*Journal of Interdisciplinary Mathematics*
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 4, pp. 667–673
DOI : 10.47974/JIM-1738

# Regularity and normality of nano topological spaces modeled by pre-open sets structures

Ghufran Ali Hussein *
Z. D. Al-Nafie †
*Department of Mathematics*
*College of Education for Pure Sciences*
*University of Babylon*
*Babylon*
*Iraq*

**Abstract**

A new class of Nano topological spaces, termed nano pre-open sets, are presented and explored in this article. Since the Nano topology contains at most five sets, we establish new structures of nano open sets and nano open covers and new requirement concepts are discussed like Nano pre-open covers, Nano pre- T_0, T_1, T_2. In this work, regularity and normality are defined as follows: Nano pre-regular and Nano pre-normal spaces. Many new theorems are stated and proved.

***Subject Classification:*** *54A05, 54C10.*

***Keywords:*** *Nano topology, Regularity space, Normality space, Pre-open, Separation axioms.*

## 1. Introduction

Nano topologies proposed by Lellis Thivagar et al. [10],[11] are introduced in terms of a subset $X$ of the universal set $U$ defined by the lower and upper approximations of this subset. The pair $(U, R)$ containing an equivalence relation $R$ on $U$ is called the space of approximation. In the same equivalence class, elements are called indistinguishable from each other. Let $X \subseteq U$. For each set R, the set of objects that satisfy the classification criterion X, is the set $L_R\ (X) = \bigcup_{x \in U}\{R(x): R(x) \subseteq X\}$. The x-provided equivalence class, R(x), is said to be the more conservative calculation for X for the collection of objects that, relative to R, may be characterized as X is the best approximation we have

* *E-mail:* **ghufran.hussein.pure378@student.uobabylon.edu.iq**
**(Corresponding Author)**
† *E-mail:* **pure.zahir.dobeas@uobabylon.edu.iq**

$U_R(X)$ and it is equal to $\bigcup_{x \in U} \{R(x): R(x) \cap X \neq \emptyset\}$, in addition to the set represented by, which includes objects that cannot be either classified as X or non-X concerning R, $B_R$(X) = $U_R$(X) - $L_R$(X), Specifically, it is referred to as X's border concerning R.This setting generates a Nano topology $\tau_R(X)$ on $U$ containing Nano open sets and their complements are called closed nano sets. $(U, \tau_R(X))$ called a Nano topological space [13,14]. Nano pre-interior Nano pre-closure in nano topological spaces has been defined by Sathishmohan et al. [9], and different related characteristics were examined. If $A \subseteq U$, The largest nano open set included in A is called the nano interior of A, denoted by N. $Int(A)$, while the smallest nano closed set N that contains A is its nano closure. $Cl(A)$ . Since the nano topology has applications in some medical and physics models used to solve real-world problems [1] and [2], we tried to modify these topologies by adding a new class of topologies to these topologies, hopping to get new results ca applicable in medicine and life in the future.

Tietze [5] and [4] are credited with the original definition of the term "normal property" in topological space. Aarts [6] then focused their attention on this matter. The expansions of the concepts of pairwise regularity, pairwise normality, and total normality were recently introduced by Dorsett [3] and Jasiml [12].

## 2. Preliminaries

**Definition 2.1: [10]** Let $U$ be the universe and $X \subseteq U, R$ be a relationship on U that is equivalent to U. Then

$$\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$$

To achieve this, we will define the Nano topology on U in terms of X.

$(U, \tau_R(X))$ called a Nano topological space denoted by $NTS$, and its elements are called Nano open sets.

**Definition 2.2: [9]** A $\sqsubseteq$ NTS $(U, \tau_R(X))$ is called a Nano Pre-open in $U$, if $A \subseteq N - Int\,(N - Cl(A))$ and denoted by $N_{\mathcal{PO}}$, its complement is called a Nano Pre-closed *(* $N_{\mathcal{PC}}$).

