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Analytical approximate solutions of random integro differential equations with laplace decomposition method

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Abstract

An efficient combination of Adomian Decomposition iterative technique coupled with Laplace transformation to solve non-linear Random Integro differential equation (NRIDE) is introduced in a novel way to get an accurate analytical solution. This technique is an elegant combination of theLaplace transform, and the Adomian polynomial. The suggested method will convert differential equations into iterative algebraic equations, thus reducing processing and analytical work. The technique solves the problem of calculating the Adomian polynomials. The method's efficiency was investigated using some numerical instances, and the findings demonstrate that it is easier to use than many other numerical procedures. It has also been established that (LTADM) is a trustworthy technique for solving differential equations. Using the Mathematica 13.3 programme, the graphs of the approximate solutions and consecutive error are presented. Two applications are presented as examples of how the proposed technique can be utilised to obtain analytical or numerical solutions for certain kinds of Random Integro Differential Equations (RIDEs)in order to demonstrate its efficacy and potential.

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Keywords: *Laplace transform, Random differential equations, Integral equations, Decomposition adomian technique, Analytical technique.*

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1. Introduction

Numerous Nonlinear Random Integral Equations (NRIEs) play a major role in modern modeling phenomena. Several biological, engineering, and physical challenges involve these phenomena.. Also, the requirement to get the accurate solutions is often seen as a challenge for these problems.Several analytical techniques which deal to find solutions are used either idealization or simplification of the original problem. These techniques have shown to be an effective and efficient tool for supplying accurate results for nonlinear problems. Several approaches to solving RIDEs have been found recently. Various theorems that aid in the handling and resolution of integral equations, PDEs, ODEs, and differential equations of all types. The Laplace transform is an effective transformation method that is frequently employed as a mathematical model for numerous engineering or physical models. Notably, the Laplace transform has features that preserve units, making it a useful tool for problem-solving. . The major goal of this study is to compute the approximate solutions by a mixture Laplace transform and Adomian decomposition method in order to find the effective solutions of the RODEs [1-12] .

2. Some Basic Concepts of Stochastic Calculus

Definition 2.1: A real valued function $\chi(\omega)$, $\omega \in \Omega$ that is measurable with regard to the probability measure $\boldsymbol{\rho}$, is called a random variable.

Definition 2.2: Stochastic process $\chi(\tau, \omega), \tau \in [\tau_0, \infty), \omega \in \Omega$ indicate by a collection of random variables on a probability space $(\ \Omega, \mathfrak{F}, \rho\ )$ that is $\chi_\tau(\omega)$ and suppose real values and is $\boldsymbol{\rho}$-measurable as a function of ω for each fixed τ. The parameter τ is regarded as a time and $\chi_\tau(.)$ denotes a random variable on the probability space Ω while $\chi(\omega)$ named a sample mammer or trajectory of the stochastic process.

Definition 2.3: For a stochastic process $\mathcal{W}_{\tau,}\ \tau \geq 0$, is said to be a Brownian motion or Wiener process, if: a. $\rho\{\mathcal{W} \in \Omega | \mathcal{W}_0(\omega) = 0\}) = 1$ b. For $0 < \tau_0 < \tau_1 < \cdots < \tau_n$ the increments $\mathcal{W}_{\tau 1} - \mathcal{W}_{\tau 0} - \mathcal{W}_{\tau 2} - \mathcal{W}_{\tau 1 \ldots,}\ \mathcal{W}_{\tau n} - \mathcal{W}_{\tau n-1}$ are independent. c. For an arbitrary τ and $h > 0, \mathcal{W}_{\tau+h} - \mathcal{W}_\tau$ has a Normal distribution with mean 0 and variance h.

3. Numerical experiments

In order to demonstrate the precision and ease of use of the suggested approach, we provide multiple examples in this section. The following examples show how to use LTADM iteration to solve nonlinear Volterra random integro differential equations (NVRDE) and nonlinear Fredholm random integro differential equations (NFRIDE). To compare the graphs of the answers found with LTADM, utilize Mathematica 13.3. It is important to remember that various discretized Brownian motion production across the unit interval [0,1] will be examined. The total numbers N=100, 500, and 1000 are assigned to these generations as shown in Figure 1.

