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Convex fuzzy distance between two convex fuzzy compact set

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Abstract

The first step of this study is to recall the definition and basic theorems for convex fuzzy metric space $(\mathcal{Z}, \mathcal{M})$. Then, the definition of the convex fuzzy distance between two sets $\mathcal{F}$ and $\mathcal{Y}$ as well as the convex fuzzy distance from a point z to a set $\mathcal{Y}$ are introduced. Furthermore, the set of all nonempty convex fuzzy compact sets $\mathrm{CFC}(\mathcal{Z})$ are proved as a convex fuzzy complete when the convex fuzzy metric space $(\mathcal{Z}, \mathcal{M})$ is convex fuzzy complete.

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1. Introduction

A generalization of the statistical metric space introduced by Kramosil and Michálek, which is known as a fuzzy metric space [1]. A necessary and sufficient condition for a fuzzy metric space to be complete presented by A. George and P. Veeramani [2]. Z. Q. Xia and F. F. Guo used real numbers to define fuzzy metric space also; they proved the equivalence between complete metric space and complete fuzzy metric space [3]. Y. Shen, D. Qiu and W. Shen prove several fixed-point theorems for a new class of self-maps in M-complete fuzzy metric spaces [4]. S. Ferhan introduces the notion of soft fuzzy metric and he proved some basic properties of the soft fuzzy metric spaces [5].

Further, M. Dinarvand introduced the fuzzy geraghty contraction type mapping and fuzzy contractive mapping then proved results on the existence and uniqueness of fixed points for these two types of mappings on fuzzy metric spaces [6].

The fuzzy T-metric and fuzzy S-metric are introduced by M. Ralevic [7]. A. B. Khalaf and M. Waleed introduce a new type of fuzzy metric spaces and they obtained several properties of it [8]. Furthermore, F. S. Fadhel et al. proved two contraction-mapping theorems on partial fuzzy metric spaces by using the suggested distance function that is defined over fuzzy points rather than fuzzy sets and then studied the relationship between the usual distance function defined over fuzzy metric spaces and the distance function defined over partial metric space [9]. K. Javed, A. Asif, and E. Savas presented orthogonal picture fuzzy metric space also they proved basic theorems for this space [10]. The investigate of two fixed point theorems related with two different contractive mappings in addition to some theoretical results concerning fuzzy metric spaces, as well as, studying their relationship with nonfuzzy or crisp metric spaces where given by J. H. Eidi et al. [11].

Let us give a brief outline of the contents of the paper: in Section 2 we recall some basic concept for convex fuzzy metric space. In Section 3 we study the convex fuzzy distance between two sets as well as point to set and give several properties. Finally, we prove that the convex fuzzy compact sets are convex fuzzy complete when the convex fuzzy metric space is convex fuzzy complete**2.**

2. Convex Fuzzy Metric Space

Let us now introduce some definitions, theorems and remarks, which help to improve the following results. The notion of convex fuzzy metric as well as several properties can be found in [12]. Therefore, all the following definitions and theorems were taken from [12].

Definition 1: If the fuzzy set $\mathcal{M}: \mathcal{Z} \times \mathcal{Z} \to [0, 1]$ and $\mathcal{Z} \neq \emptyset$ satisfies

(i) $\mathcal{M}(y, v) \in [0, 1]$ if $y \neq v$;

(ii) $y = v \Leftrightarrow \mathcal{M}(y, v) = 0$;

(iii) $\mathcal{M}(y, v) = \mathcal{M}(v, y)$;

(iv) $\alpha\mathcal{M}(y, w) + \beta\mathcal{M}(w, v) \geq \mathcal{M}(y, v)$
$\forall$ 0< α, β <1 with $\alpha + \beta$ =1 and $\forall$ $y, v, w \in \mathcal{Z}$.
Hence, $(\mathcal{Z}, \mathcal{M})$ is said to be **c-**FMS (or convex fuzzy metric space)

Remark 1: If $\delta, \sigma \in [0, 1]$ then $(\alpha\delta + (1-\alpha)\sigma) \in [0, 1]$ $\forall$ $\alpha \in [0, 1]$.
In general if $\sigma_1, \sigma_2, \ldots, \sigma_k \in [0, 1]$ then $(\alpha_1\sigma_1 + \alpha_2\sigma_2 + \ldots + \alpha_k\sigma_k) \in [0, 1]$ $\forall$ $\alpha_1, \alpha_2, \ldots, \alpha_k \in [0, 1]$ with $\alpha_1 + \alpha_2 + \ldots + \alpha_k = 1$.

Definition 2: If $(\mathcal{Z}, \mathcal{M})$ is c-FMS then
(1) $\mathfrak{B}(u, \alpha) = \{y \in \mathcal{Z}: \mathcal{M}(u, y) < \alpha\}$ is open fuzzy ball;
(2) $\overline{\mathfrak{B}}(u, \alpha) = \{y \in \mathcal{Z}: \mathcal{M}(u, y) \leq \alpha\}$ is closed fuzzy ball;
with $u \in \mathcal{Z}$ is the center as well as $\alpha \in (0, 1)$ is the radius.

Definition 3: If (($\mathcal{Z}, \mathcal{M}$) is a c-FMC then $\mathcal{F} \subseteq \mathcal{Z}$ is convex fuzzy open if $\mathfrak{B}(f, \alpha) \subseteq \mathcal{F}$ for $0 < \alpha < 1$ and for any $f \in \mathcal{F}$.

Definition 4: If $(\mathcal{Z}, \mathcal{M})$ is a c-FMC then
(i) $\mathfrak{C} \subseteq \mathcal{Z}$ is convex fuzzy closed if $\mathfrak{C}^C$ is fuzzy;
(ii) $\overline{\mathfrak{C}} = \cap\{\mathfrak{D}$ is fuzzy closed and $\mathfrak{C} \subseteq \mathfrak{D}\}$;

Definition 5: The set $\mathcal{D}$ of $\mathcal{Z}$ is convex fuzzy dense in $\mathcal{Z}$ if $\overline{\mathcal{D}} = \mathcal{Z}$ when $(\mathcal{Z}, \mathcal{M})$ is a c-FMS.

