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Some certain properties on analytic p-valent functions connected to fractional differential operator

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Abstract

This research paper introduces a new generalization of the normalized fractional differential operator, which expands a new direction in our research area. We utilize this new operator to establish a subclass of p-valent functions. The study focuses on various characteristics of this subclass, including coefficient inequality, growth, and distortion properties.

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1. Introduction

Let $\mathbb{A}(p)$ denote the class of functions of the form:

$$f(s) = s^p + \sum_{w=p+1}^{\infty} a_w s^w, \quad (p \in \aleph = \{1,2,3,\dots,\}), \tag{1}$$

are analytic in $\mathfrak{D} = \{s: s \in \mathbb{C} \text{ such that } |s| < 1\}$. By assuming that $p = 1$, then $\mathbb{A}(1) := \mathbb{A}$, and let $f(s)$ belongs to the class $\mathbb{A}(p)$ then the *convolution* $f * \tau$ of two analytic functions is given by:

$$(f * \tau)(s) = s^p + \sum_{w=p+1}^{\infty} a_w b_w s^w = (\tau * f)(s),$$

where $\tau(s) \in \mathbb{A}(p)$ and

$$\tau(s) = s^p + \sum_{w=p+1}^{\infty} b_w s^w.$$

Studies were conducted on the fractional calculus Tremblay operators not long ago. These studies have various applications, including the univalent and

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analytical functions of the open unit disk. Researchers are still keen on studying these types of series involving Tremblay fractional (integral and differential) operators [1-3]. In [4] the authors defined the following modification fractional differential Tremblay operator in terms of integral [2]:

$$\mathcal{T}_s^{\alpha,\theta} f(s) := \frac{\Gamma(\sigma)}{\Gamma(\alpha)\Gamma(1-\alpha+\theta)} \left(s^{1-\theta} \frac{d}{d\xi}\right) \int_0^{\xi} \frac{t^{\alpha-1} f(t)}{(s-t)^{\alpha-\theta}} dt,$$

$$(0 < \theta \leq 1, 0 < \alpha \leq 1 \text{ such that } 1 > \alpha - \theta \geq 0, f(s) \in \mathbb{A}, s \in \mathfrak{D}).$$

Then the generalized fractional differential operator $\mathcal{T}_s^{\alpha,\theta,\kappa}$ of three parameters studied and introduced in term of integral [5] and given by:

$$\mathcal{T}_s^{\alpha,\theta,\kappa} f(s) := \frac{(\kappa+1)^{\alpha-\theta}\Gamma(\sigma)}{\Gamma(\alpha)\Gamma(1-\alpha+\theta)} \left(s^{1-\theta-\kappa} \frac{d}{d\xi}\right) \int_0^{\xi} \frac{t^{\kappa+\alpha-1} f(t)}{(s^{1+\kappa}-t^{1+\kappa})^{\alpha-\theta}} dt, \tag{2}$$

$$(0 < \theta \leq 1, 0 < \alpha \leq 1 \text{ such that } 1 > \alpha - \theta \geq 0, \kappa > -1, f(s) \in \mathbb{A}, s \in \mathfrak{D}).$$

From (2) and for $\theta > \alpha$ and $\kappa > -1$, the normalization of fractional differential operator $\mathrm{T}_s^{\alpha,\theta,\kappa}: \mathbb{A} \to \mathbb{A}$ defined in terms of series and presented by [6]:

$$\mathrm{T}_s^{\alpha,\theta,\kappa} f(s) = s + \sum_{w=2}^{\infty} \frac{\Gamma\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)\Gamma\left(\frac{w+\kappa+\alpha}{\kappa+1}\right)}{\Gamma\left(\frac{w+\kappa+\alpha}{\kappa+1}-\alpha+\theta\right)\Gamma\left(\frac{\alpha}{\kappa+1}+1\right)} a_w s^w,\ s\, \mathfrak{D}. \tag{3}$$

Here, for $f \in \mathbb{A}(p)$, we define the extension normalization of $\mathrm{T}_s^{\alpha,\sigma,\kappa} f(s)$ as follows:

Definition 1: For $f(s) \in \mathbb{A}(p)$, let $0 < \theta \leq 1,\ 0 < \alpha \leq 1$ such that $1 > \theta - \alpha \geq 0$ and $\kappa > -1$, we define the following new linear operator $\mathcal{M}_{s,p}^{\alpha,\theta,\kappa} f(s): \mathbb{A}(p) \to \mathbb{A}(p)$ given by:

$$\mathcal{M}_{s,p}^{\alpha,\theta,\kappa} f(s) := s^p \left(T_s^{\alpha,\theta,\kappa} f(s)\right) * f(s)$$

$$= s^p + \sum_{w=1+p}^{\infty} \frac{\Gamma\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)\Gamma\left(\frac{w-p+\alpha}{\kappa+1}+1\right)}{\Gamma\left(\frac{w-p+\alpha}{\kappa+1}-\alpha+\theta+1\right)\Gamma\left(\frac{\alpha}{\kappa+1}+1\right)} a_w s^w,\ s\, \mathfrak{D}. \tag{4}$$

The operator $\mathcal{M}_{s,p}^{\alpha,\theta,\kappa}$ defined in (4) can be presented in terms of Pochhammer symbols, as follows:

$$\mathcal{M}_{s,p}^{\alpha,\theta,\kappa} f(s) := s^p + \sum_{w=2}^{\infty} \frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}} a_w s^w \tag{5}$$

From (5), we can note that the operator $\mathcal{M}_{s,p}^{\alpha,\theta,\kappa}$ is considered as a special case of the Srivastava-Wright operator [7] provided with fractional parameters, and

$$\mathcal{M}_{s,1}^{1,1,0} f(\xi) = f(s).$$

Corresponding to $\mathcal{M}_{s,p}^{\alpha,\theta,\kappa}$ which is given in (5), we introduce the generalization fractional differential operator $\mathbb{K}_{\kappa,\lambda,\varphi,p}^{m,\alpha,\theta}: \mathbb{A}(p) \to \mathbb{A}(p)$ by the form

$$\mathbb{K}_{\kappa,\lambda,\varphi,p}^{0,\alpha,\theta} f(s) = \mathcal{M}_{s,p}^{\alpha,\theta,\kappa} f(s),$$

$$\mathbb{K}^{1,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s) = (\varphi-\lambda)\mathcal{M}^{\alpha,\theta,\kappa}_{s,p}f(s) + \frac{\lambda}{p}\, s(\mathcal{M}^{\alpha,\theta,\kappa}_{s,p}f(s))' + (1-\varphi)s^p \tag{6}$$

$$= s^p + \sum_{w=p+1}^{\infty}\left(\varphi + \frac{\lambda(w-p)}{p}\right)^1 \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right] a_w s^w$$

$$= \mathbb{K}^{\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s),$$

$$\mathbb{K}^{2,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s) = \mathbb{K}^{\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)(\mathbb{K}^{1,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s))$$

$$= s^p + \sum_{w=p+1}^{\infty}\left(\varphi + \frac{\lambda(w-p)}{p}\right)^2 \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right] a_w s^w$$

$$\vdots$$

$$\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s) = \mathbb{K}^{1,\alpha,\theta}_{\kappa,\lambda\varphi,,p}\left(\mathbb{K}^{m-1,\alpha,\theta}_{\kappa,\lambda,\varphi,p} f(s)\right). \tag{7}$$

If $f(s)$ given in (1), then from (6) and (7) we obtain

$$\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s) = s^p + \sum_{w=p+1}^{\infty}\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right] a_w s^w, \tag{8}$$

$$(0<\theta\le 1, 0<\alpha\le 1, 1>\alpha-\theta\ge 0, \lambda \ge 0, \varphi\ge 1, m\in \aleph_0 = \aleph\cup\{0\}).$$

Remark 1: The operator in (8) generalizes the following different types of operators:

- $\mathbb{K}^{0,\alpha,\theta}_{0,\lambda,\varphi,1}f(s) = \mathbb{K}^{\alpha,\theta} f(s) \Rightarrow$ [4];
- $\mathbb{K}^{0,\alpha,\theta}_{\kappa,\lambda,\varphi,1}f(s) = \mathbb{K}^{\alpha,\theta}_{\kappa} f(s) \Rightarrow$ [5, 6];
- $\mathbb{K}^{0,1,1-\theta}_{\kappa,\lambda,\varphi,1}f(s) = \mathbb{K}^{1-\theta}_{\kappa} f(s) \Rightarrow$ [8];
- $\mathbb{K}^{m,1,1}_{0,\lambda,1,1}f(s) = \mathbb{K}^{m}_{\lambda} f(s) \Rightarrow$ [9];
- $\mathbb{K}^{m,1,1}_{0,1,1,1}f(s) = \mathbb{K}^{m} f(s) \Rightarrow$ [10].

Utilizing the operator $\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)$, then new subclass $¥^{m}_{\alpha,\theta}(\lambda,\kappa,\varphi;\mathcal{E},\eta,p)$ of the function is introduced by the following definition:

Definition 2: Let $¥^{m}_{\alpha,\theta}(\lambda,\kappa,\varphi;\mathcal{E},\eta,p)$ be the subclass of function $f\in\mathbb{A}(p)$ that satisfies the following condition:

$$\Re e\left\{1+\frac{1}{\mathcal{E}}\left[p(1-\eta)\frac{\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)}{s} + \eta\left(\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)\right)' - p\right]\right\} > 0, \tag{9}$$

where $\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)$ is given in (8).

This shows that it fulfills the following inequality:

$$\left|\frac{p(1-\eta)\frac{\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)}{s} + \eta\left(\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)\right)' - p}{p(1-\eta)\frac{\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)}{s} + \eta\left(\mathbb{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)\right)' - 1 + 2\mathcal{E}}\right| < 1,$$

where $p \in N, m \in \aleph_0 = \aleph \cup \{0\}$, $\mathcal{E} \in \mathbb{C}\backslash\{0\}, \lambda \geq 0$, $\eta \geq 0$ *and* $s \in \mathfrak{D}$.

Among the characteristics of the subclass $¥^{m}_{\alpha,\theta}(\lambda, \kappa, \varphi; \mathcal{E}, \eta, p)$ that are proven in this study are some of the substantial studies in geometric function theory, including growth, distortion theorems, and coefficient estimates.

2. Main Results

We begin by estimating the upper bound of the coefficients for the functions belonging to the class $f \in ¥^{m}_{\alpha,\theta}(\lambda, \kappa, \varphi; \mathcal{E}, \eta, p)$.

Theorem 1: *For* $s \in \mathfrak{D}$ *and* $m \in \aleph_0$, $\lambda \geq 0$. *let* f *given in (1) and satisfies*

$$\sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w| \leq |\mathcal{E}|, \quad (10)$$

then $f \in ¥^{m}_{\alpha,\theta}(\lambda, \kappa, \varphi; \mathcal{E}, \eta, p)$.

Proof: Let the inequality (9) is true, we obtain

$$\left|p(1-\eta)\frac{\mathrm{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)}{s} + \eta\left(\mathrm{K}^{m,\alpha,\theta}_{\kappa,\lambda,p}f(s)\right)' - p\right|$$

$$- \left|p(1-\eta)\frac{\mathrm{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi p}f(s)}{s} + \eta\left(\mathrm{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi p}f(s)\right)' - 1 + 2\mathcal{E}\right|$$

$$= \left|\sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]\right| a_w\ |s^{w-1}|$$

$$- \left|2\mathcal{E} + \sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]\right| a_w\ |s^{w-1}|$$

$$\leq \sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w||s^{w-1}|$$

$$-2|\mathcal{E}| - \sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w||s^{w-1}|$$

$$\leq \sum_{w=p+1}^{\infty}[p + \eta(w - p)]\left(\varphi + \frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w| -$$

$|\mathcal{E}| \leq 0$,

where $\mathrm{K}^{m,\alpha,\theta}_{\kappa,\lambda,\varphi,p}f(s)$ is given in (8), then $f \in ¥^{m}_{\alpha,\theta}(\lambda, \kappa, \varphi; \mathcal{E}, \eta, p)$.

