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Modular metric spaces : Some fixed-point theorems and application of secure dynamic routing for WSN

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Abstract

Data exchange within a Wireless Sensor Network (WSN) needs to adhere to high security protocols. The shortest possible route for the transfer of data between two interconnected packets is selected by dynamic routing algorithms, which increases privacy while requesting additional data to be transmitted. Several applications involving remote sensing of environmental requirements, such as WSN, are becoming more prevalent. Highly dense WSNs are evolving into a vital sensor framework enabling the Internet of Things (IoT). WSNs have the potential to synthesize a great deal of actual data, and in order to guarantee that data communication is being performed successfully, they need network architectures that have successful links between Sensor Nodes (SN). The collection of data is a term WSN utilises to define a data fusion/compression technique to minimize the volume of data sent throughout

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the network's bandwidth. The process of iteration of Fixed Points (FP) of contractions will be examined in the present paper, which is scheduled to take part under a Modular Metric Space (M-MS) context. The main objective of this research is to arrive at an algorithm for secure dynamic data transmission routing in WSN by integrating the demonstrated results of the Functional Equation (FE) with randomization. An implementation is a tool for evaluating and modelling a network's architecture in which sensors can be aggregated with a high level of security and least risk in a more secure area (Sensor Field), thereby improving the accuracy of the findings collected by the entire network.

Subject Classification: *Primary 93A30, Secondary 49K15.*

Keywords: *Fixed point theorem, Modular metric space, WSN, IoT, Accuracy, Security.*

1. Introduction

The acronym "sensor" can be employed for virtually any device used to connect the distance between digital data and real-world people [1]. Wireless sensor networks, frequently referred to as WSNs, are geographical networks of sensors that use radio signals to link to other sensors to identify problems and send the data collected to a sink situated within a specific geographic area. Sensors in WSNs are considered via limited energy consumption and computation power. In addition, they also possess the main challenges in that they include Sensor Node (SN), Energy Consumption (EC), End-to-End Delay (EED), and security (communication and data) while adhering to typical WSN functionalities 2-3]. Yet, data integrity, network stability, and resource availability are vulnerable to WSN malware. They can interrupt processes, compromise sensitive data, and spread quickly, requiring robust detection and prevention. Strategic security must address WSN concerns, including resource restrictions and changeable topology. Malware detection in WSNs depends on efficient, decentralized algorithms because of limited resources.

Approaches such as anomaly detection [4-6], performance analysis, and reputation-built systems are used to detect uncertain activities. WSNs are used in many applications, such as environmental monitoring, smart farming, and smart cities. In order to cluster and analyze the resulting information, it is vital to set up technologies that are reliable and that secure the availability of network resources when needed. This article will note the iteration of Fixed Points (FP) of contractions in a Setting in Modular Metric Space (M-MS). This paper also focuses on applying the results of the Functional Equation (FE) recognized with randomization to develop a secure dynamic data transmission routing solution in WSN.

Next, discuss with the FP. One of the most valuable and usable divisions of non-linear analysis is Fixed Point Theory (FPT), which has wide applications in many real-world applications in the normal, environmental sciences, industrial, and other mathematical notions. The principle of Banach Contraction Mapping (BCM) is the most well-known metric FPT outcome [7]. Many fields, including FE, Integral Equations (IE), and Differential Equations (DE), can benefit from this idea. In recent years, [8] developed a statement addressing the principles of Metric Modular (MM) and M-MS, which they call M-MS.

MM produces metric spaces through the use of a less robust shift operation, which is commonly referred to as modular shift with a non-metrizable topology. The spaces of integrable functions referred to as Lebesgue, Riesz, and Orlicz are generalizations of "metric spaces," "metric linear spaces," and "classical modular linear spaces." These improvements were generated by Nakano through the implementation of versions of "metric spaces" [9], presenting an in-depth description of MM and M-MS spaces. This contribution aims to study the existence and uniqueness of fixed point theorems in M-MS. Applying the theoretical results, secure dynamic routing in WSN is discussed. Now, we remember specific recognized definitions and outcomes from the literature, which support the proof of our main results.