Theorem 1: *In c-FMS $(\mathcal{Z}, \mathcal{M})$ every open fuzzy ball $\mathfrak{B}(w, \alpha)$ is a fuzzy open set.*

Definition 6: In c-FMS $(\mathcal{Z}, \mathcal{M})$, a sequence (u_k) is convex fuzzy converge to u in $\mathcal{Z}$(or simply $u_k \to u$, $\lim_{k\to\infty} u_k = u$) if for each 0< α <1, $\exists$ $\mathcal{N}$ with $\mathcal{M}(u_k, u) < \alpha$, $\forall$ k$\geq \mathcal{N}$.

Proposition 1: In c-FMS $(\mathcal{Z}, \mathcal{M})$, $u_k \to u$ if and only if $\mathcal{M}(u_k, u) \to 0$.

Definition 7: In c-FMS $(\mathcal{Z}, \mathcal{M})$, a sequence (u_k) is convex fuzzy Cauchy if $\forall$ 0< α <1, $\exists$ $\mathcal{N}$ satisfying

$$\mathcal{M}(u_n, u_k) < \alpha, \forall \text{ k, n} \geq \mathcal{N}.$$

Definition 8: If (u_k) is fuzzy Cauchy sequence in c-FMS $(\mathcal{Z}, \mathcal{M})$ satisfying $\exists$ $y \in \mathcal{Z}$ with $u_k \to y$ then $\mathcal{Z}$ is convex fuzzy complete.

Proposition 2: In c-FMS $(\mathcal{Z}, \mathcal{M})$, if $u_k \to y \in \mathcal{Z}$ then (u_k) is fuzzy Cauchy.

Definition 9: In c-FMS $(\mathcal{Z}, \mathcal{M})$
(i) The set $\mathcal{D} \neq \emptyset$ in $\mathcal{Z}$ is convex fuzzy bounded if $\exists$ $\alpha \in (0, 1)$ with $\mathcal{D} \subset \mathfrak{B}(u, \alpha)$ for some $u \in \mathcal{Z}$.

(ii) The sequence (d_k) in $\mathcal{Z}$ is convex fuzzy bounded if $\exists\ \alpha \in (0, 1)$ with $(d_k) \in \mathfrak{B}(u,\alpha)$ for some $u \in \mathcal{Z}$

Theorem 2: *In c-FMS* $(\mathcal{U},m)$

(i) if $(u_k) \in \mathcal{Z}$ with $u_k \to u \in \mathcal{Z}$ then (u_k) is convex fuzzy bounded.

(ii) if $(u_k) \in \mathcal{Z}$ with $u_k \to u \in \mathcal{Z}$ and $u_k \to d \in \mathcal{Z}$ as $k \to \infty$. Then $u=d$.

Theorem 3: *In c-FMS* $(\mathcal{Z}, \mathcal{M})$ *if* $\mathfrak{D} \subset \mathcal{Z}$ *then*

(i) $\exists\ (d_k) \in \mathfrak{D}$ satisfying $d_k \to d$.;

(ii) $d \in \overline{\mathfrak{D}}$;

Are equivalent.

Theorem 4: *In c-FMS* $(\mathcal{Z}, \mathcal{M})$ *if* $\mathfrak{D} \subset \mathcal{U}$ *then*

(i) $\forall\ u \in \mathcal{M}, \exists\ d \in \mathfrak{D}$ satisfies $\mathcal{M}(u, d) < \alpha$ for some $\alpha \in (0, 1)$.

(ii) $\overline{\mathfrak{D}} = \mathcal{Z}$.

Are equivalent.

Definition 10: Assume that $(\mathcal{Z},\mathcal{M}_{\mathcal{Z}})$ and $(\mathcal{F}, \mathcal{M}_{\mathcal{F}})$ are two c-FMS. If $\forall\ 0 < \alpha < 1, \exists\ 0 < \sigma < 1$, satisfying $\mathcal{M}_{\mathcal{F}}[\mathcal{T}(u), \mathcal{T}(b)] < \alpha$ as $b \in \mathcal{Z}$ and if $\mathcal{M}_{\mathcal{Z}}(w, b) < \sigma$ then the function $\mathcal{T}:\mathcal{Z}\to\mathcal{F}$ is convex fuzzy continuous at $u \in \mathcal{Z}$.

If $\forall\ z \in \mathcal{Z}$ this is satisfied then $\mathcal{T}$ is fuzzy continuous on $\mathcal{Z}$.

Theorem 5: *In* $c - FMS\ (\mathcal{Z},\mathcal{M})$ *if* $(u_k) \in \mathcal{Z}$ *with* $u_k \to u \in Z$ *as* $k \to \infty$ *and* $(w_k) \in \mathcal{Z}$ *with* $w_k \to w \in \mathcal{Z}$ *as* $k \to \infty$. *Then* $M(u\_k, w\_k) \to M(u, w)$ *as* $k \to \infty$.

3. Results and discussion

First, let us introduce the following two definitions:

Definition 11: A c-FMS $(\mathcal{Z},\mathcal{M})$ is convex fuzzy compact if and only if $\forall\ (y_k)$ in $\mathcal{Z}$ has a subsequence (y_{k_i}) such that $y_{k_i} \to z \in \mathcal{Z}$.

Definitions 12: Let $\mathcal{K}$ be a subset of c-FMS $(\mathcal{Z},\mathcal{M})$ and let $\alpha \in (0, 1)$.

(i) A set $\mathcal{B}_\alpha \subseteq \mathcal{K}$ is a **convex α-fuzzy net** for $\mathcal{K}$ if $\forall\ h \in \mathcal{K}\ \exists\ b \in \mathcal{B}_\alpha$ satisfying $\mathcal{M}(h, b) < \alpha$.