Example 3.1: Consider the Nonlinear Fredhlom Random Integro- Differential Equation (NFRIDEs) [13,14] as follows:

$$X'(t,\omega) = e^{w_t(\omega)} - \frac{1}{3}te^{w_t(\omega)} + \int_0^1 ts\, X(s,\omega)ds \text{ and } X(0,\omega) = 0$$

Solution: Applying the LTADM and using the initial condition to solve above equation, we get: $X_1\,(t,\omega) = -\frac{1}{6}e^{\omega}(-6+t)t$

$$X_2\,(t,\omega) = -\frac{1}{48}te^{\omega}(-48+8t-8t^2+t^3)$$

$$X_3\,(t,\omega) = -\frac{te^{W}(-2880+480t-480t^2+60t^3-48t^4+5t^5)}{2880}$$

$$X_4\,(t,\omega) = -\frac{e^{W}(-322560t+53760t^2-53760t^3+6720t^4-5376t^5+560t^6-384t^7+35t^8)}{322560}$$

$$X_5\,(t,\omega) = -\frac{1}{3870720}e^{W}(-3870720t+645120t^2-645120t^3+80640t^4-64512t^5+6720t^6-4608t^7+420t^8-256t^9+21t^{10})$$

By the same way we can get $X_6\,(t,\omega)$, ... The following tables and figure represents the conscuitive error using LTADM to solve Random Volterra integro differential equation as explained on the following Table 1.

Table 1

Show some results of the consecutive error for N=1000

t	E2	E3	E4	E5	E6
0.1	0.000166901	1.67254×10^{-7}	1.19624×10^{-10}	6.65301×10^{-14}	1.38778×10^{-17}
0.2	0.000945229	3.79707×10^{-6}	1.08776×10^{-8}	2.42174×10^{-11}	4.40759×10^{-14}
0.3	0.0051544	0.0000466909	3.01366×10^{-7}	1.51107×10^{-9}	6.19449×10^{-12}
0.4	0.011811	0.000190633	2.1905×10^{-6}	1.95448×10^{-8}	1.42546×10^{-10}
0.5	0.0241562	0.000610616	0.0000109787	1.53211×10^{-7}	1.74724×10^{-9}
0.6	0.0540727	0.00197292	0.0000511551	1.02902×10^{-6}	1.69112×10^{-8}
0.7	0.0494563	0.00246209	0.0000870204	2.38503×10^{-6}	5.33908×10^{-8}
0.8	0.147803	0.00963459	0.000445443	0.0000159626	4.67083×10^{-7}
0.9	0.0785812	0.00649955	0.000380908	0.000017294	6.40965×10^{-7}
1	0.0965975	0.00988974	0.000716678	0.0000402147	1.84155×10^{-6}

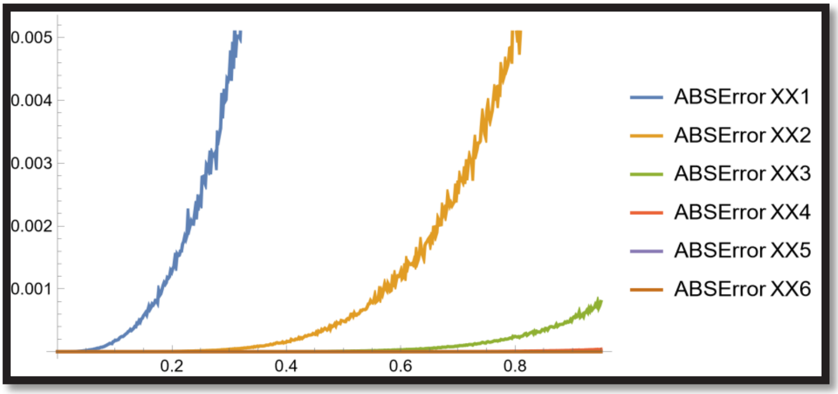

Figure 1
2Dplots of results of the conscutive error when N =1000

Example 3.2: Consider the integro-differential RVIDEs as follows:

$$\mathrm{D}''\Upsilon(\xi,\omega) = 0.5\ \omega_{\xi}^{2}(\omega) + \int_0^{\xi}(\xi-\tau)[\Upsilon(\tau,\omega)]^2\ d\xi \text{ with } \Upsilon(0,\omega)=1 \text{ and } \Upsilon'(0,\omega)=0$$

Solution: By taking LTADM on both sides of above equation,with N = 100, 500, 1000 generations of Brownian motion, thus we get the approximate solutions which are presented as in Table 2, and represented in Figure 2.