2. Preliminaries

Definition 2.1: A function $w: (0, \infty) \times X \times X \to [0, \infty]$ is said to be integrated on X if it proved that the following states, Eq. (1) to Eq. (2)

$$a = b \text{ iff } w_\lambda(a, b) = 0 \text{ for all } \lambda > 0 \tag{1}$$

$$w_\lambda(a, b) = w_\lambda(b, a) \text{ for all } \lambda > 0 \tag{2}$$

$$w_{\lambda+\mu}(a, c) \leq w_\lambda(a, b) + w_\mu(b, c) \text{for all } \lambda > 0 \text{ for all } a, b, c \in X \tag{3}$$

If an example of (i), the function w proves only $w_\lambda(a, a) = 0$ for all $\lambda > 0$. Then w is hypothetical to be a pseudo-modular on X.

Example 2.2: [10] Let X be a set, Eq. (4)

Then,
$$w_\lambda^0(a, b) = \begin{cases} \infty & a \neq b \\ 0 & a = b \end{cases} \tag{4}$$

Example 2.3: [10] Let (X, d) be metric space with metric d and at least two points. The indexed set is w best cases of MM on a set X, Eq. (5) to Eq. (8).

$$w_\lambda^1 = d(x, y)$$

$$w_\lambda^2 = \frac{d(x,y)}{\phi(\lambda)} \tag{6}$$

where $\phi: (0, \infty) \to (0, \infty)$ is a non-decreasing continuous function.

$$w_\lambda^3(a, b) = \begin{cases} \infty & d(a, b) \geq \lambda \\ 0 & d(a, b) < \lambda \end{cases} \tag{7}$$

$$w_\lambda^4(a, b) = \begin{cases} 0 & d(a, b) \leq \lambda \\ \infty & d(a, b) > \lambda \end{cases} \tag{8}$$

Definition 2.2: [10] Assume w be a pseudo-modular on X and $a_0 \in X$. Then, the sets Eq. (9) and Eq. (10)

$$X_w = X_w(a_0) = \{x \in X: w_\lambda(a, a_0) \to 0 \text{ as } \lambda \to \infty\} \tag{9}$$

$$X_w^* = X_w^*(a_0) = \{x \in X: \exists \lambda = \lambda(x) > 0, \tag{10}$$

such that $w_\lambda(a, a_0) < \infty\}$ are said to be M-MS (around a_0)

Definition 2.3: [9-10] Let X_w and X_w^* be M-MS. The sequence $\{\xi_n\}$ in X_w (or X_w^*) is theoretically to be w-convergent to $\xi \in X$ if and only if $w_\lambda(\xi_n, \xi) \to 0$ as $n \to \infty$ for some $\lambda > 0$. Then a is named the modular perimeter of $\{\xi_n\}$.

The classification $\{\xi_n\}$ in X_w is referred to as w-Cauchy if $w_\lambda(\xi_n, \xi_m) \to 0$ as $m, n \to \infty$ for approximately $\lambda > 0$.

A subclass $M$of X_w or X_w^* is proved to be w-complete if any w-Cauchy Sequence (CS) in M is an w-convergent sequence and its w parameters are in M

Definition 2.4: Let w be a MM on X. A map $T: X_w^* \to X_w^*$ is proved to be an original type w-contraction, if there exists $0 < k < 1$ and λ_0 conditional on k, such that Eq. (11) and Eq. (12).

$$w_{k\lambda}(Ta, Tb) \le \alpha_1 w_\lambda(a, b) + \alpha_2 w_\lambda(a, Ta) + \alpha_3 w_\lambda(b, Tb) + \alpha_4 w_\lambda(a, Ta) + \alpha_5 w_\lambda(b, Ta) + \alpha_6[w_\lambda(b, Ta) + w_\lambda(a, Tb)] \tag{11}$$