**Example 2.3:** Let $U = \{\alpha, \beta, \delta\}$, $X = \{\alpha\}$, $\frac{U}{R} = \{\{\alpha\}, \{\beta\}, \{\beta, \delta\}\}$. $\tau_R(X) = \{U, \emptyset, \{\alpha\}\}$. The nano closed sets $F = \{\emptyset, U, \{\beta, \delta\}\}$. $= \{U, \emptyset, \{\alpha\}, \{\alpha, \beta\}, \{\alpha, \delta\}\}$. $N_{\mathcal{PC}}(U, X) = \{\emptyset, U, \{\beta, \delta\}, \{\delta\}, \{\beta\}\}$.

**Definition 2.4: [9]** A $\sqsubseteq NTS$ $(U, \tau_R(X))$ is called a Nano-regular open in $U$, *if* $N - Int\,(N - Cl(A)) = A$ and it is denoted by $N_{RO}$.

The complement of $N_{RO}$ is called a Nano-regular closed, and it is denoted by $N_{RC}$.

**Definition 2.5: [8]** $\cup\, N_{\mathcal{PO}}$ Nano Pre–interior of A is the set of $\forall$ sets that are included in A, and it is symbolized by $N_{\mathcal{PO}}\ Int(A)$ or $N_{\mathcal{PO}}(\mathring{A})$.

**Definition 2.6: [8]** $\cap N_{\mathcal{P}C}$ Nano Pre-closure of A is the set of $\forall$ sets that include A and is denoted by $N_{\mathcal{P}C}\ Cl(A)$ or $N_{\mathcal{P}C}(\overline{A})$.

**Definition 2.7: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano-$T_0$ (or $N - T_0$) space for $a, b \in U$ and $a \neq b$, there exists a nano-open $\mathcal{G} \ni a$ in $H$ and $b$ in not in $\mathcal{G}$.

**Definition 2.8: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano Pre-$T_0$ (or $N_{\mathcal{P}O} - T_0$) space for $a, b \in U$ and $a \neq b$, $\exists$ a $N_{\mathcal{P}O}$ set $\mathcal{G} \ni a \in \mathcal{G}$ and $b \notin \mathcal{G}$.

**Example 2.9:** Let $U = \{\alpha, \beta, \delta\}$, $\tau_R(X) = \{\emptyset, U, \{\alpha\}, \{\alpha, \beta\}, \{\beta\}\}$, $N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{1\alpha\}, \{1\beta\}, \{\alpha, \beta\}\}$

$\alpha \neq 1\beta \Rightarrow \alpha \in \{1\alpha\} \wedge \beta \notin \{1\alpha\}$. $\alpha \neq \delta \Rightarrow \alpha \in \{1\alpha\} \wedge \delta \notin \{1\alpha\}$. $\beta \neq \delta \Rightarrow 1\beta \in \{1\beta\} \wedge \delta \notin \{1\beta\}$.

$\therefore (U, \tau_R(X))$ *is a* $N_{\mathcal{P}O} - T_0$ space.

**Definition 2.10: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano-$T_1$(or $N - T_1$) space if for $a, b \in U \ni a \neq b$,$\exists$ a Nan-open sets $\mathcal{G}\ \&\ \mathcal{H} \ni a \in \mathcal{G}, b \notin \mathcal{G}\ \&\ b \in \mathcal{H}, a \notin \mathcal{H}$.

**Definition 2.11: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano Pre-$T_1$ (or $N_{\mathcal{P}O} - T_1$) space for $a, b \in U$ and $a \neq b$, there exists a$N_{\mathcal{P}O}$sets $\mathcal{G}\ \&\ \mathcal{H} \ni a \in \mathcal{G}, b \notin \mathcal{G}\ \&\ b \in \mathcal{H}, a \notin \mathcal{H}$.