(ii) The set $\mathcal{K}$ is called totally convex fuzzy bounded if $\forall\ \alpha \in (0, 1), \exists$ a finite convex α-fuzzy net $\mathcal{B}_\alpha = \{b_1, b_2, \ldots, b_k\} \subseteq \mathcal{K}$.

Now, we introduce some properties for the convex fuzzy metric space.

Lemma 1: *Let* $(\mathcal{Z},\mathcal{M})$ *be a c-FMS and* $\mathcal{K} \subseteq \mathcal{Z}$. *If* $\mathcal{Z}$ *is totally convex fuzzy bounded then* $\mathcal{K}$ *is totally convex fuzzy bounded.*

Proof: If $\mathcal{K}=\emptyset$ is clear. Let $\mathcal{K} \neq \emptyset$ and let $\alpha \in (0, 1)$ be given, there is a finite α_1-fuzzy net $B_{\alpha_1} = \{z_1, z_2, \dots, z_k\} \subseteq \mathcal{Z}$ for $\mathcal{Z}$ where $\alpha_1 = \frac{\alpha}{2}$. Hence $\mathcal{K} \subseteq \cup_{j=1}^{k}$ cfb(z_j, α_1). Let B_j= cfb(z_j, α_1) for j=1,2,…,k, which intersect $\mathcal{K}$. Let $\hbar_j \in (\mathcal{K} \cap B_j)$. Thus $B_\alpha = \{\hbar_1, \hbar_2, \dots, \hbar_k\} \subseteq \mathcal{K}$ is an convex α-fuzzy net for $\mathcal{K}$ because for every $\hbar \in \mathcal{K}$ there is $z_j \in B_j$ and $\mathcal{M}(\hbar, \hbar_j) \leq \delta_1 \mathcal{M}(\hbar, z_j) + \delta_2 \mathcal{M}(z_j, \hbar_j) \leq \delta_1\alpha_1 + \delta_2\alpha_1 = \alpha_1 < \alpha$, where $\delta_1 + \delta_2 = 1$

Theorem 6: *Let $(\mathcal{Z},\mathcal{M})$ be a c-FNS. If $\mathcal{Z}$ is totally convex fuzzy bounded and convex fuzzy complete then $\mathcal{Z}$ is convex fuzzy compact.*

Proof: Let (z_k) be a sequence in $\mathcal{Z}$ since $\mathcal{Z}$ is totally convex fuzzy bounded then $\forall\ \alpha \in (0, 1)$ there is a finite convex α-fuzzy net $\mathcal{B}_\alpha = \{b_1, b_2, \dots, b_k\} \subseteq \mathcal{Z}$ satisfying $\mathcal{M}(z_{k_j}, b_j) < \alpha$. Now $\mathcal{M}(z_{k_j}, z_{k_m}) < \gamma\ \mathcal{M}(z_{k_j}, b_j) + \delta\ \mathcal{M}(b_j, z_{k_m}) = \gamma\alpha + \delta\alpha = \alpha$ where $\gamma + \delta = 1$.

Thus, the subsequence (z_{k_j}) is convex fuzzy Cauchy in $\mathcal{Z}$ Cauchy but $\mathcal{Z}$ is convex fuzzy complete. Thus $z_{k_j} \to z \in \mathcal{Z}$. Hence $\mathcal{Z}$ is convex fuzzy compact.

Notation 1: Suppose that $(\mathcal{Z},\mathcal{M})$ is a c-FMS and let CFC($\mathcal{Z}$)= $\{\mathcal{Y} : \mathcal{Y} \neq \emptyset, \mathcal{Y} \subset \mathcal{Z}, \mathcal{Y}$ is convex fuzzy compact$\}$.

Definition 13: Suppose that $(\mathcal{Z},\mathcal{M})$ is a c-FMS then

(i) Define $\mathcal{M}(p, \mathcal{K}) = \inf\{\mathcal{M}(p, \hbar) : \hbar \in \mathcal{K}\}$ where $\mathcal{M}(p, \mathcal{K})$ is called the **convex fuzzy distance from p to $\mathcal{K}$** . It is clear that $\mathcal{M}(p,\mathcal{K}) = \mathcal{M}(\mathcal{K}, p) = \inf\{\mathcal{M}(\hbar, p) : \hbar \in \mathcal{K}\}$ since $\mathcal{M}(p,\hbar) = \mathcal{M}(\hbar, p)$.

(ii) Define $\mathcal{M}(\mathcal{F}, \mathcal{K}) = \max\{\mathcal{M}(f, \hbar) : f \in \mathcal{F}$ and $\hbar \in \mathcal{K}\}$ where $\mathcal{M}(\mathcal{F}, \mathcal{K})$ is called the **convex fuzzy distance from the set $\mathcal{F}$ to $\mathcal{K}$**. It is clear that $\mathcal{M}(\mathcal{F}, \mathcal{K}) = \mathcal{M}(\mathcal{K}, \mathcal{F}) = \max\{\mathcal{M}(\hbar, f) : \hbar \in \mathcal{K}$ and $f \in \mathcal{F}\}$ since $\mathcal{M}(f, \hbar) = \mathcal{M}(\hbar, f)$

Remarks 2:

(i) For existence of $\hbar \in \mathcal{K}$ satisfying $\mathcal{M}(f, \mathcal{K}) = \mathcal{M}(f, \hbar)$ notice that the set of real numbers $\Gamma = \{\mathcal{M}(f, b) : b \in \mathcal{K}\} \subset [0, 1]$ has a infinmum point we prove $(\inf\Gamma) \in \Gamma$, define $\mathcal{L}_f : \mathcal{K} \to [0, 1]$ by $\mathcal{L}_f(b) = \mathcal{M}(f, b)$ for all $b \in \mathcal{K}$ then $\mathcal{L}_f$ is convex fuzzy continuous since $\mathcal{M}$ is convex fuzzy continuous. Consider $\alpha = \inf\{\mathcal{L}_f(b) : b \in \mathcal{Y}\}$ for existence of $\hbar \in \mathcal{K}$ satisfying $\mathcal{M}(f, \hbar) = \alpha$ we can find sequence (b_k) in $\mathcal{K}$ such that $A_{\mathbb{R}}(\mathcal{L}_f(b_k), \hbar)) < \frac{1}{k}$ [13].