For all $0 < \lambda < \lambda_0, a, b \in X_w^*$ and $\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5 + 2\alpha_6 < 1$ (12)

3. Main Results

Theorem 3.1: *Let X be a non-empty set, w be a MM on X and $T: X_w^* \to X_w^*$ be well-defined as follows: EQU (13)*

$$w_{k\lambda}(Ta, Tb) \le \alpha_1 w_\lambda(a, b) + \alpha_2 w_\lambda(a, Ta) + \alpha_3 w_\lambda(b, Tb) + \alpha_4 w_\lambda(a, Ta) + \alpha_5 w_\lambda(b, Ta) + \alpha_6[w_\lambda(b, Ta) + w_\lambda(a, Tb)] \tag{13}$$

Assume that all $\lambda > 0$, there occurs an $a_0 \in X_w^*$ such that $w_\lambda(a_0, Ta_0) < \infty$ then the mapping T has a FP in X_w^*. Also, the FP of T is exceptional.

Proof: Eq. (14) to Eq. (20)

Let $\xi_0 \in X$ and $\{\xi_n\}_{n=1}^{\infty}$ be a sequence in X well-defined as $\xi_n = T\xi_{n-1} = T^n\xi_0$,

$$n = 1,2,3, .. w_\lambda(\xi_n, \xi_{n+1}) = w_\lambda(T\xi_{n-1}, T\xi_n)$$
$$\le \alpha_1 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_2 w_\lambda(\xi_{n-1}, T\xi_{n-1}) + \alpha_3 w_\lambda(\xi_n, T\xi_n) + \alpha_4 w_\lambda(\xi_{n-1}, T\xi_n) + \alpha_5 w_\lambda(\xi_n, T\xi_{n-1}) + \alpha_6[w_\lambda(\xi_n, T\xi_{n-1}) + w_\lambda(\xi_{n-1}, T\xi_n)] \tag{14}$$

$$\le \alpha_1 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_2 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_3 w_\lambda(\xi_n, \xi_{n+1}) + \alpha_4 w_\lambda(\xi_{n-1}, \xi_{n+1}) + \alpha_5 w_\lambda(\xi_n, \xi_n) + \alpha_6[w_\lambda(\xi_n, \xi_n) + w_\lambda(\xi_{n-1}, \xi_{n+1})] \tag{15}$$

$$\le \alpha_1 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_2 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_3 w_\lambda(\xi_n, \xi_{n+1}) + \alpha_4 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_4 w_\mu(\xi_n, \xi_{n+1}) + \alpha_6 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_6 w_\mu(\xi_n, \xi_{n+1}) \tag{16}$$

$$\le \alpha_1 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_2 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_3 w_\lambda(\xi_n, \xi_{n+1}) + \alpha_4 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_4 w_\lambda(\xi_n, \xi_{n+1}) + \alpha_6 w_\lambda(\xi_{n-1}, \xi_n) + \alpha_6 w_\lambda(\xi_n, \xi_{n+1}) \tag{17}$$

$$(1 - \alpha_3 - \alpha_4 - \alpha_6) w_\lambda(\xi_n, \xi_{n+1}) \le (\alpha_1 + \alpha_2 + \alpha_4 + \alpha_6) w_\lambda(\xi_{n-1}, \xi_n) \tag{18}$$

$$w_\lambda(\xi_n, \xi_{n+1}) \le \frac{\alpha_1 + \alpha_2 + \alpha_4 + \alpha_6}{1 - \alpha_3 - \alpha_4 - \alpha_6} w_\lambda(\xi_{n-1}, \xi_n) \tag{19}$$

$$w_\lambda(\xi_n, \xi_{n+1}) \leq \eta w_\lambda(\xi_{n-1}, \xi_n) \tag{20}$$

Where $\eta = \frac{\alpha_1+\alpha_2+\alpha_4+\alpha_6}{1-\alpha_3-\alpha_4-\alpha_6}$,

$$w_\lambda(\xi_n, \xi_{n+1}) \leq \eta w_\lambda(\xi_{n-1}, \xi_n) \leq \eta^2 w_\lambda(\xi_{n-2}, \xi_{n-1}) \tag{21}$$

Continuing this method, obtained, $\leq \eta^n w_\lambda(\xi_0, \xi_1)$

Now, verified that $\{\xi_n\}_{n=1}^{\infty}$ is a CS in X.