In next results, we obtain the growth and distortion theorems for function $f(s) \in ¥^{m}_{\alpha,\theta}(\lambda, \kappa, \varphi; \mathcal{E}, \eta, p)$, respectively:

Theorem 2: *Let* $f \in ¥_{\alpha,\theta}^{m}(\lambda,\kappa,\varphi;\mathcal{E},\eta,p)$*. Then for* $|s| = r < 1$*, we obtain*

$$r^p - \frac{|\mathcal{E}|\, r^{p+1}}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]} \le |f(s)| \le r^p + \frac{|\mathcal{E}|\, r^{p+1}}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]}$$

Proof: From inequality (10), we have

$$\sum_{w=p+1}^{\infty}[p+\eta(w-p)]\left(\varphi+\frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w| \le |\mathcal{E}|$$

and

$$[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]\sum_{w=p+1}^{\infty}|a_w| \le$$

$$\sum_{w=p+1}^{\infty}[p+\eta(w-p)]\left(\varphi+\frac{\lambda(w-p)}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{(w-p)}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{(w-p)}{\kappa+1}}}\right]|a_w| \le |\mathcal{E}|.$$

Hence, we get

$$\sum_{w=p+1}^{\infty}|a_w| \le \frac{|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]}. \tag{11}$$

From equation (1) and applying (11), we have

$$|f(s)| = \left|s^p + \sum_{w=p+1}^{\infty} a_w\, s^w\right| \le |s^p| + r^{p+1}\sum_{w=p+1}^{\infty}|a_w|$$
$$\le r^p + \frac{|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]} r^{p+1}.$$

While, the other assertion can be proved as follows

$$|f(s)| = \left|s^p + \sum_{w=p+1}^{\infty} a_w\, s^w\right| \ge |s^p| - r^{p+1}\sum_{w=p+1}^{\infty}|a_w|$$
$$\ge r^p - \frac{|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]} r^{p+1}.$$

The proof is concluded.

Likewise, it is possible to demonstrate the following theorem:

Theorem 3: *If* $f \in ¥_{\alpha,\theta}^{m}(\lambda,\kappa,\varphi;\mathcal{E},\eta,p)$*. Then for* $|s| = r < 1$ *we have*

$$pr^{p-1} - \frac{(p+1)|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^m \left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]} r^p \le |f'(s)| \le$$

$$pr^{p-1}+\frac{(p+1)|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^{m}\left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]}r^{p}\ .$$

Proof: Let $f\in ¥_{\alpha,\theta}^{m}(\lambda,\kappa;\mathcal{E},\eta,p)$. Then from (10), obtain

$$\sum_{w=p+1}^{\infty}|a_w|\leq\frac{|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^{m}\left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]},$$

also, from (1), we have

$$|f'(s)|=\left|ps^{p-1}+\sum_{w=p+1}^{\infty}wa_w\,s^{w-1}\right|\leq pr^{p-1}+(p+1)r^{p}\sum_{w=p+1}^{\infty}|a_w|$$

$$\leq pr^{p-1}+\frac{(p+1)|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^{m}\left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]}r^{p}.$$

Similarly, we can prove that

$$\left|f'(s)\right|\geq pr^{p-1}-\frac{(p+1)|\mathcal{E}|}{[p+\eta]\left(\varphi+\frac{\lambda}{p}\right)^{m}\left[\frac{\left(\frac{\alpha}{\kappa+1}+1\right)_{\frac{1}{\kappa+1}}}{\left(\frac{\alpha}{\kappa+1}-\alpha+\theta+1\right)_{\frac{1}{\kappa+1}}}\right]}r^{p}.$$

This finishes the theorem assertion.

3. Conclusion

The results of this study are applicable to the fractional differential operator in a complex domain within the open unit disk U. We have introduced a normalized fractional differential operator based on the generalized Tremblay operator of p-valent functions. Additionally, we have included growth and distortion theorems, as well as coefficient estimates for a class of p-valent functions defined by using the new fractional differential operator.

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