

Nano M. and nano IM. sets in nano topological spaces

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Abstract

The topic of Nano topology is considered one of the important topics in the field of general topology because it has practical applications in the engineering, medical, economic and other fields. In this research, a Nano mildly closed sets were studied, and its important properties and relationship with both closed and open Nano sets were investigated. The new Nano sets were linked to the concept of Nano ideal, the development of nano ideal mildly closed set and it has been studied its properties. In addition to the applied aspect of the research, a sample was taken from patients infected with viral hepatitis, and by examining the infected people and using closed and open (nano mildly. and nano ideal mildly) sets, the important symptoms that constitute the core of this dangerous disease.

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1. Introduction

An ideal I [1,2,3] on a set Z is a selection of subsets of Z that satisfy the following (1) If $E \in I$ and $D \subseteq E$, then $D \in I$ and (2) If $E \in I$ and $D \in I$ then $D \cup E \in I$. Lellis [4] was the first to introduce the idea of nano sets as follows : Let $Z \neq \emptyset$ be a set and R be an equivalence relation defined on Z and let $D \subseteq Z$, then $L_R(D) = \bigcup_{t \in D} \{R(t); R(t) \subseteq D\}$, $U_R(D) = \bigcup_{t \in D} \{R(t); R(t) \cap D \neq \emptyset\}$ and $B_R(D) = L_R(D) - U_R(D)$, where $L_R(D)$, $U_R(D)$ and $B_R(D)$ are abbreviation of the lower approximation, Upper approximation and the boundary of D with respect to R . By taking an equivalence relation R on an empty set Z , then the subsets of the power set of Z and its symbol $\tau_R(D) =$

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$\{Z, \emptyset, L_R(D), U_R(D), B_R(D)\}$ will be a topology on Z respecting D , where D is any subset of Z and $(Z, \tau_R(D))$ is called nano topological space respecting D [for instance see 5, 6, 7], any member w in $\tau_R(D)$ (resp. $Z - w$) is called nano open (resp. nano closed) and for short n . open (resp. n . closed)

Kuratowski [2], using his closure operator generated a topology, which he called $*$ -topology, which connects the topology τ and the ideal I defined on the same set Z . The triple (Z, τ, I) is called ideal topological space or ideal space see [8,9,10]. M. Parimala et. al. [11] Presented the notation of nano ideal topological spaces and they gave many of its important properties. Of course the nano ideal topological spaces follows the usual topology example of having nano ideal open sets and nano ideal closed sets and other concepts.

K. Bhuvaneshwari [12] present nano generalized sets as follows : A subset E of $(Z, \tau_R(D))$ is termed nano generalized closed (resp. nano generalized open) set if $n.clo(E) \subseteq W$ (resp. $W \subseteq n.Int(E)$) for each W in $\tau_R(D)$ (resp. for each W in $(Z - \tau_R(D))$, containing E (resp. contain in E), where $n.clo(E)$ and $n.Int(E)$ denoted the nano closure and nano interior operators [11,13] of E in $(Z, \tau_R(D))$.

Recall that a subset B of a topological space (Z, τ) is termed mildly closed set [14] if $clo(Int(B)) \subseteq W$ for each generalized open set W [15] in Z which contain B , where $clo(B)$ and $Int(B)$ are the closure and interior operator respectively [16].

In our research, the concept of mildly closed was inserted into nano topology and nano ideal topology where the characteristics of both concepts were studied, the relationship between them were studied, and ample examples were given, supported by tables for clarification. The main goal is the applied aspect, where an important disease, which is viral hepatitis, was studied and the results were compared in nano. topology and nano ideal and the improvement that occurred in them with certainty using the nano mildly sets.

Finally, the symbol $ng.O(Z, \tau_R(D))$ means the collection of all nano generalized open sets in nano, topology on Z respecting D [17].

2. Nano M. closed sets

In this section, we will offer a new closed nano topological and nano.Ideal topological sets. The major properties achieved by these sets will be highlighted. Throughout this section, we will suffice with the symbol $(Z, \tau_R(D))$ to indicate nano topological space respecting a subset D of Z .

Definition 2.1: A subset S of $(Z, \tau_R(D))$ is said to be nano M. closed (briefly; nM .closed) if $n.clo(n.Int(S)) \subseteq L$ if $S \subseteq L$ for each $L \in ng.O(Z, \tau_R(D))$, the complement of nM .closed is termed nM .open. The assortment of all nM .closed (resp. nM . open) sets will be denoted by $nM.C(Z, \tau_R(D))$ (resp. $nM.O(Z, \tau_R(D))$).