Now using convex fuzzy compactness of $\mathcal{K}$ we have (b_k) contains (b_{k_j}) such that $b_{k_j} \to \hbar \in \mathcal{K}$. At last, the convex fuzzy continuity of $\mathcal{L}_f$ implies that $\mathcal{L}_f(\hbar) = \alpha$.

(ii) Using the convex fuzzy compactness of $\mathcal{K}$ and $\mathcal{F}$ the definition of $\mathcal{M}(\mathcal{K},\mathcal{F})$ is well defined. That is $\exists\ k \in \mathcal{K}$ and $f \in \mathcal{F}$ satisfy $\mathcal{M}(\mathcal{K},\mathcal{F})$=$\mathcal{M}(k, f)$.

Theorem 7: *If $(\mathcal{Z},\mathcal{M})$ is a c-FMS then (CFC($\mathcal{Z}$), $\mathcal{M}$) is c-FMS.*

Proof:

(1) If $\mathcal{Y} \neq \mathcal{F}$ then $0< \mathcal{M}(\mathcal{Y},\mathcal{F}) <1$;

(2) $\mathcal{M}(\mathcal{Y},\mathcal{F})=0 \Leftrightarrow \max\{\ \mathcal{M}(f, y): f \in \mathcal{F}$ and $y \in \mathcal{Y}\ \}=0 \Leftrightarrow \mathcal{M}(f, y)=0\ \forall f \in \mathcal{F}$ and $\forall\ y \in \mathcal{Y} \Leftrightarrow f= y\ \forall\ f \in \mathcal{F}$ and $\forall\ y \in \mathcal{Y} \Leftrightarrow \mathcal{Y} = \mathcal{F}$;

(3) $\mathcal{M}(\mathcal{Y},\mathcal{F}) = \max\{\ \mathcal{M}(y\ , f) = \mathcal{M}(f, y): y \in \mathcal{Y}$ and $f \in \mathcal{F}\}= \mathcal{M}(\mathcal{F},\mathcal{Y})$;

(4)$\mathcal{M}(\mathcal{Y},\mathcal{F}) \leq \gamma\ \mathcal{M}(\mathcal{Y},\mathcal{K}) + \delta\ \mathcal{M}(\mathcal{K},\mathcal{Y})$ for all $\mathcal{K} \in$ H($\mathcal{Z}$) with $\gamma+\delta=1$.

$\mathcal{M}(\mathcal{Y},\mathcal{F}) = \max\{\ \mathcal{M}(f, y) : f \in \mathcal{F}$ and $y \in \mathcal{Y}\ \}$
$\leq \max\{\ \gamma\ \mathcal{M}(f, k) + \delta\ \mathcal{M}(k, y): f \in \mathcal{F}$ and $y \in \mathcal{Y}\ , k \in \mathcal{K}\}$
$\leq \gamma\ \max\{\ \mathcal{M}(f, k): f \in \mathcal{F}$ and $k \in \mathcal{K}\}+ \delta\ \max\{\ \mathcal{M}(k, y) : y \in \mathcal{Y}$ and $k \in \mathcal{K}\}$
$\leq \gamma\ \mathcal{M}(\mathcal{Y},\mathcal{K}) + \delta\ \mathcal{M}(\mathcal{K},\mathcal{Y})$

Hence (CFC($\mathcal{Z}$), $\mathcal{M}$) is c-FMS.

Lemma 2: *If $(\mathcal{Z},\ \mathcal{M})$ is a c-FMS and $\mathcal{K}$, $\mathcal{F} \in$ CFC($\mathcal{Z}$) with $\mathcal{K} \subseteq \mathcal{F}$ then $\mathcal{M}(p\ ,\mathcal{F}) \leq \mathcal{M}(p, \mathcal{K})$ for any $p \in \mathcal{Z}$.*

Proof: $\mathcal{M}(p,\mathcal{F}) = \min\{\mathcal{M}(p, f) : f \in \mathcal{F} - \mathcal{K}\} \leq \min\{\ \mathcal{M}(p, k) : k \in \mathcal{K}\ \}$ $= \mathcal{M}(p, \mathcal{K})$ since $\mathcal{M}(p, f) \leq \mathcal{M}(p, k)$ for all $f \in \mathcal{F} - \mathcal{Y}$ and for all $k \in \mathcal{K}$.

Lemma 3: *Suppose that $(\mathcal{Z},\ \mathcal{M})$ is a c-FMS and $\mathcal{Y}$, $\mathcal{F} \in$ CFC($\mathcal{Z}$). We have*

(i) If $\mathcal{Y} \subseteq \mathcal{F}$ then $\mathcal{M}(\mathcal{F},\mathcal{Y})$ =0.

(ii) If $\mathcal{Y} \neq \mathcal{F}$ then $\mathcal{M}(\mathcal{Y},\mathcal{F}) \neq 0$ and $\mathcal{M}(\mathcal{F},\mathcal{Y}) \neq 0$,

Proof: The proof this theorem is direct.

Lemma 4: *Suppose that $(\mathcal{Z},\mathcal{M})$ is a c-FMS and $\mathcal{Y}$, $\mathcal{F}$, $\mathcal{K} \in$ CFC($\mathcal{Z}$) then*

(i) $\mathcal{M}[(\mathcal{Y} \cup \mathcal{F}), \mathcal{K}] = \max\ \{\mathcal{M}(\mathcal{Y},\mathcal{K})\ , \mathcal{M}(\mathcal{F},\mathcal{K})\}$.