**Definition 2.12: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano-$T_2$ (or $N - T_2$) space if for $a, b \in U$ and $a \neq b$,$\exists$ disjoint Nan-open sets$\mathcal{G}$ and $\mathcal{H} \ni a \in \mathcal{G}$ and $b \in \mathcal{H}$.

**Definition 2.13: [7]** A $NTS\ (U, \tau_R(X))$ is called a Nano Pre-$T_2$(or $N_{\mathcal{P}O} - T_2$) space for $a, b \in U$ and $a \neq b$, $\exists$ disjoint $N_{\mathcal{P}O}$ sets$\mathcal{G}$ and $\mathcal{H} \ni a \in \mathcal{G}$ and$b \in \mathcal{H}$.

**Example 2.14:** Let $U = \{\alpha, \beta, \delta\}, U/R = \{\{\alpha\}, \{\beta\}, \{\beta, \delta\}\}, X = \{\alpha, \delta\}$ and $\tau_R(X) = \{\emptyset, U, \{\alpha\}, \{\beta, \delta\}\}$.

The nano closed set

$$F = \{U, \emptyset, \{\beta, \delta\}, \{\alpha\}\}. N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{\alpha\}, \{\beta\}, \{\delta\}, \{\alpha, \beta\}, \{\alpha, \delta\}, \{\beta, \delta\}\}.$$

$$\alpha \neq \beta \Rightarrow \alpha \in \{1\alpha\} \wedge \beta \in \{\beta\} \Rightarrow \{1\alpha\} \cap \{1\beta\} = \emptyset.$$

$$\alpha \neq \delta \Rightarrow \alpha \in \{1\alpha\} \wedge \delta \in \{\delta\} \Rightarrow \{1\alpha\} \cap \{1\delta\} = \emptyset.$$

$$\beta \neq \delta \Rightarrow \beta \in \{1\beta\} \wedge \delta \in \{\delta\} \Rightarrow \{1\beta\} \cap \{1\delta\} = \emptyset.$$

$\therefore (U, \tau_R(X))$is a $N_{\mathcal{P}O} - T_2$ space.

## 3. Nano Pre-regularity and Nano Pre-normality

**Definition 3.1:** A $NTS\ (U, \tau_R(X))$ is said to be $N_{\mathcal{P}O}$ –regular space iff $\forall\ N_{\mathcal{P}C}$, $\forall\ p \notin N_{\mathcal{P}C}\ \exists\ \mathcal{G}, \mathcal{H} \in N_{\mathcal{P}O}(U, X),\ p \in \mathcal{G} \wedge N_{\mathcal{P}C} \subseteq \mathcal{H},\ \mathcal{G} \cap \mathcal{H} = \emptyset$.

**Example 3.2:** Let $U = \{\alpha_1, \alpha_2, \alpha_3\}$ and $U/R = \{\{\alpha_1\}, \{\alpha_2\}, \{\alpha_2, \alpha_3\}\}$, $X = \{\alpha_1, \alpha_3\}$, $\tau_R(X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_2, \alpha_3\}\}$. The nano closed set

$$F = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_1\}\}.$$

$$N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_2\}, \{\alpha_3\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}, \{\alpha_2, \alpha_3\}\}.$$

$$N_{\mathcal{P}C}(U, X) = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_1, \alpha_3\}, \{\alpha_1, \alpha_2\}, \{\alpha_3\}, \{\alpha_2\}, \{\alpha_1\}\}.$$

$$\alpha_1 \notin \{\alpha_2, \alpha_3\} \Rightarrow \alpha_1 \in \{\alpha_1\} \wedge \{\alpha_2, \alpha_3\} \subseteq \{\alpha_2, \alpha_3\} \ni \{\alpha_1\} \cap \{\alpha_2, \alpha_3\} = \emptyset.$$

$$\alpha_2 \notin \{\alpha_1\} \Rightarrow \alpha_2 \in \{\alpha_2, \alpha_3\} \wedge \{\alpha_1\} \subseteq \{\alpha_1\} \ni \{\alpha_2, \alpha_3\} \cap \{\alpha_1\} = \emptyset.$$

$$\alpha_3 \notin \{\alpha_1\} \Rightarrow \alpha_3 \in \{\alpha_2, \alpha_3\} \wedge \{\alpha_1\} \subseteq \{\alpha_1\} \ni \{\alpha_2, \alpha_3\} \cap \{\alpha_1\} = \emptyset.$$

$\therefore$ $(U, \tau_R(X))$ is a $N_{\mathcal{P}O}$ −regular space.

