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Solutions of random ordinary differential equations with laplace transform - Adomian decomposition method

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Abstract

Combine the Laplace Adomian Decomposition method (LTADM) for non-linear random ordinary differential equations is utilised in a novel way to obtain accurate solutions.The iterative implementation of the Laplace transform decomposition is employed to approximate the solutions of ordinary differential equations with random components. We use a Normal distribution to analyse the parameters and initial conditions of ordinary differential equations with random components. Using the Mathematica 13.3 programme, the graphs of the approximate solutions and absolute error are presented. An investigation is conducted into the effects of the normal distribution on the results of random component differential equations. According to the results of the numerical investigations, this method is extremely effective. Two applications are presented as examples of how the proposed technique can be utilised to obtain analytical or numerical solutions for certain kinds of random differential equations in order to demonstrate its efficacy and potential.

Subject Classification: *35R45, 35R70.*

Keywords: *Laplace transform, Random differential equations, Normal distribution, Semi analytic approach, Decomposition technique.*

1. Introduction

Random Ordinary Differential Equations (RODEs) are currently significant topics that involve ordinary differential equations (ODEs) with random variables or constants. In physics, engineering, and mathematical biology, random differential equations (RDEs) used to analyse some problems.

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In recent times, many scientific challenges spanning domains such as control theory, fluid mechanics, climate models, and physics have been successfully resolved. In the literature, there are remarkably few studies discussing random nonlinear differential equations (RNDEs). When simulating disparate phenomena, noise is crucial. Differential equations with some essential difficulty due to uncertainty are specifically produced by the mathematical modeling of physical systems and a number of real-world situations. Uncertainty can be included in differential systems in a variety of ways. especially in the past few decades in a variety of fields, such as the social sciences, mathematical biology, and engineering. Ordinary Differential Equations (ODEs) with a multidimensional stochastic process $\mathcal{W}_{\tau,}$ are named RDEs. In numerous applications across numerous scientific domains, RNDEs are essential. These issues are frequently unsolvable analytically. Consequently, numerous numerical techniques for solving ODEs,PDEs, RPDEs and RODEs have been created. Decomposition method, differential transform method, finite difference method and numerous other methods are among the many numerical techniques available [1-6].Many authors studied some types of differential equations. The major goal of this study is to compute the approximate solutions by a mixture Laplace transform and Adomian decomposition method in order to find the effective solutions of the RODEs [7-12] .

2. Some Basic Concepts of Stochastic Calculus

Definition 1: A real valued function $\chi(\omega)$, $\omega \in \Omega$ that is measurable with regard to the probability measure $\boldsymbol{\rho}$, is called a random variable.

Definition 2: Stochastic process $\chi(\tau,\omega), \tau \in [\tau_0,\infty), \omega \in \Omega$ indicate by a set of random variables on a space (Ω, F, ρ) that is $\chi_\tau(\omega)$ and suppose real values and is $\boldsymbol{\rho}$-measurable as a function of ω for each fixed τ. The parameter τ is regarded as a time and $\chi_\tau(.)$ denotes a random variable on the probability space Ω while $\chi(\omega)$ called a sample mammer or (path) trajectory of the stochastic process.

Definition 3: Stochastic process $\mathcal{W}_{\tau,}$ $\tau \geq 0,$ is said to be Wiener process, or Brownian motion if: a. $\rho\{\mathcal{W} \in \Omega | \mathcal{W}_0(\omega) = 0\}) = 1$ b. For $0 < \tau_0 < \tau_1 < \cdots < \tau_n$ the increments $\mathcal{W}_{\tau 1} - \mathcal{W}_{\tau 0} - \mathcal{W}_{\tau 2} - \mathcal{W}_{\tau 1 \dots,}$ $\mathcal{W}_{\tau n} - \mathcal{W}_{\tau n-1}$ are independent. c. For an arbitrary τ and $h > 0, \mathcal{W}_{\tau+h} - \mathcal{W}_\tau$ has a Normal distribution with mean 0 and variance h.