$$\Upsilon(0,\omega) = 1\ ,\ \Upsilon_1(\xi,\omega) = 1 + \frac{\xi^4}{24} + \xi^3 w$$

$$\Upsilon_2(\xi,\omega) = 1 + \frac{\xi^4}{24} + \frac{\xi^8}{1344} + \frac{\xi^{12}}{152064} + \xi^3 w + \frac{\xi^7 w}{42} + \frac{\xi^{11} w}{2640} + \frac{\xi^{10} w^2}{180}$$

$$\Upsilon_3(\xi,\omega) = 1 + \frac{\xi^4}{24} + \frac{\xi^8}{1344} + \frac{13\xi^{12}}{1064448} + \frac{\xi^{16}}{6386688} + \frac{197\xi^{20}}{135908720640} + \frac{\xi^{28}}{34962671665152} + \xi^3 w + \frac{\xi^7 w}{42} + \frac{\xi^{11} w}{1680} + \frac{67\xi^{15} w}{6652800} + \frac{1493\xi^{19} w}{12741442560} + \frac{\xi^{23} w}{1154165760} + \frac{\xi^{27} w}{281817169920} + \frac{\xi^{10} w^2}{180} + \frac{37\xi^{14} w^2}{229320} + \frac{2081\xi^{18} w^2}{712514880} + \frac{\xi^{22} w^2}{35126784} + \frac{163\xi^{26} w^2}{978531840000} + \frac{\xi^{17} w^3}{48960} + \frac{\xi^{21} w^3}{3175200} + \frac{\xi^{25} w^3}{285120000} + \frac{\xi^{24}(25+78848w^4)}{2820361420800}$$

By the same way, we can get $\Upsilon_4(\xi,\omega), \Upsilon_5(\xi,\omega), \ldots \ldots \ldots$

Table 2
Show Some Results of the consecutive error for N=1000

t	$E1$	$E2$	$E3$	$E4$
0.1	1.34317×10^{-10}	1.11022×10^{-16}	0.	0.
0.2	2.93129×10^{-8}	4.30767×10^{-13}	1.11022×10^{-16}	0.
0.3	1.02801×10^{-6}	7.609×10^{-11}	2.9976×10^{-15}	0.
0.4	7.38779×10^{-7}	1.90736×10^{-10}	2.4647×10^{-14}	0.
0.5	6.83296×10^{-6}	4.14814×10^{-9}	1.28297×10^{-12}	3.33067×10^{-16}
0.6	0.0000561077	6.42515×10^{-8}	3.90181×10^{-11}	1.46549×10^{-14}

0.7	0.000255664	5.54891×10^{-7}	6.3565×10^{-10}	4.48974×10^{-13}
0.8	0.00152356	6.02899×10^{-6}	1.24599×10^{-8}	1.57985×10^{-11}
0.9	0.00155128	9.07631×10^{-6}	2.7613×10^{-8}	5.13869×10^{-11}
1	0.00320488	0.0000299155	1.47293×10^{-7}	4.47145×10^{-10}

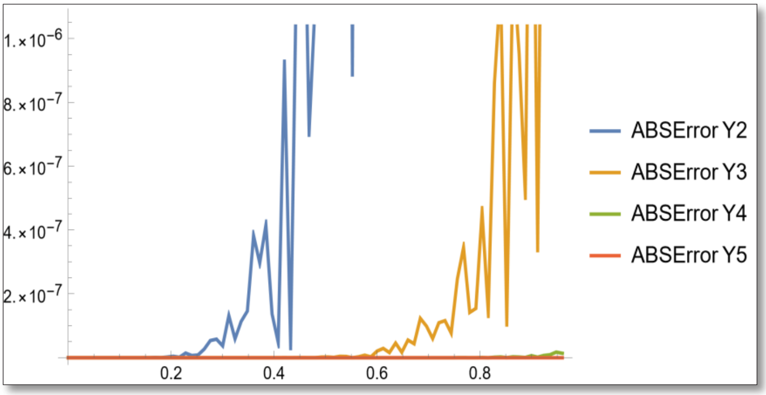

Figure 2
2Dplots of results of the conscutive error when N =1000

5. Conclusion

We have successfully employed LTADM to obtain an approximate analytical solution for several applications of NRIDEs . The proposed technique is simple to use appropriate and successful for getting the desired results for these issues, and LTADM provides the converging series results quickly. RIDEs' approximate analytical solution was obtained quickly and effectively. The LTADM is more powerful and dependable in obtaining approximate solutions to several classes of random component integral equations. The error used here, when the exact solution is not exists to check the accuracy and convergence of the sequence of approximate results to the exact solution.

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