Example 2.2: Let $Z = \{k, p, t\}$ and $R = \{(k, k), (p, p), (t, t), (p, t), (t, p)\}$, so $R(k) = \{k\}$ and $R(p) = R(t) = \{p, t\}$. Thus $Z/R = \{\{k\}, \{p, t\}\}$. Consider

$S = \{p\}$, then $L_R(S) = \phi$, $U_R(S) = \{p, t\}$ and $B_R(S) = \{p, t\}$, it follows from this $\tau_R(S) = \{Z, \phi, \{p, t\}\}$. Thus $ng.C = \{Z, \phi, \{k\}, \{k, p\}, \{k, t\}\}$, hence $ng.O(S) = \{Z, \phi, \{p, t\}, \{t\}, \{p\}\}$. Now it is not difficult to determine $nM.C(Z, \tau_R(S))$ which equals $P(Z)$ (The power set of Z).

Remark 2.3:

- (1) Every n . closed set is nM . closed set
- (2) Every n . open set is nM . open

Proof: (1) Let S be a nano. closed subset of $(Z, \tau_R(D))$. Since $n.clo(S) = S$ [18], hence S is $ng.closed$. Thus if $S \subseteq L$ and L is $ng.open$ implies $n.clo(S) \subseteq L$. But $n.Int(S) \subseteq S$ [18], hence $n.clo(n.Int(S)) \subseteq n.clo(S) \subseteq L$. Proof (2) follows from (1) directly.

$nM.closed$ sets have feature as the following theorem clarify.

Theorem 2.4: Let S be a $nM.closed$ subset of $(Z, \tau_R(D))$, then $n.clo(n.Int(S)) - S$ does not contain non. empty $n.closed$ set.

Proof: Let $W \neq \emptyset$ be a $n.closed$ set such that $W \subseteq n.clo(n.Int(S)) - S$. Thus $W \subseteq n.clo(n.Int(S)) \cap S^c$ [19] that's mean $W \subseteq n.clo(n.Int(S))$ and $S \subseteq W^c$. Now since W^c is $n.open$, hence it is $ng.open$ [20] implies $n.clo(n.Int(S)) \subseteq W^c$ and of course $W \subseteq (n.clo(n.Int(S)))^c$. But $W \subseteq n.clo(n.Int(S))$ this is a strak contradiction.

3. Nano ideal M. closed sets

In this section, the two concepts nano ideal and M.closed will be linked. In the remainder of this research, we mean by the symbol $(Z, \tau_R(D), I)$ is nano ideal topological space of a universal set Z respecting a subset D relative to an ideal I on Z . Now we beginning with the following definition.

Definition 3.1: A subset S of $(Z, \tau_R(D), I)$ is called nano ideal M. closed set (briefly: $nIM.closed$) if $n.clo(n.Int(S)) - V \in I$ for each $nM.open$ V of $(Z, \tau_R(D))$. The complement of S is called nano ideal M.open (briefly: $nIM.open$). We will suffice with symbol $nIM.C(Z, \tau_R(D), I)$ to represent the collection of $nIM.closed$ sets, while the symbol $nIM.O(Z, \tau_R(D), I)$ expresses the assortment of all $nIM.open$ sets.

Example 3.2: The universal set Z in $(Z, \tau_R(D), I)$ is $nIM.closed$ since $n.clo(n.Int(Z)) = Z$.

Proposition 3.3: Each $n.closed$ set in $(Z, \tau_R(D))$ is $nIM.closed$ in $(Z, \tau_R(D), I)$.

Proof: Assuming S is $\mathcal{N}.closed$ and $S - V \in I$, where $V \in \mathcal{N}g.O(Z, \tau_R(D))$, then $\mathcal{N}.clo(S) = S$ [21] which leads to $\mathcal{N}.clo(S) - V \in I$. But $\mathcal{N}.clo(\mathcal{N}.Int(S)) \subseteq \mathcal{N}.clo(S)$ [22], hence $\mathcal{N}.clo(\mathcal{N}.Int(S)) - V \subseteq \mathcal{N}.clo(S) - V \in I$, and since I is an ideal implies $\mathcal{N}.clo(\mathcal{N}.Int(S)) - V \in I$, hence S is $\mathcal{N}IM.closed$.