Let $m, n > 0$ with $m > n$, Eq. (22) to Eq. (24).

$$w_\lambda(\xi_n, \xi_m) \leq w_\lambda(\xi_n, \xi_{n+1}) + w_\lambda(\xi_{n+1}, \xi_{n+2}) + w_\lambda(\xi_{n+2}, \xi_{n+3}) + \tag{22}$$

$$\leq \eta^n w_\lambda(\xi_1, \xi_0) + \eta^{n+1} w_\lambda(\xi_1, \xi_0) + \cdots + \eta^{n+m-1} w_\lambda(\xi_1, \xi_0) \tag{23}$$

$$\leq \eta^n w_\lambda(\xi_1, \xi_0) \tag{24}$$

Take $m, n \to \infty$, $\lim_{n\to\infty} w_\lambda(\xi_n, \xi_m) = 0$

Hence $\{\xi_n\}_{n=1}^{\infty}$ is a CS in X.

Since $\{\xi_n\}_{n=1}^{\infty}$ is a CS $\{\xi_n\}$ converges to $\xi^* \in X$

Now, proved that x^* is the exceptional FP of T, Eq. (25) to Eq. (29).

$$w_\lambda(\xi^*, T\xi^*) \leq w_\lambda(\xi^*, \xi_{n+1}) + w_\lambda(\xi_{n+1}, T\xi^*) \tag{25}$$

$$\leq w_\lambda(\xi^*, \xi_{n+1}) + w_\lambda(T\xi_n, T\xi^*) \tag{26}$$

$$\leq w_\lambda(\xi^*, \xi_{n+1}) + [\alpha_1 w_\lambda(\xi_n, \xi^*) + \alpha_2 w_\lambda(\xi_n, T\xi_n) + \alpha_3 w_\lambda(\xi^*, T\xi^*)$$
$$+\alpha_4 w_\lambda(\xi_n, T\xi^*) + \alpha_5 w_\lambda(\xi^*, T\xi_n) + \alpha_6[w_\lambda(\xi^*, T\xi_n) + w_\lambda(\xi_n, T\xi^*)]] \tag{27}$$

$$\leq w_\lambda(\xi^*, \xi_{n+1}) + \alpha_1 w_\lambda(\xi_n, \xi^*) + \alpha_2 w_\lambda(\xi_n, \xi^*) + \alpha_2 w_\lambda(\xi^*, \xi_{n+1})$$
$$+\alpha_3 w_\lambda(\xi^*, T\xi^*) + \alpha_4 w_\lambda(\xi^*, \xi_n) + \alpha_4 w_\lambda(\xi^*, T\xi^*) + \alpha_5 w_\lambda(\xi^*, \xi_{n-1})$$
$$+\alpha_6 w_\lambda(\xi^*, \xi_{n-1}) + \alpha_6 w_\lambda(\xi_n, \xi^*) + \alpha_6 w_\lambda(\xi^*, T\xi^*) \tag{28}$$

$$(1 - \alpha_3 - \alpha_4 - \alpha_6) w_\lambda(\xi^*, T\xi^*) \leq (1 + \alpha_2) w_\lambda(\xi^*, \xi_{n+1})$$
$$+(\alpha_1 + \alpha_2 + \alpha_4 + \alpha_6) w_\lambda(\xi_n, \xi^*) + (\alpha_5 + \alpha_6) w_\lambda(\xi^*, \xi_{n+1}) \tag{29}$$