(ii) If $\mathcal{F} \subseteq \mathcal{K}$ then $\mathcal{M}(\mathcal{Y},\mathcal{K}) \leq \mathcal{M}(\mathcal{Y},\mathcal{F})$.

Proof:

(i) $\mathcal{M}[(\mathcal{Y} \cup \mathcal{F}), \mathcal{K}] = \max\{\mathcal{M}(z, k): z \in (\mathcal{Y} \cup \mathcal{F})$ and $k \in \mathcal{K}\}$
$= \max\{\max\{\mathcal{M}(z, k): : z \in \mathcal{Y}$ and $k \in \mathcal{K}\ \}\ , \max\{\ \mathcal{M}(z, k):: z \in \mathcal{F}$ and $k \in \mathcal{K}\ \}\}$
$= \max\{\mathcal{M}(\mathcal{Y},\mathcal{K})\ , \mathcal{M}(\mathcal{F},\mathcal{K})\}$.

(ii) By Lemma 2, $\mathcal{M}(y, k) \leq \mathcal{M}(y, f)$ for any $y \in \mathcal{Y}\ , k \in \mathcal{K}$and $f \in \mathcal{F}$so $\mathcal{M}(\mathcal{Y},\mathcal{K})$ =max$\{\ \mathcal{N}(y, k) : y \in \mathcal{Y}$ and $k \in \mathcal{K}\} \leq \max\{\ \mathcal{M}(y, f): y \in \mathcal{Y}$ and $f \in \mathcal{F}\} = \mathcal{M}(\mathcal{Y},\mathcal{F})$.

Definition 14: Suppose that $(\mathcal{Z},\mathcal{M})$ is a c-FMS and $\mathcal{Y} \in$ CFC$(\mathcal{Z})$ then we define **the extension of $\boldsymbol{\mathcal{Y}}$** by $B(Y,\alpha) = \{z \in Z: M(z,y) < \alpha$ for all $y \in \mathcal{Y}$, and $\alpha \in (0,1)\}$.

Theorem 8: *Suppose that $(\mathcal{Z},\mathcal{M})$ is a c-FMS and $\mathcal{Y}$, $\mathcal{F} \in$ CFC$(\mathcal{Z})$ and $\alpha \in (0, 1)$. Then $\mathcal{M}(\mathcal{Y},\mathcal{F}) < \alpha$ if and only if $\mathcal{Y} \subset \mathfrak{B}(\mathcal{F},\alpha)$.*

Proof: To prove $\mathcal{M}(\mathcal{Y}, \mathcal{F}) < \alpha$ if and only if $\mathcal{Y} \subset \mathfrak{B}(\mathcal{F}, \alpha)$. Suppose that $\mathcal{M}(\mathcal{Y},\mathcal{F}) < \alpha$ then max$\{ \mathcal{M}(y, f): y \in \mathcal{Y}$ and $f \in \mathcal{F}\} < \alpha$ implies $\mathcal{Y} \subset \{y \in \mathcal{Y}: \mathcal{M}(y, f) < \alpha$, for all $f \in \mathcal{F}$, $\alpha \in (0, 1)\}$. Hence $\mathcal{Y} \subset \mathfrak{B}(\mathcal{F}, \alpha)$.

Now suppose that $\mathcal{Y} \subset \mathfrak{B}(\mathcal{F},\alpha)$ and let $y \in \mathcal{Y}$ this implies that $y \in \mathfrak{B}(\mathcal{F}, \alpha)$ so for all $f \in \mathcal{F}$ we have $\mathcal{M}(y, f) < \alpha$ for all $y \in \mathcal{Y}$. Hence max$\{ \mathcal{M}(y, f): y \in \mathcal{Y}$ and $f \in \mathcal{F}\} < \alpha$. Thus $\mathcal{M}(\mathcal{Y},\mathcal{F}) < \alpha$. Hence $\mathcal{M}(\mathcal{F},\mathcal{Y}) < \alpha \iff \mathcal{F} \subset \mathfrak{B}(\mathcal{Y}, \alpha)$.

Corollary 1: *Suppose that $(\mathcal{Y}_k)$ is a sequence in CFC$(\mathcal{Z})$ which is convex fuzzy converge to $\mathcal{Y} \in$ CFC$(\mathcal{U})$ then for any $\alpha \in (0, 1)$ there is $N \in \mathbb{N}$ such that $\mathcal{M}(\mathcal{Y}_k, \mathcal{Y}) < \alpha$ if and only if $\mathcal{Y}_k \subset \mathfrak{B}(\mathcal{Y},\alpha)$ for all $k \geq N$.*

Proof: Follows from Theorem 8.

Corollary 2: *Suppose that $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC$(\mathcal{U})$ so $\forall \alpha \in (0, 1)$ $\exists$ N satisfy $M(Y_m, Y_k) < \alpha \iff Y\_m \subset B(Y_k,\alpha)$ $\forall m, k \geq N$.*

Proof: Follows from Theorem 8.

Theorem 9: *Consider that the c-FMS $(\mathcal{Z}, \mathcal{M})$ is a convex fuzzy complete. Let $(\mathcal{Y}_k)$ be a convex fuzzy Cauchy sequence in (CFC$(\mathcal{Z})$, $\mathcal{M}$). Assume that (k_n) is an increasing sequence with $0 < k_1 < k_2 < \dots < k_n < \cdots$. Let $(y_{k_j}) \in \mathcal{Y}_{k_j}$ be a convex fuzzy Cauchy sequence in $\mathcal{Z}$ then there is a convex fuzzy Cauchy sequence $(\hat{y}_k) \in \mathcal{Y}_k$ where $\hat{y}_{k_j} = y_{k_j}$ $\forall j \in \mathbb{N}$.*