**Example 3.3:** Let $U = \{\alpha_1, \alpha_2, \alpha_3\}$ and $U/R = \{\{\alpha_1\}, \{\alpha_2\}, \{\alpha_2, \alpha_3\}\}$, $X = \{\alpha_1\}$, $\tau_R(X) = \{\emptyset, U, \{\alpha_1\}\}$.

The nano closed set $F = \{U, \emptyset, \{\alpha_2, \alpha_3\}\}$.

$$N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}\}.$$

$$N_{\mathcal{P}C}(U, X) = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_3\}, \{\alpha_2\}\}.$$

$$\alpha_1 \notin \{\alpha_2, \alpha_3\} \Rightarrow \exists \{\alpha_1\}, \{\alpha_1, \alpha_2\} \in N_{\mathcal{P}O}, \{\alpha_1\} \cap \{\alpha_2, \alpha_3\} \neq \emptyset, \alpha_1 \in \{\alpha_1\} \wedge \{\alpha_2, \alpha_3\} \nsubseteq \{\alpha_1, \alpha_2\}.$$

$\therefore$ $(U, \tau_R(X))$ is not a $N_{\mathcal{P}O}$ −regular space.

**Theorem 3.4:** $\forall$ *Nano regular-space is* $N_{\mathcal{P}O}$ − *regular-space.*

***Proof:*** A Nano closed set, F, is assumed $x \notin \mathcal{F}$ Point the Nano regular space at a certain location $(U, \tau_R(X))$, $\exists$ Two different open nanosets $\mathcal{G}$ and $\mathcal{H}$ such that $x \in \mathcal{G}$ and $\mathcal{F} \subset \mathcal{H}$. since every Nano open set is$N_{\mathcal{P}O}$ set, $\mathcal{G}$ and $\mathcal{H}$ are $N_{\mathcal{P}O}$ sets ,$x \notin N_{\mathcal{P}C}$ such that $x \in \mathcal{G}$ and $N_{\mathcal{P}C} \subset \mathcal{H}$. Hence $(U, \tau_R(X))$is a $N_{\mathcal{P}O}$ − regular space.

**Remark 3.5:** Every $N_{\mathcal{P}O}$ − regular space is not Nano regular

**Example 3.6:** Let $U = \{\alpha, \beta, \delta, \Omega\}$ with $U/R = \{\{\alpha, \delta\}, \{\beta\}, \{\Omega\}\}$ and $X = \{\alpha, \delta\}$. Then the Nano topology is $\tau_R(X) = \{U, \emptyset, \{\alpha, \delta\}\}$. Then $(U, \tau_R(X))$ is $N_{\mathcal{P}O}$ − regular space but not Nano regular space.$\tau_R(X) = \{U, \emptyset, \{\alpha, \delta\}\}$.The nano closed set $F = \{\emptyset, U, \{\beta, \Omega\}\}$ .

$$\alpha \notin \{\beta, \Omega\} \Rightarrow \alpha \in \{\alpha, \delta\} \wedge \{\beta, \Omega\} \subseteq U \ni \{\alpha, \delta\} \cap U \neq \emptyset.$$

$\therefore$ $(U, \tau_R(X))$is not nano regular space.