$$w_\lambda(\xi^*, T\xi^*) \leq 0 \text{ as } n \to \infty$$

Now, the expression that ξ^* is the FP of T. Accept that ξ' is an additional FP of T. Then have $T\xi' = \xi'$, Eq. (31) to Eq. (33)

$$w_\lambda(\xi^*, \xi') = w_\lambda(T\xi^*, T\xi') \tag{30}$$

$$\leq \alpha_1 w_\lambda(\xi^*, \xi') + \alpha_2 w_\lambda(\xi^*, T\xi^*) + \alpha_3 w_\lambda(\xi', T\xi') + \alpha_4 w_\lambda(\xi^*, T\xi')$$
$$+\alpha_5 w_\lambda(\xi', T\xi^*) + \alpha_6[w_\lambda(\xi', T\xi^*) + w_\lambda(\xi^*, T\xi')] \tag{31}$$

$$\leq \alpha_1 w_\lambda(\xi^*, \xi') + \alpha_2 w_\lambda(\xi^*, \xi^*) + \alpha_3 w_\lambda(\xi', \xi') + \alpha_4 w_\lambda(\xi^*, \xi') + \alpha_5 w_\lambda(\xi', \xi^*)$$
$$+\alpha_6[w_\lambda(\xi', \xi^*) + w_\lambda(\xi^*, \xi')] \tag{32}$$

$$w_\lambda(\xi^*, \xi') \leq (\alpha_1 + \alpha_4 + \alpha_5 + 2\alpha_6) w_\lambda(\xi^*, \xi') \tag{33}$$

$\Rightarrow \xi^* = \xi'$, $\therefore$ Thas a unique FP.

Corollary 3.2: *Let X position a non-empty set, w position a MM on X and $T: X_w^* \to X_w^*$ be defined as follows: Eq. (34).*

$$w_{k\lambda}(Ta, Tb) \le \alpha_1 w_\lambda(a, b) + \alpha_2 w_\lambda(a, Ta) + \alpha_3 w_\lambda(b, Tb) + \alpha_4 w_\lambda(a, Ta) + \alpha_5 w_\lambda(b, Ta) \tag{34}$$

Assuming that each $\lambda > 0$, there occurs an $a_0 \in X_w^*$ such that $w_\lambda(a_0, Ta_0) < \infty$ then the mapping T has a FP in X_w^*. Also, the FP of T is exceptional.

Proof: Assume that $\alpha_6 = 0$ in the preceding theorem attained the outcome.

4. Applications

Implementing the FP mode confirms that Dynamic Programming (DP) will achieve the expected outcome. As the outcome, it will be extracted as an end result of dynamic routing, which incorporates unpredictability for an anonymous WSN. Dual state procedures — a State Space (SS) and a Decision Space (DS) — constitute a Decision Process (DP). SS can be divided into three separate states: the initial state, the transitional state, and the action state. On the contrary, DS describes a technique that is recommended with the objective of reproducing the result of a specific issue. Both optimization and machine learning make a great deal of algorithms that are highly identical to their respective fields.

The total number of detector hits in a WSN that are utilized ranges between 100 to 1000. Frequently, they share information with one another via satellite, and at other times, they proceed immediately to a Base Station (BS). If there are more WSNs, it is possible to collect data on a broad region precisely, decreasing the risk of attacks. In most cases, SN is distributed over a significant sensor region and transmits data of the highest level. Data will be received from multiple comparable SNs, followed by it will be sent directly to the BS (Figure 1).