Proof: We construct $(\hat{y}_k) \in \mathcal{Y}_k$ as follows for $1 \leq k \leq k_1$ let $\hat{y}_k \in \{ y \in \mathcal{Y}_k: \mathcal{M}(y, y_{k_j}) = \mathcal{M}(y_{k_j}, \mathcal{Y}_k)\}$ then $\hat{y}_k$ is exists because $\mathcal{Y}_k$ is convex fuzzy compact. Also for $j \geq 2$ and $\forall$ k $\in \{ k_j+1, k_j+2, \dots, k_{j+1} \}$ let $\hat{y}_k \in \{ y \in \mathcal{Y}_k: \mathcal{M}(y, y_{k_j}) = \mathcal{M}(y_{k_j}, y_k)\}$. Clearly $\hat{y}_{k_j} = y_{k_j}$ by our construction. Since $(y_{k_j}) \in \mathcal{Y}_{k_j}$ is a convex fuzzy Cauchy in $\mathcal{Z}$ let $0 < \alpha < 1$ $\exists$ $k_j, k_n \geq N_1$with $\mathcal{M}(y_{k_j}, y_{k_n}) < \alpha$.

Moreover, because $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC$(\mathcal{Z})$, then $\exists$ N_2 satisfy $\mathcal{M}(\mathcal{Y}_j, \mathcal{Y}_n) < \alpha$ $\forall$ k, n $\geq N_2$. Now, consider N =min$\{ N_1$, N_2, then for i, n $\geq$ N implies that $\mathcal{M}(\hat{y}_i, \hat{y}_n) \leq \delta_1 \mathcal{M}(\hat{y}_i, y_{k_j}) + \delta_2 \mathcal{M}(y_{k_j}, y_{k_n}) + \delta_3 \mathcal{M}(y_{k_n}, \hat{y}_n)$, where $\delta_1 + \delta_2 + \delta_3 = 1$ when i $\in \{ k_{j-1}+1, k_{j-1}+2, \dots, k_j \}andk_{n-1}+1 \leq$ n

$\leq$ k_n. Since $\mathcal{M}(\mathcal{Y}_m, \mathcal{Y}_{k_j}) < \alpha$ then there exists $\hat{y}_m \in \mathcal{Y}_m \cap [\mathfrak{B}((y_{k_j}), \alpha)]$, then $\mathcal{M}(\hat{y}_m, y_{n_j}) < \alpha$.

Similarly, we can show that $\mathcal{M}(y_{k_n}, \hat{y}_k) \leq$ t. Hence for all i, n $\geq$ N, $\mathcal{M}(\hat{y}_i, \hat{y}_n) \leq \delta_1\ \alpha+\delta_2\ \alpha+\delta_3\ \alpha = \alpha$.

Thus $(\hat{y}_k) \in \mathcal{Y}_k$ is a convex fuzzy Cauchy.

Notation 2: If $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC($\mathcal{Z}$) with $\mathcal{Y}_k \to \mathcal{Y} \in$ CFH($\mathcal{U}$) then $\mathcal{Y}$ = {$z \in \mathcal{Z}$: there is a Cauchy sequence $(y_k) \in \mathcal{Y}_k$ such that $y_k \to z$ }.

Lemma 5: *If $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC($\mathcal{Z}$) with $\mathcal{Y}_k \to \mathcal{Y}$ then $\mathcal{Y} \neq \emptyset$. When the c-FMS ($\mathcal{Z}$,$\mathcal{M}$) is a convex fuzzy complete*

Proof: Consider $N_k \in \mathbb{N}$ with $N_1 < N_2 < \ldots < N_k < \ldots$ satisfies $\mathcal{M}(\mathcal{Y}_j, \mathcal{Y}_n) < (\frac{1}{2^k})$ for all j, n $\geq N_k$. Choose $y_{N_1} \in \mathcal{Y}_{N_1}$ but $\mathcal{M}(\mathcal{Y}_{N_1},\mathcal{Y}_{N_2}) < (\frac{1}{2})$ so we can find $y_{N_2} \in \mathcal{Y}_{N_2}$ such that $\mathcal{M}(y_{N_1}, y_{N_2}) < (\frac{1}{2})$. Pick $y_{N_j} \in \mathcal{Y}_{N_j}$ with $1 \leq j \leq k$ satisfies $\mathcal{M}(y_{N_{j-1}}, y_{N_j}) \leq (\frac{1}{2^{j-1}})$. Thus because $\mathcal{M}(\mathcal{Y}_{N_k}, \mathcal{Y}_{N_{k-1}}) < (\frac{1}{2^k})$ and $y_{N_k} \in \mathcal{Y}_{N_k}$ we can find $y_{N_{k+1}} \in \mathcal{Y}_{N_{k+1}}$ such that $\mathcal{M}(y_{N_k}, y_{N_{k-1}}) < (\frac{1}{2^k})$. Consider $y_{N_{k-1}}$ be in $\mathcal{Y}_{N_{k+1}}$ that nearest to y_{N_k}. Continuo in this manner $(y_{N_j}) \in \mathcal{Y}_{N_j}$ such that $\mathcal{M}(y_{N_j}, y_{N_{j-1}}) < (\frac{1}{2^j})$. Then for $N_j, N_n > N_m$

$\mathcal{M}(y_{N_j}, y_{N_n}) \leq \delta_1\ \mathcal{M}(y_{N_j}, y_{N_{j+1}}) + \delta_2\ \mathcal{M}\ (y_{N_{j+1}}, y_{N_{j+2}}) + \ldots + \delta_{N_j-N_n}\ \mathcal{M}\ (y_{N_{n-1}}, y_{N_n})$,

where $\delta_1 + \delta_2 + \ldots + \delta_{N_j-N_n} = 1$.

Now

$$\mathcal{M}(y_{N_j}, y_{N_n}) < \delta_1\ (\tfrac{1}{2}) + \delta_2\ (\tfrac{1}{2^2}) + \ldots + \delta_{N_j-N_n}\ (\tfrac{1}{2^{N_j-N_n}}) < \beta.$$

For some β, $0 < \beta < 1$.Using Theorem 9 $\exists$ $(w_{N_i}) \in \mathcal{Y}_i$ satisfying $w_{N_i} = y_{N_i}$. Hence lim $w_i \in \mathcal{Y}$. Therefore, $\mathcal{Y} \neq \emptyset$.