$$N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{\alpha\}, \{\delta\}, \{\Omega\}, \{\alpha, \beta\}, \{\alpha, \delta\}, \{\alpha, \Omega\}, \{\beta, \delta\}, \{\beta, \Omega\}, \{\delta, \Omega\}, \{\alpha, \beta, \delta\}, \{\alpha, \beta, \Omega\}, \{\alpha, \delta, \Omega\}, \{\beta, \delta, \Omega\}\}.$$

$$N_{\mathcal{P}C}(U, X) = \{U, \emptyset, \{\beta, \delta, \Omega\}, \{\alpha, \beta, \Omega\}, \{\alpha, \beta, \delta\}, \{\delta, \Omega\}, \{\beta, \Omega\}, \{\beta, \delta\}, \{\alpha, \Omega\}, \{\alpha, \delta\}, \{\alpha, \beta\}, \{\Omega\}, \{\delta\}, \{\beta\}, \{\alpha\}\}.$$

$$\alpha \notin \{\beta, \Omega\} \Rightarrow \alpha \in \{\alpha\} \wedge \{\beta, \Omega\} \subseteq \{\beta, \Omega\} \ni \{\alpha\} \cap \{\beta, \Omega\} = \emptyset.$$
$$\beta \notin \{\alpha, \Omega\} \Rightarrow \beta \in \{\beta, \delta\} \wedge \{\alpha, \Omega\} \subseteq \{\alpha, \Omega\} \ni \{\beta, \delta\} \cap \{\alpha, \Omega\} = \emptyset.$$
$$\delta \notin \{\alpha\} \Rightarrow \delta \in \{\beta, \delta\} \wedge \{\alpha\} \subseteq \{\alpha\} \ni \{\beta, \delta\} \cap \{\alpha\} = \emptyset .$$
$$\Omega \notin \{\alpha, \beta\} \Rightarrow \Omega \in \{\Omega\} \wedge \{\alpha, \beta\} \subseteq \{\alpha, \beta\} \ni \{\Omega\} \cap \{\alpha, \beta\} = \emptyset.$$

$\therefore (U, \tau_R(X))$ is$N_{\mathcal{PO}}$ $-$regular space.

**Lemma 3.7:** *If every open cover of a $N_{\mathcal{PO}}$ $-$regular spaceX has a nano locally finite $N_{\mathcal{PO}}$ refinement, then for every open cover $\{U_a\}_{a\in\beta}$ of the nano space X there exists a nano closed locally finite cover $\{V_a\}_{a\in\beta}$ of X such that $V_a \subset U_a$, for every $a \in \beta$.*

***Proof:*** By a $N_{\mathcal{PO}}$ $-$regular space of $X$ there exists an open cover $\mathcal{W}$ of the nano space $X$ such that $\{N\,\overline{W}: W \in \mathcal{W}\}$ is a $N_{\mathcal{PO}}$ refinement of $\{U_a\}_{a\in\beta}$. Take a nano locally finite $N_{\mathcal{PO}}$ refinement $\{\mathcal{A}_t\}_{t\in\tau_R}$ of the cover $\mathcal{W}$, for every $t \in \tau_R$ choose an $a(t) \in \beta$ such that $N\,\overline{\mathcal{A}_t} \subset U_{a(t)}$, and let $V_a = \bigcup_{a(t)=a} N\,\overline{\mathcal{A}_t}$. From Theorems 3.2 and 3.4 it follows readily that $\{V_a\}_{a\in\beta}$ is a nano closed locally finite cover of $X$ and the definition of the $V_a$'s implies that $V_a \subset U_a$, for every $a \in \beta$ .

**Example 3.8:** Let $U = \{\alpha, \beta, \delta\}$ and $U/R = \{\{\alpha\}, \{\beta\}, \{\beta, \delta\}\}$ , $X = \{\alpha, \delta\}$, $\tau_R(X) = \{\emptyset, U, \{\alpha\}, \{\beta, \delta\}\}$ .