3. Convergence Analysis

Theorem 1.1: *Let A be an operator from* $A : H \to H$ *Hilbert space. Then the series solution* $\mathcal{X}(t;\omega) = \sum_{k=0}^{\infty} \mathbb{w}_k(t,\omega)$ *converges if there exists* $0 < \boldsymbol{\vartheta} < 1$, *suchthat* $\|\mathbb{w}_0(t;\omega) + \mathbb{w}_1(t;\omega) + \dots + \mathbb{w}_{k+1}(t;\omega)\| \leq \vartheta \|\mathbb{w}_0(t;\omega) + \mathbb{w}_1(t;\omega) + \cdots + \mathbb{w}_{k+1}(t;\omega)\|$ *Thus* $\|\mathbb{w}_{k+1}(t;\omega)\| \leq \vartheta \|\mathbb{w}_k(t;\omega)\|$ *for all* $k = 0,1,\dots$

Proof: Define the sequence $\sum_{k=0}^{\infty} \mathbb{W}_k(t,\omega)$,$\mathbb{r}_0 = \mathbb{W}_0(t;\omega)$,

$\mathbb{r}_1 = \mathbb{W}_0(t;\omega) + \mathbb{W}_1(t;\omega)$,$\mathbb{r}_2 = \mathbb{W}_0(t;\omega) + \mathbb{W}_1(t;\omega) + \cdots + \mathbb{W}_2(t;\omega)$

$$\mathbb{r}_m = \mathbb{W}_0(t;\omega) + \mathbb{W}_1(t;\omega) + \mathbb{W}_3(t;\omega) + \cdots + \mathbb{W}_m(t;\omega)$$

Consider

$$\|\mathbb{r}_{m+1} - \mathbb{r}_m\| = \|\mathbb{W}_{m+1}(t;\omega)\| \le \gamma\|\mathbb{W}_m(t\,;\,\omega)\|\gamma^2\|\mathbb{W}_{m-1}(t\,;\,\omega)\|$$
$$\le \gamma^{m+1}\,\|\mathbb{W}_0\,(t;\omega)\|.$$

For $m\,,j\,\in\,N\,,m\,\ge j$

$$\|\mathbb{r}_m - \mathbb{r}_j\| = \|(\mathbb{r}_m - \mathbb{r}_{m-1}) + (\mathbb{r}_{m-1} - \mathbb{r}_{m-2}) + \cdots + \mathbb{r}_{m+1} - \mathbb{r}_m\|$$
$$\le \gamma^m\|\mathbb{W}_0(t\,;\,\omega)\| + \gamma^{m-1}\|\mathbb{W}_0(t\,;\,\omega)\| + \gamma^{m-2}\|\mathbb{W}_0(t\,;\,\omega)\|$$
$$+ \cdots + \gamma^{j+1}\|\mathbb{W}_0(t\,;\,\omega)\| \le \frac{1-\gamma^{m-1}}{1-\gamma}\gamma^{j+1}\|\mathbb{W}_0(t\,;\,\omega)\|$$

And since $0 < \gamma < 1$, then $\lim_{m,j\to\infty}\|\mathbb{r}_m(t\,;\,\omega) - \mathbb{r}_j(t\,;\,\omega)\| = 0$

Therefore $\{\mathbb{r}_m(t\,;\,\omega)\}$ is a Cauchy sequence , then the series solution $\mathcal{X}(t;\omega) == \sum_{k=0}^{\infty} \mathbb{W}_k(t;\omega)$converge.

4 Numerical experiments

The Laplace Transform Adomian Decompositi technique will be utilized to solve the RODEs in this part. It is essential to note that the generation of various discretized Brownian motion throughout the unit period [0,1] will be investigated. The total number N=100, 500, and 1000 denotes these generations as shown in Figure 1 and 2.

