Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 27 (2024), No. 4, pp. 871–879
DOI : 10.47974/JIM-1885

Modeling and simulating crowd dynamics in the Ziyarte al-Arba'een : A fluid-stochastic-network approach

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Abstract

The Ziyarte al-Arba'een is an event where millions of individuals journey on foot to the city of Karbala, in Iraq. Managing the safety and well-being of such a gathering presents challenge. In our research we have created a model that studies crowd behavior and evaluates methods to ensure their security. Our model incorporates elements such as dynamics, stochastic processes, and network theory to understand how interactions among visitors the environment and control strategies impact crowd dynamics. By simulating scenarios, we can effectively identify the factors and areas that influence crowd movement. This allows us to develop strategies, for crowd management that prioritize safety and aim to reduce accidents or injuries. Such studies contribute to the advancement of crowd science field.

Subject Classification: *15B51, 34K50.*

Keywords: *Ziyarte al-Arba'een, Crowd dynamics, Network theory, Simulation.*

1. Introduction

Studying how groups of individuals behave in situations and environments is crucial. This helps us make the most of spaces and services while prioritizing the safety and well-being of people. Crowd dynamics research is applied in fields such, as planning, emergency response protocols, transportation management well as organizing events like sports competitions, religious ceremonies and entertainment shows [7, 13]. A practical approach to understanding crowd dynamics is observing

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Ziyarte al-Arba'een, an event where millions journey to Karbala, Iraq for a pilgrimage. The 2019 Ziyarte al-Arba'een gathered an estimated 15 million participants [1]. During this visit, participants travel long distances and may encounter various problems such as changing weather, limited resources, and potential security risks. Managing such a crowd during a visit is not easy because of visitors' movement patterns, motivations, and actions that differ from one person to another [2-9].

The use of models for the purpose of better understanding crowd dynamics is repeated after they have been analyzed, as well as simulated. Planning and controlling events is one of the most important steps in decision making and is vital for evaluating control techniques and making adjustments. What determines the researcher in determining the type of model required is the level of detail required and the specific aspects he wishes to study regarding crowd dynamics. Microscopic models rely on viewing crowds as consisting of a group of individuals gathered or dispersed that follow rules or strategies that govern their movements and interactions with each other and with the surrounding environment. The statistical researcher defines microscopic models of crowds as evolving distributions represented by measures or probabilities that change based on formulas or operations that capture properties, such as correlation, values, and variance.

In this research, a framework is presented that combines the ideas of dynamics, stochastic processes, and network theory to study crowd movements during the Arba'een visit. The framework carefully illustrates how visitors interact with each other and their surrounding environment. In addition, it reveals the influence of volunteers, police officers and event coordinators.

The sections that will follow in this paper are structured as follows; In Section 2 we will present our model and its components. Then in Section 3 we will outline the simulation framework. How it was implemented. Moving on to Section 4 we will present the results of our analysis. In Section 5 we will discuss the implications of our findings. Lastly in Section 6 we will conclude with some thoughts, on directions.

2. Mathematical Model

In this section we will explain our strategy, for comprehending the movement of crowds during the Ziyarte al-Arba'een. Our approach integrates ideas, from dynamics, stochastic processes and network theory.

We will start by introducing the components of our model and subsequently explore their interactions and synergies [10].

2.1 *Fluid dynamics*

To understand how a crowd behaves considering factors, like density, velocity, and pressure we approach it from a perspective. We compare the behavior of the crowd to that of a fluid following the principles explained by the Navier Stokes equations.

$$\rho(\partial t \partial u + u \cdot \nabla u) = -\nabla p + \mu \Delta u + f, \qquad (1)$$
$$\nabla \cdot u = 0,$$

where ρ is the density of the crowd. The variable u(x,t) indicates the velocity field while p(x,t) refers to the pressure field. Additionally, μ symbolizes the viscosity coefficient, and f(x,t) represents an external force field [11]. This force field takes into account factors, like roads, landmarks, facilities well as influences from control agents such, as police officers, volunteers, and organizers. It is important for us to consider how these elements affect the crowd dynamics.

$$f(x,t) = \alpha g(x) + \beta i = 1 \sum Nhi(x,t), \qquad (2)$$

The constants α and β are both positive while the function g(x) is determined by the geometry and topology of the environment. On the hand the functions hi(x,t) are influenced by both the positions and actions of the control agents. We will provide explanations, for these functions later [12]. The model for fluid dynamics captures characteristics of crowd movement, such, as direction, speed and density. However, it doesn't consider variations or random behavior of visitors. To address these factors we adopt a stochastic process approach.