Lemma 6: *If $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC($\mathcal{Z}$) with $\mathcal{Y}_k \to \mathcal{Y}$ then $\mathcal{Y}$ is a convex fuzzy complete.*

When the c-FMS ($\mathcal{Z}$,$\mathcal{M}$) is a convex fuzzy complete.

Proof: Suppose $(y_i) \in \mathcal{Y}_i$ with $y_i \to y$ to prove $y \in \mathcal{Y}$ $\forall$ I, $\exists$ $(y_{i,k}) \in \mathcal{Y}_k$ satisfies $y_{i,k} \to y_i$. $\exists N_i \in \mathbb{N}$ satisfies $\mathcal{M}(y_{N_i}, y) < (\frac{1}{i})$. Also, $\exists$ $n_i \in \mathbb{N}$ satisfies $\mathcal{M}(y_{N_i,n_i}, y_{N_i}) < (\frac{1}{i})$. Therefore

$\mathcal{M}(y_{N_i,n_i}, y) \leq \delta_1 \mathcal{M}(y_{N_i,n_i}, y_{N_i}) + \delta_2\ \mathcal{M}(y_{N_i}, y) < \delta_1\ (\frac{1}{i}) + \delta_2\ (\frac{1}{i}) = (\frac{1}{i})$,

where $\delta_1 + \delta_2 = 1$. Put $d_{n_i} = y_{N_i,n_i}$ we see that $d_{n_i} \in \mathcal{Y}_{n_i}$ and $d_{n_i} \to y$.

Now by Theorem 9, (d_{n_i}) is extended to $(z_i) \in \mathcal{Y}_i$ thus $y \in \mathcal{Y}$ hence $\mathcal{Y}$ is convex fuzzy closed. Therefore, $\mathcal{Y}$ is convex fuzzy complete because $\mathcal{Z}$ is convex fuzzy complete.

Lemma 7: *Suppose that c-FMS $(\mathcal{Z}, \mathcal{M})$ is a convex fuzzy complete. If $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC($\mathcal{Z}$) with $\mathcal{Y}_k \to \mathcal{Y}$ then $\forall\ \alpha \in (0, 1)\ \exists\ N \in \mathbb{N}$ such that $\mathcal{Y} \subset \mathfrak{B}(\mathcal{Y}_k, \alpha)$ for all $k \geq N$.*

Proof: $\forall\ \alpha \in (0, 1)\ \exists$ N $\in \mathbb{N}$ satisfies $\mathcal{M}(\mathcal{Y}_k, \mathcal{Y}_n) < \alpha\ \forall$ k, n $\geq$ N. Thus for k $\geq$ n $\geq$ N, $\mathcal{Y}_k \subset \mathfrak{B}(\mathcal{Y}_n, \alpha)$. To prove that $\mathcal{Y} \subset \mathfrak{B}(\mathcal{Y}_n, \alpha)$ let $y \in \mathcal{Y}$ so $\exists\ (y_i) \in \mathcal{Y}_i$ with $y_i \to y$. Therefore, with k $\geq$ N, $\mathcal{M}(y_k, y) < \alpha$. Thus $y_k \in \mathfrak{B}(\mathcal{Y}_n, \alpha)$ because $\mathcal{Y}_n$ is convex fuzzy compact of $\mathcal{Y}_n$ this implies that $\mathfrak{B}(\mathcal{Y}_n, \alpha)$ is convex fuzzy closed. Thus $y_k \in \mathfrak{B}(\mathcal{Y}_n, \alpha)\ \ \forall$ k $\geq$ N so $y \in \mathfrak{B}(\mathcal{Y}_n, \alpha)$. Hence $\mathcal{Y} \subset \mathfrak{B}(\mathcal{Y}_n, \alpha)\ \forall$ n $\geq$ N.

Lemma 8: *Suppose that c-FMS $(\mathcal{Z}, \mathcal{M})$ is a convex fuzzy complete. If $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy in CFC($\mathcal{Z}$) with $\mathcal{Y}_k \to \mathcal{Y}$ then $\mathcal{Y}$ is convex fuzzy compact.*

Proof: If $\mathcal{Y}$ is not totally convex fuzzy bounded then for some $0 < \alpha < 1$ so does not exists a finite α-convex fuzzy net. Thus $\exists\ (y_i) \in \mathcal{Y}$ satisfies $\mathcal{M}(y_i, y_j) \geq \alpha$ for i $\neq$j. This present a contradiction.

Using Lemma 7, n $\geq$ N satisfies $\mathcal{Y} \subset \mathfrak{B}(\mathcal{Y}_n, \alpha)$. For each w_i there is a corresponding $y_i \in \mathcal{Y}_n$ for which $\mathcal{M}(w_i, y_i) < \beta$ with $\beta < \alpha$. Since $\mathcal{Y}_n$ is convex fuzzy compact and $(y_{n_i}) \subseteq (y_i)$ convex fuzzy converges. Thus $\exists\ y_{n_i}$ and y_{n_j} satisfies $\mathcal{M}(y_{n_i}\ y_{n_j}) < \sigma$ where $0 < \sigma < 1$ with $\sigma < \alpha$. Now

$$\mathcal{M}(w_{n_i}, w_{n_j}) \leq \delta_1 \mathcal{M}(w_{n_i}, y_{n_i}) + \delta_2\ \mathcal{M}(y_{n_i}, y_{n_j},) + \delta_3\ \mathcal{M}(y_{n_j}, \mathrm{w}_{n_j}),$$

where $\delta_1 + \delta_2 + \delta_3 = 1$. Then

$$\mathcal{M}(w_{n_i}, w_{n_j}) \leq \delta_1 \beta + \delta_2\ \alpha + \delta_3 \alpha < \alpha.$$

Thus $\mathcal{Y}$ is totally convex fuzzy bounded and by Lemma 7, $\mathcal{Y}$ is convex fuzzy compact.