Example 2.1: Let $\frac{d^3y\,(t,\omega)}{dt^3} + \frac{dy\,(t\,,\omega)}{dt} = y^2(t,\omega) + W_t(\omega),$ and initial condition

$$y_0(0,\omega) = 0.5\,, \frac{dy(0,\omega)}{dt} = 0\,, \frac{d^2y(0,\omega)}{dt} = 0 \qquad (1)$$

Solution: By using Laplace transform Adomain decomposition method for Eq.(1) and taking the inverse of Laplace transform for both sides , the successive approximate numerical solutions may be obtain as fallows

$y_0(t\,,\omega) = 0.5$

$$y_1(t\,,\omega) = \mathcal{L}^{-1}\left\{\frac{y\,(0,\omega)}{u^3} - \frac{1}{u^3}\,\mathcal{L}\left\{\frac{dy_0(t,\omega)}{dt} - \,y_0^2\,(t;\omega) - \,W_t(\omega)\right\}\right\}$$

$$y_1\,(t,\omega) = 0.25t^2 + 0.04166666664t^3 + 0.16666t^3W_t(0)$$
$$y_2(t,\omega) = 0.25t^2 - 0.0208333332t^4 + 0.002083333333t^5$$
$$+ 0.0003472224t^6 + 0.1666666t^3W_t(\omega)$$
$$- 0.008333333t^5W_t(\omega)$$

And similarly, we get

$$-0.00014880382t^7 + 6.8893273 \times 10^{-7}t^9$$
$$+ 0.16666666666666666t^3W_t(\omega) - 0.00833333t^5W_t(\omega)$$
$$+ 0.00138889t^6W_t(\omega) + 0.0001984126984t^7W_t(\omega)$$
$$- 0.0000496746t^8W_t(\omega) + 0.0000028589t^9W_t(\omega)$$

The following Figures show the approximate solutions and iterative error of RODE usingLTADM with N=1000 generations of Brownian motion

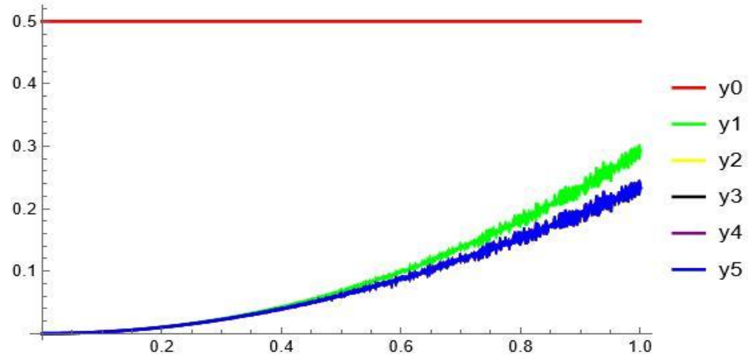

Figure 1
Approximate solutions of RODE using LTADM with N =1000 generations of Brownian motion.

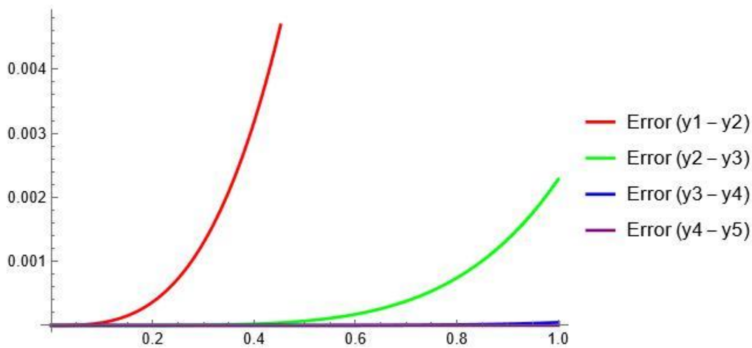

Figure 2
Error of RODE usingLTADM with N =1000 generations of Brownian motion.

Example 2.2: Consider the nonlinear random ordinary differential equation as:

$$\frac{dx}{dt} + x^2 = \mathrm{Cos}(W_t(\omega))\,,\ x(t_0,\omega) = 1,\ t \in [0,1] \qquad \dots(2)$$

Solution: Using Laplace transform Adomain decomposition method for Eq.(2), and taking inverse Laplace for both sides, the successive approximate numerical solutions may be obtain as fallows:

$$x_1(t\,,\omega) = 1 - \frac{t^2}{2} + \frac{1}{2}t^2\mathrm{Cos}[w]$$

$$x_2(t\,,\omega) = 1 - t + t^2 - \frac{t^3}{3} + t\cos[w] - t^2\cos[w] + \frac{2}{3}t^3\cos[w] - \frac{1}{3}t^3\cos[w]^2$$