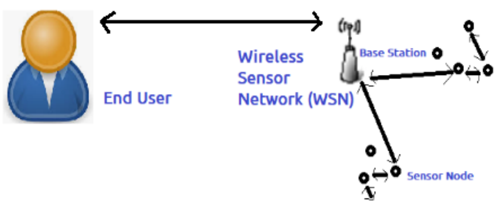

Figure 1
Data aggregation of SN using WSN

For the objective of performing dynamic routing with selection in a WSN, which includes *'N'* nodes while maintaining the network's safety concerning transmitting data, Bellman developed a technique in 1958 that connects in N–1 (or lower) duplicate data. The approach was created via the FE principle. By using Eq. (35) and Eq. (36), one can attempt to describe the challenge of DP in the framework of a FE.

$$k_1(s) = \max_{t\in T}\left\{K(s,t) + f_1\left(s,t,k_1\big(\delta(s,t)\big)\right)\right\} \text{ for } s \in T \tag{35}$$

$$k_2(s) = \max_{t\in T}\left\{K(s,t) + f_2\left(s,t,k_1\big(\delta(s,t)\big)\right)\right\} \text{ for } s \in T \tag{36}$$

Now, $\delta: T_1 \times T_2 \to T_1$; $K: T_1 \times T_2 \to T_3$; $f_1, f_2: T_1 \times T_2 \times T_3 \to T_3$

Assume T_1 and T_2 are the DS and State space, respectively. This study observes a comparable result point frequency for the below equations. In order to express a set of true-valued limited mappings on 'T', it's appropriate to apply the sign '*W(T)*'. Supposing an arbitrary element $j \in W(T)$. Assume the below-mentioned theory set (S1) K, f_1, f_2 are limited (S2) for $s \in T, j \in W(T)$ state $C, D: W(T) \to W(T)$ by Eq. (37) to Eq. (40).

$$Cj(u) = \max_{t\in T}\left\{K(s,t) + f_1\left(s,t,j\big(\delta(s,t)\big)\right)\right\} \text{ for } s \in T \tag{37}$$

$$Dj(u) = \max_{t\in T}\left\{K(s,t) + f_1\left(s,t,j\big(\delta(s,t)\big)\right)\right\} \text{ for } s \in T \tag{38}$$

Proved that the functions K, f_1 and f_2 are all limited. (S3) For $(s,t) \in T_1 \times T_2, j, k \in W(T)$ and $h \in T$ $|f_1(s,t,j(h) - f_1(s,t,k(h)| \le M(j,k)$

Where,

$$M(j,k) = \alpha_1 w_\lambda(j,k) + \alpha_2 w_\lambda(j,Tj) + \alpha_3 w_\lambda(k,Tk) + \alpha_4 w_\lambda(j,Tj) + \alpha_5 w_\lambda(k,Tj) + \alpha_6[w_\lambda(k,Tj) + w_\lambda(j,Tk)] \tag{39}$$

For all

$$0 < \lambda < \lambda_0 \text{ and } \alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 + \alpha_5 + 2\alpha_6 < 1. \tag{40}$$

5. Conclusion and Future Work

A selection of significant Fixed Point (FP) proofs for different dimensions have been provided in this article, which highlighted the significance and feasibility of the Modular Metric space (M-MS). For example, it can be stated that the FP and average M-MS of a contractive mapping are highly useful, even if the implied scenario has no impact on the complete M-MS. The conclusion is that the FP and typical M-MS of a contractive mapping are valuable even if the contract condition is not forced on the entire M-MS. Nodes may be used in some Wireless Sensor Network (WSN) applications to track specific physical events or environmental factors. In this context, a "Fixed Point" could be a location where Sensor Nodes (SN) are deployed to detect the environment uninterruptedly. An FP in network stability can be defined as a state or configuration where network throughput, power consumption, or reliability are maintained at or within acceptable levels. Founding and maintaining an FP could be a vital goal in WSN design and management, mainly in dynamic environments. There is a practical corollary derived from the demonstrated results. In the context of the implications being researched in the real world, the software's application of these results to multi-stage algorithm optimization will also be addressed. In conclusion, there is a conflict in the use-case sector involving developing and formulating the Functional Equation (FE), which results in dynamic routing with selection and enhances security functions within a WSN.

The present model's focus is to develop this idea further into the framework for bipolar fuzzy MS and to predict the probable existence of a bipolar fuzzy metric space in future studies.

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