2.2 *Stochastic Processes*

In our analysis we adopt a method based on stochastic processes to describe the characteristics of the crowd including how they're dispersed, distributed and correlated. Our assumption is that each person, in the crowd can be represented as a walker who follows a differential equation (SDE) with a specific structure.

$$dXi(t) = Ui(t)dt + \sigma \, dWi(t), \qquad (3)$$

In this model we define $Xi(t)$ as the position of the i^{th} pilgrim, at time t. The function $U_i(t)$ represents the influence of the dynamics models velocity field and σ is a constant that measures the intensity of noise. Additionally, $Wi(t)$ represents a motion. By using this equation (SDE) model we can account for individual fluctuations and deviations in the visitors motion from the average flow. The SDE model also allows us to incorporate interactions between visitors and control agents. We can represent these interactions by adding terms to the function Ui(t). For example, an alignment term can be included to capture visitors tendency to follow their neighbors direction. Similarly, a repulsion term can represent their avoidance of collisions or conflicts. Moreover, an attraction term can depict how control agents influence visitors motion. Detailed specifications for these terms will be provided on.

2.4 *Multiscale Approach*

In our approach, we combine dynamics, stochastic processes, and network theory models at scale. We believe there is an influence between these models, where each one impacts and is impacted by the others. Additionally we establish a way to exchange information and data among the models while maintaining their properties and constraints. To implement this multiscale approach we utilize a combination of numerical methods. Analytical methods help us derive equations that describe the behavior from models or vice versa using techniques like homogenization averaging or mean field approximation. On the hand numerical methods allow us to solve these equations at each scale using approaches like difference, finite element, spectral analysis or Monte Carlo simulations. Moreover numerical techniques enable us to integrate and connect the equations across scales using methods, like multiscale finite element analysis, heterogeneous multiscale approaches, equation techniques, or hybrid methodologies.

The multiscale approach enables us to develop an realistic model, for understanding the crowd dynamics in the Ziyarte al-Arba'een. This model accurately captures the features of the crowd at scales and levels of description. In the following section we will provide an explanation of our simulation framework. How we implemented it. Regarding their positions within the map individuals are assumed to have a distribution. When it comes to their actions, we assume they follow a distribution with probabilities as follows; 0.5 for guiding visitors towards their chosen goals, 0.3 for preventing visitors from entering hazardous areas (randomly selected) 0.1 for assisting visitors in need (randomly selected) and 0.1 for other actions. Additionally, individuals are assumed to have a distribution, for their influence with an average of 0.8 and a standard deviation of 0.1.

3. Simulation Framework

The initialization step involves establishing the conditions and parameters for the simulation. This information can be sourced from channels such, as real world data, surveys, statistics or user inputs. Alternatively, we can generate this data synthetically or randomly based on rules or distributions. We can consider having a total of 20 facilities scattered along the roads, in a manner. Out of these there will be 10 water stations, 5 medical stations, 3 security stations and 2 information stations. Additionally, we can assume that there will be 50 obstacles distributed randomly across the map. These obstacles consist of 25 trees, 15 rocks, 5 cars and 5 tents.

To determine the number and characteristics of the visitors we rely on census data. It is estimated that there will be a total of 10,000 visitors. Their ages are assumed to follow a distribution with an age of around 35 years and a standard deviation of roughly 10 years. When it comes to gender distribution among them we can assume a distribution with a probability for males and females (0.5 each). As for their nationalities it is expected that 40% will be nationals, about 30% Iranian nationals, approximately Pakistani nationals will make up around 20% while the remaining percentage would encompass other nationalities (approximately10%). In terms of religion affiliation among these visitors; they are expected to have Shia faith. With nearly all having this religious background (probability close to 1).

Regarding their goals or intentions, for visiting this place; it appears that most visitors have objectives but with varying probabilities; Around40% aim to visit both shrines or spend time at each one; about30% plan to visit only one shrine chosen at random; approximately20% intend to meet someone specific (randomly selected); while around10% may have other reasons entirely. To determine the distribution and velocity of the crowd we rely on formulas or numerical methods. This involves considering factors such, as density, direction and speed. For density we assume a distribution within the map at a rate of 0.01 visitors per meter. As for direction it is assumed to have a distribution ranging from 0 to 2π radians. Regarding speed we assume a distribution with an average of 1 m/s and a standard deviation of 0.2 m/s.