$x_3(t\,,\omega) = 1 - t + t^2 - t^3 + \frac{2t^4}{3} - \frac{t^5}{3} + \frac{t^6}{9} - \frac{t^7}{63} + t\mathrm{Cos}[w] + \frac{2}{3}t^3\mathrm{Cos}[w] - \frac{5}{6}t^4\mathrm{Cos}[w] + \frac{2}{3}t^5\mathrm{Cos}[w] - \frac{1}{3}t^6\mathrm{Cos}[w] + \frac{4}{63}t^7\mathrm{Cos}[w] + \frac{1}{6}t^4\mathrm{Cos}[w]^2 - \frac{1}{3}t^5\mathrm{Cos}[w]^2 + \frac{1}{3}t^6\mathrm{Cos}[w]^2 - \frac{2}{21}t^7\mathrm{Cos}[w]^2 - \frac{1}{9}t^6\mathrm{Cos}[w]^3 + \frac{4}{63}t^7\mathrm{Cos}[w]^3 - \frac{1}{63}t^7\mathrm{Cos}[w]^4 - 2t^2\mathrm{Cos}[w] + t^3\mathrm{Cos}[w] - \frac{2}{3}t^4\mathrm{Cos}[w] + \frac{1}{6}t^5\mathrm{Cos}[w] + \frac{2}{3}t^4\mathrm{Cos}[w]\mathrm{Cos}[w] - \frac{1}{3}t^5\mathrm{Cos}[w]\mathrm{Cos}[w] + \frac{1}{6}t^4\mathrm{Cos}[w]^2\mathrm{t\ Cos}[w] - t^3\mathrm{Cos}[w]^2$. By the same way , we get $x_4(t\,,\omega)$, $x_5(t\,,\omega)$, ...

The following Figures show the approximate solutions and iterative error of RODE usingLTADM with N =500 generations of Brownian motion as shown in Figure 3 and 4.

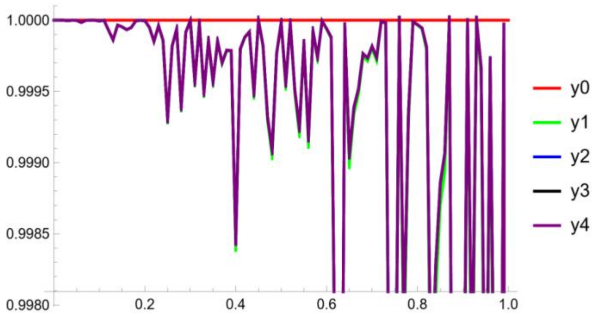

Figure 3
Approximate solutions of RODE usingLTADM with N =500 generations of Brownian motion.

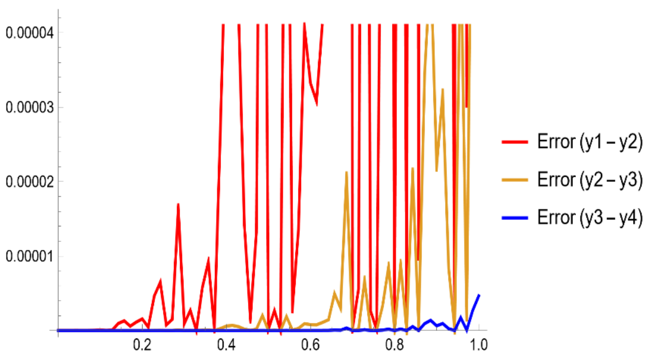

Figure 4
Error of RODE usingLTAD with N=500

5. Conclusion

Here, We've used this effectively LTADM to obtain an approximate analytical solution for several applications of RODEs that are randomized with the help of Normal distribution. The proposed technique is simple to use, appropriate, and successful for getting the desired results for these issues, and LTADM provides the converging series results quickly. RODEs' approximate analytical solution was obtained quickly and effectively. The LTADM is more powerful and dependable in obtaining approximate solutions to several classes of random component linear and nonlinear ordinary differential equations. The error utilized in this case is to verify the accuracy and convergence of the series of approximations to the precise solution in the absence of the exact solution.

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