Ahmed J. Obaid. Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020). DOI: 10.1080/09720502.2020.1727611.

[13] Shelash, Hader Baqer, Ahmad, Hasanain Hamid & Obaid, Ahmed J. On characterizations some of subgroups of the dihedral group D2n, *Journal of Discrete Mathematical Sciences and Cryptography*, 1-6, DOI: 10.47974/JDMSC-1553.

Received October, 2023