Corollary 3.4: Each $\mathcal{N}.open$ subset of $(Z, \tau_R(D))$ is $\mathcal{N}IM.open$ in $(Z, \tau_R(D), I)$.

Proof: Direct acquisition from (3.3)

Example 3.5. Let $Z = \{e_1, e_2, e_3\}$ and $R = \{(e_1, e_1), (e_2, e_2), (e_3, e_3), (e_1, e_3), (e_3, e_1)\}$. Clear that R is equivalence relation on Z such that $R(e_1) = \{e_1, e_3\} = R(e_3)$, $R(e_2) = \{e_2\}$. Thus the equivalence class is $Z/R = \{\{e_1, e_3\}, \{e_2\}\}$. Therefore the all nano. topological spaces of the universal set Z with respect to all subsets of it are listed in the following Table 1.

Table 1
Nano topological spaces with respect of $P(Z), Z = \{e_1, e_2, e_3\}$

D	$L_R(D)$	$U_R(D)$	$B_R(D)$	$\tau_R(D)$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\{Z, \emptyset\}$
Z	Z	Z	$\emptyset$	$\{Z, \emptyset\}$
$\{e_1\}$	$\emptyset$	$\{e_1, e_3\}$	$\{e_1, e_3\}$	$\{Z, \emptyset, \{e_1, e_3\}\}$
$\{e_2\}$	$\{e_2\}$	$\{e_2\}$	$\emptyset$	$\{Z, \emptyset, \{e_2\}\}$
$\{e_3\}$	$\emptyset$	$\{e_1, e_3\}$	$\{e_1, e_3\}$	$\{Z, \emptyset, \{e_1, e_3\}\}$
$\{e_1, e_2\}$	$\{e_2\}$	Z	$\{e_1, e_3\}$	$\{Z, \emptyset, \{e_2\}, \{e_1, e_3\}\}$
$\{e_1, e_3\}$	$\{e_1, e_3\}$	$\{e_1, e_3\}$	$\emptyset$	$\{Z, \emptyset, \{e_1, e_3\}\}$
$\{e_2, e_3\}$	$\{e_2\}$	Z	$\{e_1, e_3\}$	$\{Z, \emptyset, \{e_2\}, \{e_1, e_3\}\}$

Assuming that $I = \{\emptyset, \{e_3\}\}$ is an ideal on Z and $D = \{e_2\}$. Now the above table is used to obtain the following:

$$\mathcal{N}g.O(Z, \tau_R(D)) = \{\{e_2, e_3\}, \{e_1, e_2\}, \{e_3\}, \{e_2\}, \{e_1\}, Z, \emptyset\}$$

$$\mathcal{N}M.O(Z, \tau_R(D)) = \{\{e_2, e_3\}, \{e_1, e_2\}, \{e_2\}, Z, \emptyset\}$$

$$\text{And } \mathcal{N}IM.O(Z, \tau_R(D), I) = \{\{e_1\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}, Z, \emptyset\}$$

It is obviously the sets $\mathcal{N}g.open$, $\mathcal{N}M.open$ and $\mathcal{N}IM.open$ of the universal set Z according to subset D are not identical.

The concept of kernel via $\mathcal{N}M.open$ and $\mathcal{N}IM.open$ sets will be discussed in this section as a generalization of this concept in the general topological spaces

Definition 3.6: [18] Suppose S is a subset of $(Z, \tau_R(D))$, then the nano. kernel of S and his symbol $\mathcal{N}.ker(S) = \cap \{V; S \subset V, V \in \tau_R(D)\}$.

Definition 3.7: For $(Z, \tau_R(D))$ if $S \subseteq Z$, then we define the nano M. kernal of S, symbolized by $\text{NM}.ker(S) = \cap \{V; S \subseteq V, V \in \text{NM}.O(Z, \tau_R(D))\}$

Definition 3.8: For $(Z, \tau_R(D), I)$ if $S \subseteq Z$, then the nano ideal M. kernal of S, which symbolized by $\text{NIM}.ker(S) = \cap \{V; S \subseteq V, V \in \text{NIM}.O(Z, \tau_R(D), I)\}$

Example 3.9: From example (3.5) if selected the subset $D = \{e_2\}$, so $\tau_R(D) = \{Z, \emptyset, \{e_2\}\}$ and $\text{NIM}.O(Z, \tau_R(D), I) = \{\{e_1\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}, Z, \emptyset\}$

Thus $\text{NIM}.ker(S)$ can be described and listed in the following Table 2, where $S \subseteq Z$.