The nano closed set $F = \{U, \emptyset , \{\beta, \delta\}, \{\alpha\}\}$.

$$N_{\mathcal{PO}}\,(U, X) = \{\emptyset, U, \{\alpha\}, \{\beta\}, \{\delta\}, \{\alpha, \beta\}, \{\alpha, \delta\}, \{\beta, \delta\}\}.$$
$$N_{\mathcal{PC}}\,(U, X) = \{U, \emptyset, \{\beta, \delta\}, \{\alpha, \delta\}, \{\alpha\}, \{\beta\}, \{\delta\}, \{\alpha, \beta\}, \{\alpha, \delta\}, \{\beta, \delta\}\}.$$

$A = \{U, \{\beta, \delta\}\}$ , $B$ is a $N_{\mathcal{PO}}$ refinement of $A$, $B = \{\{\alpha, \beta\} , \{\beta, \delta\}\}$.

$\alpha \notin \{\beta, \delta\} \Rightarrow \alpha \in \{\alpha\} \wedge \{\beta, \delta\} \subseteq \{\beta, \delta\} \ni \{\alpha\} \cap \{\beta, \delta\} = \emptyset.$

$$\beta \notin \{\alpha\} \Rightarrow \beta \in \{\beta, \delta\} \wedge \{\alpha\} \subseteq \{\alpha\} \ni \{\beta, \delta\} \cap \{\alpha\} = \emptyset.$$
$$\delta \notin \{\alpha\} \Rightarrow \delta \in \{\beta, \delta\} \wedge \{\alpha\} \subseteq \{\alpha\} \ni \{\beta, \delta\} \cap \{\alpha\} = \emptyset.$$

**Definition 3.9:** A $NTS\,(\mathrm{U}, \tau_{\mathrm{R}}(\mathrm{X}))$ is said to be a $N_{\mathcal{PO}}$ $-$normal space iff $\forall \mathcal{A}, \beta$ $N_{\mathcal{PC}}$ sets of $U \ni \mathcal{A} \cap \beta = \emptyset$ . Then $\mathcal{G}, \mathcal{H} \in N_{\mathcal{PO}}$ , $\mathcal{A} \subseteq \mathcal{G}$ and $\beta \subseteq \mathcal{H}$, $\mathcal{G} \cap \mathcal{H} = \emptyset$ .

**Example 3.10:** Let $U = \{\alpha_1, \alpha_2, \alpha_3\}$ and $U\backslash R = \{\{\alpha_1\}, \{\alpha_2\}, \{\alpha_2, \alpha_3\}\}$, $X = \{\alpha_1, \alpha_3\}$.

$\tau_R(X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_2, \alpha_3\}\}$. Is $(U, \tau_R(X))$ is a $N_{\mathcal{PO}}$ $-$normal space. The nano closed sets $F = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_1\}\}$.

$$N_{\mathcal{PO}}\,(U, X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_2\}, \{\delta\alpha_3\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}, \{\alpha_2, \alpha_3\}\}.$$
$$N_{\mathcal{PC}}\,(U, X) = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_1, \alpha_3\}, \{\alpha_1, \alpha_2\}, \{\alpha_3\}, \{\alpha_2\}, \{\alpha_1\}\}.$$

$$\{\alpha_1\} \subseteq \{\alpha_1\} \text{ and } \{\alpha_2, \alpha_3\} \subseteq \{\alpha_2, \alpha_3\}, \{\alpha_1\} \cap \{\alpha_2, \alpha_3\} = \emptyset.$$
$$\{\alpha_2\} \subseteq \{\alpha_2\} \text{ and } \{\alpha_1\} \subseteq \{\alpha_1\}, \{\alpha_2\} \cap \{\alpha_1\} = \emptyset.$$
$$\{\alpha_3\} \subseteq \{\alpha_3\} \text{ and } \{\alpha_2\} \subseteq \{\alpha_2\}, \{\alpha_3\} \cap \{\alpha_2\} = \emptyset.$$

$\therefore (U, \tau_R(X))$ is a $N_{\mathcal{P}O}$ $-$normal space.