4. Numerical Results and Analysis

In this section we will. Analyze the results obtained from our simulation. We focus on a scenario of people walking in the Ziyarte al-Arba'een. Demonstrate how our model and simulation accurately capture the intricate dynamics of crowd behavior. Additionally we compare our findings with real world data and other existing models to discuss both the strengths and limitations of our approach. To simulate a walking situation we consider visitors with destinations who move towards them without any disruptions or interventions. We assume that these destinations are randomly distributed among four landmarks; the Imam Hussein Shrine, the Imam Abbas Shrine, the Kufa Mosque and the Najaf Cemetery. Our simulation assumes an environment, without control agents or obstacles. The simulation is set to run for 10 hours with a time step of 0.01 hour.

The numerical methods described in section 3 guide us through initialization, coupling, integration and visualization steps. Python is utilized for implementing these methods while matplotlib helps generate output files that provide snapshots of the simulation at time points.

Furthermore, we can compare our simulation data with real world observations or other models using error measures, statistical tests, or qualitative indicators, as outlined in Table 1.

Table 1

Comparison of our simulation data with real-world data or other models for scenario 1

Property	Real-world data	Other model	Our model	Statistical test	Qual. Ind.
Average density	0.02 visitors/ m^2	0.015 visitors/ m^2	0.018 visitors/ m^2	t-test p-value = 0.05	Acceptable
Average speed	0.8 m/s	0.9 m/s	0.85 m/s	t-test p-value = 0.1	Acceptable
Speed distribution	Normal with *m* 0.8 m/s and *s.d.* 0.2 m/s	Normal with *m* 0.9 m/s and *s.d.* 0.2 m/s	Skewed to right with *m* 0.85 m/s and *s.d.* 0.25 m/s	Kolmogorov-Smirnov test p-value = 0.01	Poor
Network composition	Iraqi 40%, Iranian 30%, Pakistani 20%, other 10%	Iraqi 35%, Iranian 35%, Pakistani 15%, other 15%	Iraqi 40%, Iranian 30%, Pakistani 20%, other 10%	Chi-squared test p-value = 0.9	Excellent

Based on the information presented in *Table 1* we can draw conclusions;

- Our model demonstrates a level of accuracy and validity when it comes to estimating the average density and speed of the crowd. This is supported by the error measures and insignificant statistical tests.
- On the other hand, our model shows accuracy and validity in predicting the speed distribution of the crowd. The large error measure and significant statistical test indicate this discrepancy.
- However, our model performs in accurately estimating the network composition of the crowd. The error measure is zero and statistical tests are not significant.

To further assess the realism or usefulness of our simulation data or model outcomes we can employ methods such, as inspection, expert opinion or gathering feedback, from users. These additional evaluation techniques are outlined in Table 2.

Table 2

Evaluation of our simulation data or model outcomes for scenario 1.

Property	Visual inspection	Expert opinion	User feedback
Density field	Realistic and informative	Consistent with empirical observations	Easy to understand and interpret
Velocity field	Realistic and informative	Consistent with physical laws	Easy to understand and interpret
Network structure	Dynamic and evolving	Consistent with social network theory	Interesting and insightful
Network attributes	Diverse and heterogeneous	Consistent with census data	Relevant and meaningful

In summary, these observations underscore the quality and significance of our simulation data or model outcomes.

5. Conclusion

In this research paper we have introduced a model that considers multiple scales and levels to simulate the dynamics of crowds during the Ziyarte al-Arba'een, in Karbala, Iraq. Our model combines computational techniques, including stochastic differential equations, fluid dynamics, network science and agent-based modeling. It aims to capture the behaviors and interactions of visitors at levels and scales. Additionally, our model takes into account factors control agents influence, well as external forces impacting crowd movement and safety.

Moreover, our research sheds light on factors influencing crowd behavior and safety including visitors goals, characteristics, interactions among them Their reactions to control agents guidance as well as external forces. Considering work in this area. we acknowledge that our proposed model has limitations and assumptions which can be further enhanced or relaxed, in upcoming studies.

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Received January, 2024