Table 2
 $\text{NM}.ker(S)$ and $\text{NIM}.ker(S); S \subset Z = \{e_1, e_2, e_3\}$

S	$\text{NM}.ker(S)$	$\text{NIM}.ker(S)$
$\emptyset$	$\emptyset$	$\emptyset$
Z	Z	Z
$\{e_1\}$	Z	$\{e_1\}$
$\{e_2\}$	$\{e_2\}$	$\{e_1, e_2\}$
$\{e_3\}$	Z	$\{e_3\}$
$\{e_1, e_2\}$	Z	$\{e_1, e_2\}$
$\{e_1, e_3\}$	Z	$\{e_1, e_3\}$
$\{e_2, e_3\}$	Z	Z

Remark 3.10: If S is a subset of $(Z, \tau_R(D), I)$, then $\text{NIM}.ker(S) \subseteq \text{N}.ker(S)$.

Proof: Suppose $a \notin \text{N}.ker(S)$, so $a \notin \cap \{V; S \subseteq V, V \in \tau_R(D)\}$, that's mean there exists $V \in \tau_R(D)$ with $S \subseteq V$ and $a \notin V$. By (3.4) we have $V \in \text{NIM}.open$ with respect $(Z, \tau_R(D), I)$.

Thus $a \notin \cap \{V; S \subseteq V, V \in \text{NIM}.O(Z, \tau_R(D), I)\}$ implies $a \notin \text{NIM}.ker(S)$ which complete the proof.

Note: The term $\text{N}.ker(S) \subseteq \text{NIM}.ker(S)$ in general is bogus. From Table (2) if $S = \{e_1, e_2\}$, then $\text{N}.ker(S) = Z$ whilst $\text{NIM}.ker(S) = \{e_1, e_2\}$

It is not intricate to deduce that if $S \subseteq Z$ is $\text{NIM}.open$, then $S = \text{NIM}.ker(S)$. Now we have the following definitions.

Definition 3.11: A subset S of the universal set of $(Z, \tau_R(D), I)$ is called $\text{NIM}. \emptyset$ set if $S = \text{NIM}.ker(S)$.

Definition 3.12: In $(Z, \tau_R(D), I)$ if $V = K \cap S$ where V is any subset of $Z, K \in \text{NIM}.C(Z)$ and S is $\text{NIM}. \emptyset$ set, then V is termed nano ideal $\text{M}\lambda.closed$ (briefly: $\text{NIM}. \lambda.closed$)

A peek at the table (2) and you'll find that $\{e_1\}, \{e_3\}, \{e_1, e_2\}, \{e_1, e_3\}$ are all $\text{NIM}. \emptyset$ sets. If $S = \{e_2\}$, so from example (3.5) we have $\text{NIM}.C(Z, \tau_R(D), I) =$

$\{\{e_2, e_3\}, \{e_1, e_2\}, \{e_3\}, \{e_2\}, Z, \emptyset\}$ and by simple calculation we will find each subset V of Z is $\text{NIM.}\lambda$ closed set.

Proposition 3.13:

- (1) All of $\text{NIM.}\theta$ set is $\text{NIM.}\lambda$ set
- (2) All of $\text{NIM.}\text{closed}$ set is $\text{NIM.}\lambda$ set

Proof:

- (1) Consider S is $\text{NIM.}\theta$ set in $(Z, \tau_R(D), I)$. By reason of $Z \in \text{NIM.}C(Z)$ and $S = S \cap Z$, so it will be the direct conclusion for that S is $\text{NIM.}\lambda$ set.
- (2) In $(Z, \tau_R(D), I)$ assume $S \in \text{NIM.}C(Z)$, so by the reason $Z = \text{NIM.ker}(Z)$ and $S = S \cap Z$ it follows from that S is $\text{NIM.}\lambda$ set.

4. From life application via $\text{NIM.}\text{open}$ sets

Among the infection diseases is viral hepatitis, which is of several types A, B, C, D , and E , and there are other types of damage to the liver are temporary and disappear with recovery from the disease, same are permanent and others damage the liver and lead to death, which is considered more dangerous. Type C leads to the death of (400,000) people every year. The causes of this disease are many, including the use of the infected person's tools, drinking contaminated water, or eating foods washed with contaminated water, etc. As for the symptoms of this disease, they vary: Fever, joint pain, abdominal pain, dark urine yellowing of the skin and white of the eyes, loss of appetite, weakness and fatigue, nausea and vomiting.