**Example 3.11:** Let $U = \{\alpha_1, \alpha_2, \alpha_3\}$ and $U/R = \{\{\alpha_1\}, \{\alpha_2\}, \{\alpha_2, \alpha_3\}\}, X = \{\alpha_1\}, \tau_R(X) = \{\emptyset, U, \{\alpha_1\}\}$.

The nano closed set $F = \{U, \emptyset, \{\alpha_2, \alpha_3\}\}$.

$$N_{\mathcal{P}O}(U, X) = \{\emptyset, U, \{\alpha_1\}, \{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3\}\}.$$
$$N_{\mathcal{P}C}(U, X) = \{U, \emptyset, \{\alpha_2, \alpha_3\}, \{\alpha_3\}, \{\alpha_2\}\}.$$
$$\{\alpha_2\} \subseteq \{\alpha_1, \alpha_2\} \text{ and } \{\alpha_3\} \subseteq \{\alpha_1, \alpha_3\}, \{\alpha_1, \alpha_2\} \cap \{\alpha_1, \alpha_3\} \neq \emptyset.$$

$\therefore (U, \tau_R(X))$ is not a $N_{\mathcal{P}O}$ $-$normal space.

## 4. Conclusions

In the paper, Using Pre-open sets, I created a new class of Nano topological spaces, we looked at a few regular and normal notations in pre-open sets in nano topological spaces. Although our notion differs from the traditional nano topological concept, it nonetheless satisfies certain of its characteristics.

## References

[1] Abd El-Fattah, A.El-Atik, and Hanan Z. Hassan. "Some nano topological structures via ideals and graphs." *Journal of the Egyptian Mathematical Society* 28.1 : 1-21 (2020).

[2] Al Nafie, Z.D., Mera, A. *Journal of Advanced Research in Dynamical and Control Systems*, 10(10), pp. 156-158 (2018).

[3] C.Dorsett; "Unique, foundational properties of completely regular, normal and related properties", *J. Mathematical Sciences: Advances and Applications*, Vol 44, pp 111-117 (2017).

[4] Engelking, Ryszard. "General topology" Sigma series in pure mathematics 6 (1989).

[5] H.Tietze; "Beitragezurallgemeinen topology 1", *Math. Ann*, Vol 88, pp 290-312 (1923).

[6] J.M. Aarts and M. Mrsevic; "Pairwise complete regularity as a separation axiom", *J. Australian Mathematical Society*, Vol 48, No 2, pp 235-245 (1990).

[7] Mashhour, A. S. "On precontinuous and weak precontinuous mappings." *Proc. Math. Phys. Soc. Egypt.*. Vol. 53. (1982).

[8] Pawlak, Zdzisław. "Rough sets." *International Journal of Computer & Information Sciences* 11 : 341-356 (1982).

[9] Sathishmohan, P., et al. "More on pre-neighborhoods in nano topological spaces." *Communicated to an International Journal* (2018).

[10] Thivagar, M. Lellis, and Carmel Richard. "On nano forms of weakly open sets." *International Journal of Mathematics and Statistics Invention* 1.1 : 31-37 (2013).

[11] Thivagar, M. Lellis, and Carmel Richard. "Note on nano topological spaces." *Communicated* 1.2.2 : 2-3 (2014).

[12] T.H. Jasiml and L.H.Othman;" On completely normal spaces", *Tikrit J. Pure. Sci*, Vol 24, No 5, pp 111-114 (2018).

[13] Shubham Sharma & Ahmed J. Obaid. Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.

[14] Shelash, Hader Baqer , Ahmad, Hasanain Hamid & Obaid, Ahmed J. On characterizations some of subgroups of the dihedral group D2n, *Journal of Discrete Mathematical Sciences and Cryptography*, 1-6, DOI: 10.47974/JDMSC-1553.

*Received September, 2023*