Theorem 10: *If the c-FMS $(\mathcal{Z}, \mathcal{M})$ is a convex fuzzy complete then (CFC($\mathcal{Z}$), $\mathcal{H}$) is a convex fuzzy complete.*

Proof: Assume that $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy sequence in CFC($\mathcal{Z}$). Now by Lemma 5, $\mathcal{Y} \neq \emptyset$, also using Lemma 6 $\mathcal{Y}$ is convex fuzzy complete. Therefore by Lemma 7, $\forall\ \alpha \in (0, 1)\ \exists\ N \in N$ satisfies $Y \subset B(\mathcal{Y}_k, \alpha)\ \forall\ k \geq N$. Now Lemma 8 $\mathcal{Y}$ is convex fuzzy compact. Therefore, to prove $\mathcal{Y}_k \to \mathcal{Y}$ it is sufficient to prove for $0 < \alpha < 1\ \exists\ N$ satisfies $\mathcal{Y}_k \subset B(Y, \alpha)\ \forall\ k \geq N$. Because $(\mathcal{Y}_k)$ is a convex fuzzy Cauchy thus $\forall\ 0 < \alpha < 1\ \exists\ N \in N$ satisties $\mathcal{M}(\mathcal{Y}_k, \mathcal{Y}_n) < \alpha\ \forall\ k, n \geq N$. Thus for $k, n \geq N$, $\mathcal{Y}_k \subset \mathfrak{B}(\mathcal{Y}_n, \alpha)$.

Let $k \geq N$ we will show that $\mathcal{Y}_k \subset \mathfrak{B}(\mathcal{Y}, \alpha)$. Let $y \in \mathcal{Y}_k$ then $\exists\ n < \mathrm{N}_1 < \mathrm{N}_2 < \ldots < \mathrm{N}_k <$ ..

and for $k, n \geq \mathrm{N_j}$ $\mathcal{Y}_\mathrm{k} \subset \mathfrak{B}(\mathcal{Y}_\mathrm{n}, \frac{\alpha}{2^{j-1}})$ note that $\mathcal{Y}_\mathrm{k} \subset \mathfrak{B}(\mathcal{Y}_{\mathrm{N_i}}, \frac{\alpha}{2})$. Since $y \in \mathcal{Y}_\mathrm{k}$ there is $y_{\mathrm{N_i}} \in \mathcal{Y}_{\mathrm{N_i}}$ such that $\mathcal{M}(\mathrm{y}, \mathrm{w}_{\mathrm{N_i}}) < (\frac{\alpha}{2})$.

But $y_{\mathrm{N_1}} \in \mathcal{Y}_{\mathrm{N_1}}$ there is $y_{\mathrm{N_2}} \in \mathcal{Y}_{\mathrm{N_2}}$such that $\mathcal{M}(y_{\mathrm{N_1}}, y_{\mathrm{N_2}}) < (\frac{\alpha}{2^2})$.

Similarly $\exists$ $(y_{\mathrm{N_j}})$ satisfies $y_{\mathrm{N_j}} \in \mathcal{Y}_{\mathrm{N_j}}$ and $\mathcal{M}(y_{\mathrm{N_j}}, y_{\mathrm{N_{j-1}}}) <(\frac{\alpha}{2^{j+1}})$.

After finite steps causes $\mathcal{M}(y, y_{\mathrm{N_j}}) < (\alpha/2)\ \forall j$, as well as $(y_{\mathrm{N_j}})$ is a convex fuzzy Cauchy. Thus each $\mathcal{Y}_{\mathrm{N_j}} \subset \mathfrak{B}(\mathcal{Y}_\mathrm{k}, \frac{\alpha}{2})$ and $(y_{\mathrm{N_j}})$ converges to a point u and because $\mathfrak{B}(\mathcal{Y}_\mathrm{k}, \frac{\alpha}{2})$ is convex fuzzy closed $u \in \mathfrak{B}(\mathcal{Y}_\mathrm{k}, \frac{\alpha}{2})$. Moreover, $\mathcal{M}(u, y_{\mathrm{N_j}}) \leq (\frac{\alpha}{2})$ implies

$$\mathcal{M}(y, u) \leq \delta_1\ \mathcal{M}(y, y_{\mathrm{N_j}}) + \delta_2\ \mathcal{M}(y_{\mathrm{N_j}}, u) < \delta_1(\frac{\alpha}{2}) + \delta_2(\frac{\alpha}{2}) = \frac{\alpha}{2} < \alpha.$$

Then, $\mathcal{M}(y,u) < \alpha$. Thus $\mathcal{Y}_\mathrm{k} \subset \mathfrak{B}(\mathcal{Y}, \alpha)$ for all $\mathrm{k} \geq \mathrm{N}$. Hence $\mathcal{Y}_\mathrm{k} \to \mathcal{Y}$ consequently (CFC($\mathcal{Z}$),$\mathcal{M}$) is a convex fuzzy complete.

4. Conclusion

In this study we use the convex fuzzy metric $\mathcal{M}$ to define the convex fuzzy distance from a point z to a set $\mathcal{Y}$ as well as the convex fuzzy distance between two sets $\mathcal{F}$ and $\mathcal{Y}$ then we introduce the set CFC($\mathcal{Z}$) by $\{\mathcal{Y} : \mathcal{Y} \neq \emptyset, \mathcal{Y} \subset \mathcal{Z}, \mathcal{Y}$ is convex fuzzy compact$\}$. Thus our aim is to prove the c-FMS, CFC($\mathcal{Z}$) is convex fuzzy complete when the c-FMS $(\mathcal{Z}, \mathcal{M})$ is convex fuzzy complete.

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