In our application below, a sample of (4) people was selected out of (20) patients who suffer from symptoms similar to the above. The reason for selection is to facilitate the calculation and ensure the selection of a sample that differs in symptoms. Six effective symptoms were selected out of the eight symptoms of this dangerous disease. The details of the sample are in the following Table 3:

Table 3
A sample of patients with symptoms of viral hepatitis

S	Abdominal Pain	Dark urine	Joint Pain	Loss of appetite	Yellowing skin & whites of eyes	Fever	result
w_1	√	X	√	√	√	Very high	Yes
w_2	X	√	X	√	X	High	No
w_3	√	√	√	√	√	Normal	No
w_4	X	√	X	√	√	High	Yes

From the above table, if the set of patients $Z = \{w_1, w_2, w_3, w_4\}$, $I = \{\emptyset, \{w_1\}, \{w_3\}, \{w_1, w_3\}\}$ and $D = \{w_2, w_3\}$. Now if w_k and w_t suffer from the same symptoms, then it is certain that $R = \{(w_k, w_t); w_k, w_t \in Z\}$ forms an

equivalence relation and according to the table above it will be $Z/R = \{\{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}\}$, consequently $\tau_R(D) = \{Z, \emptyset, \{w_2, w_3\}\}$, hence

$$ng.O(Z, \tau_R(D)) = \left\{ Z, \emptyset, \{w_2, w_3, w_4\}, \{w_1, w_2, w_3\}, \{w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_4\}, \{w_2\}, \{w_3\}, \{w_1\} \right\}.$$

Thus

$$nM.O(Z, \tau_R(D)) = \left\{ \emptyset, Z, \{w_2, w_3, w_4\}, \{w_1, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_4\}, \{w_3, w_4\}, \{w_3\}, \{w_2\}, \{w_2, w_3\} \right\}$$

and from the last set with the ideal I we get

$$nIM.O(Z, \tau_R(D), I) = \left\{ Z, \emptyset, \{w_2, w_3, w_4\}, \{w_1, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_2, w_3\}, \{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\} \right\}.$$

If the loss of appetite (briefly L) column is neglected, it will be $Z/R_L(D) = \{\{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}\} = Z/R$ and with the same previous steps, we have $nM.O(Z, \tau_{R_L}(D)) = nM.O(Z, \tau_R(D))$ and $nIM.O(Z, \tau_{R_L}(D), I)$, $nIM.O(Z, \tau_R(D), I)$ are identical. If the yellowing skin column is neglected, it will be $Z/R_{yse} = \{\{w_1\}, \{w_2, w_4\}, \{w_3\}\}$, consequently

$\tau_{R_{yse}}(D) = \{Z, \emptyset, \{w_3\}, \{w_2, w_4\}, \{w_2, w_3, w_4\}\}$. Thus $ng.O(Z, \tau_{R_{yse}}(D)) = \left\{ Z, \emptyset, \{w_2, w_3, w_4\}, \{w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_4\}, \{w_2\}, \{w_3\} \right\}$, hence in this case $nIM.O(Z, \tau_{R_{yse}}(D), I) = P(Z)$ (the power set of Z). Now if the column of dark urine (du) is neglected, it will be $Z/R_{du} = \left\{ \{w_1, w_3\}, \{w_2\}, \{w_4\} \right\}$, consequently

$\tau_{R_{du}}(D) = \{Z, \emptyset, \{w_2\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$, so $ng.O(Z, \tau_{R_{du}}(D)) = \left\{ \emptyset, Z, \{w_1, w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2\}, \{w_3\}, \{w_1\} \right\}$ and from this set we get

$nM.O(Z, \tau_{R_{du}}(D)) = \left\{ Z, \emptyset, \{w_2, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2\}, \{w_3\}, \{w_1\} \right\}$ and under

the influence of the ideal itself, it will be $nIM.O(Z, \tau_{R_{du}}(D), I) = \left\{ \emptyset, Z, \{w_2, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2\}, \{w_3\}, \{w_1\} \right\}$ which is obviously distinct from $nIM.O(Z, \tau_R(D), I)$. Finally if column abdominal pain (ap) is omitted, it will be $Z/R_{ap} = Z/R$, hence $\tau_{R_{ap}}(D), ng.O(Z, \tau_{R_{ap}}(D)), nM.O(Z, \tau_{R_{ap}}(D))$ and $nIM.O(Z, \tau_{R_{ap}}(D), I)$ are similar to their counterparts in the first relation before deletion.

From everything mentioned above in this application, we conclude that: the main symptoms through which this disease may develop or the core of symptoms [23] of this disease is the set $core(R) = \{\text{Yellowing skin, Dark urine}\}$. The following Table 4 serves as an extract of the application.

Table 4
Summary of the life applications by use NIM. open sets.

Equivalence Classes	$Z/R = \{\{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}\}$
	$Z/R_L = Z/R$
	$Z/R_{yse} = \{\{w_1\}, \{w_2, w_4\}, \{w_3\}\}$
	$Z/R_{du} = \{\{w_1, w_3\}, \{w_2\}, \{w_4\}\}$
	$Z/R_{ap} = Z/R$
Nano topology	$\tau_R(D) = \{Z, \emptyset, \{w_2, w_3\}\}$
	$\tau_{R_L}(D) = \tau_R(D)$
	$\tau_{R_{yse}}(D) = \{Z, \emptyset, \{w_3\}, \{w_2, w_4\}, \{w_2, w_3, w_4\}\}$
	$\tau_{R_{du}}(D) = \{\emptyset, Z, \{w_2\}, \{w_1, w_3\}, \{w_1, w_2, w_3\}\}$
	$\tau_{R_{ap}}(D) = \tau_R(D)$
ng.O (Z, $\tau_R(D)$)	$\text{ng.O}(Z, \tau_R(D)) = \{Z, \emptyset, \{w_2, w_3, w_4\}, \{w_1, w_2, w_3\}, \{w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_4\}, \{w_2\}, \{w_3\}, \{w_1\}\}$
	$\text{ng.O}(Z, \tau_{R_L}(D)) = \text{ng.O}(Z, \tau_R(D))$
	$\text{ng.O}(Z, \tau_{R_{yse}}(D)) = \{Z, \emptyset, \{w_2, w_3, w_4\}, \{w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_4\}, \{w_2\}, \{w_3\}\}$
	$\text{ng.O}(Z, \tau_{R_{du}}(D)) = \{\emptyset, Z, \{w_1, w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2\}, \{w_3\}, \{w_1\}\}$
	$\text{ng.O}(Z, \tau_{R_{ap}}(D)) = \text{ng.O}(Z, \tau_R(D))$
NIM. O (Z, $\tau_R(D), I$)	$\text{NIM.O}(Z, \tau_{R_L}(D), I) = \{\emptyset, Z, \{w_2, w_3, w_4\}, \{w_1, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_3, w_4\}, \{w_2, w_4\}, \{w_2, w_3\}, \{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}\}$
	$\text{NIM.O}(Z, \tau_{R_L}(D), I) = \text{NIM.O}(Z, \tau_R(D), I)$
	$\text{NIM.O}(Z, \tau_{R_{yse}}(D), I) = P(Z)$
	$\text{NIM.O}(Z, \tau_{R_{du}}(D), I) = \{\emptyset, Z, \{w_2, w_3, w_4\}, \{w_1, w_2, w_4\}, \{w_1, w_2, w_3\}, \{w_2, w_3\}, \{w_1, w_3\}, \{w_1, w_2\}, \{w_2\}, \{w_3\}, \{w_1\}\}$
	$\text{NIM.O}(Z, \tau_{R_{ap}}(D), I) = \text{NIM.O}(Z, \tau_R(D), I)$

5. Conclusion

Throughout this research, nano M.open sets were studied as a generalization to nano open sets in topological spaces. The nano M. open was linked with the concept of the ideal in the second section of the research, and many of the properties enjoyed by the new sets NM and NIM and their relationships with each other and with topological nano sets were presented. These characteristics are enhanced with illustrative examples and tables to study

all subsets from a universal set. It is worth noting that the concept of NM.ker , NIM.ker for nano set was presented due to its importance in nano topology, and special cases of the latter sets, their properties and relationships were investigated, supported by illustrative examples.

When reviewing the research, the reader will find the extent to which the presented nano sets differ from the topological nano sets, especially in the last section, which concerns the medical application and calculating the core of viral hepatitis. A sample was taken from four patients suffering from symptoms similar to those of viral hepatitis, and by calculating the nano topology, the main symptoms for diagnosing the disease were not obtained, but in the open nano sets NM and NIM the main symptoms for diagnosis were identified.

References

- [1] R. Vaidyanathaswamy, "The localisation theory in set topology" *Proc. Indian Acad. Sci.*, Vol. 20. pp. 51- 61 (1945).
- [2] K. Kuratowski, "Topology" Vol. I New York : Academic Press (1966).
- [3] P. Samuels, "A topology formed from a given topology and ideal" *Journal of the London Mathematical Society*, Vol.2, no. 4, pp. 409 – 416 (1975).
- [4] T. M. Lellis and C. Richard, "Note on nano topological spaces" *Communicated* 1, Vol.2, pp. 2-3, (2014).
- [5] R. Vijayalakshmi and A. P. Mookambika, "Some new nano sets in nano topological Spaces" In AIP Conference Proceeding, AIP Publishing, Vol. 2177, No.1 (2019).
- [6] J. Saeid and M. Parimala, "On some new notions in nano ideal topological spaces" *Eurasian Bulletin of Mathematics* (Issn : 2687 – 5632), pp. 85 – 93 (2018).
- [7] H. J. Ali and S. I. Mahmood, "New generalization of soft LC.- space" *Iraqi Journal of Science*, Vol. 63, no. 11, pp. 4890 – 4900 (2022).
- [8] S. Modak, "Some new topologies on ideal topological spaces" *Proceeding of the National Academy of Science, India Section A : Physical Science*, Vol. 82, pp. 233-243 (2012).
- [9] Y.Y. Yousif and A . H. Mohammed, "Fibrewise soft ideal topological Space" *Journal of Physics : Conference Series*, Iop Publishing, Vol. 1003, pp. 1-12 (2018), doi 10. /088/1742-6596/1/012050.
- [10] R. manoharan and P. Thangavelu, "Some new sets and topologies in ideal topological spaces" *Chines Journal of Mathematics*, Vol. 2013, pp. 1-6 (2013).

- [11] M. Parimala, T. Noiri and S. Jafar, "New types of nano topological spaces via nano ideals" To appear (2016).
- [12] K. Bhuvaneshwari and K.M. Gnanapriya, "Nano generalized closed sets in nano topological spaces" *International of Scientific and Research Publications*, Vol. 4, no.5, pp. 2250 – 2258 (2014).
- [13] A. H. Hwaidi, J. H. Hussein, " λ - semi – open sets in generalized topological spaces" *AIP Conference Proceedings*, 2398, 060078 (2022).
- [14] A. R. Sadek and J.H. Hussein, "On Mildly Regular Space" *International Mathematical Forum*, Vol. 12, no. 20, pp. 967 – 972 (2017).
- [15] N. Levine, "Generalized closed sets in topology" *Rend . Circ. Math. Palermo*, Vol. 19, pp. 89 – 96 (1970).
- [16] S. A. Morris, "Topology without tears" *University of New England* (1989).
- [17] S. M. Sandhya, S. Jeyashri, S. Ganesan and C. Alexander, "Between nano closed sets and nano generalized closed sets in nano topological spaces" *Advances in mathematics : Scientific Journal*, Vol. 9, no. 3, pp. 757 – 771 (2020).
- [18] I. Rajasekaran and O. Nethaji, "On some new subsets of nano topological spaces" *Journal of New Theory*, Vol. 6, pp. 52 – 58 (2017).
- [19] G. Gladle, "Arithmetic, The Foundation of Mathematics" *The Rosen Publishing Group, Inc. Encyclopaedia Britannica* (2015).
- [20] K. Bhuvaneshwari and A. Ezhilarasi, "On nano semi – generalized and nano generalized – semi closed sets in nano topological spaces" *International Journal of Mathematics and Computer Applications Research*, Vol.4, no.3, pp.117 – 124 (2014).
- [21] A.R. Sadek and M. A. Abdel Karim, "Some games via semi – generalized regular spaces" *Journal of Interdisciplinary Mathematics*, Vol. 26, no. 5, pp. 862 – 872 (2023).
- [22] M.L. Thivagar and C. Richard, "On nano forms of weakly open sets" *International Journal of Mathematics and Statistics Invention*, Vol. 1, no. 1, pp. 31 – 37 (2013).
- [23] E. A. Mohsin and Y. Y. Yousif., "Nano perfect mappings" *Journal of Interdisciplinary Mathematics*, Vol. 26, no. 7, pp. 1431–1437 (2023).

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