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Bianchi type III inflationary universe with flat potential and stiff fluid distribution in general relativity

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Abstract

In this study, we looked at the "Bianchi Type III stiff fluid cosmological model in general relativity". To get an inflationary solution we have used appropriate condition between matric potential. Here n is non-negative integer and B and C are dependent function of time "t". The structure characteristics of model are also pointed. The behaviour of the model under different physical conditions is also discussed.

Subject Classification: *83C05.*

Keywords: *Stiff fluid, Flat potential, Bianchi type III, General relativity.*

1. Introduction

In resent framework cosmology become an extraordinary subject to understand the study about the universe. So, in order to examine the inflationary universe, we take into account the general Bianchi type III space-time.

The extraordinarily quick rapid enlargement of the universe's early stages is known as inflation by at least a factor of 10^{78} in volume, driven by a vacuum with negative pressure. The inflation hypothesis was proposed

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by Guth [1]. Afterward it was proposed by Sato [2] inflation significance is crucial to the resolution of a variety of issues in cosmology like homogeneity, isotropy, horizon, monopole in grand unified theory [3]. The Higgs field with potential and the scalar field φ are central themes in cosmological research. This paper has surveyed the extensive body of research dedicated to these concepts, shedding light on their significance within the inflationary scenario and their broader implications for our understanding of the cosmos. Future investigations into these areas hold the promise of unravelling the mysteries surrounding the origins and evolution of the universe.

The Stiff Fluid cosmological model has attracted considerable interest in the study of the universe. In this model, the speed of ray(light) is equated to the speed of sound, and its governing equations exhibit similarities with those describing gravitational fields [4]. Barrow has delved into the significance of the stiff equation of state (p=ρ) in relation to the early stages of the universe's evolution [5]. Furthermore, researchers such as Mark et al. [6], Bali and Ali [7], and Roy and Singh [8] have investigated the stiff fluid cosmological model within the framework of general relativity, each exploring different contexts. Additionally, Bianchi Type III models have been explored by Poonia and Sharma [9-17], albeit in distinct contexts.

This paper presents an exploration of the "Bianchi Type III cosmological model" incorporating stiff fluid within the realm of general relativity. To obtain a determinate solution, we utilize the condition $C = B^n$, where n is a non-negative integer, and both B and C are dependent functions of time. The paper also emphasizes the structural characteristics of this model.

2. Field Equation and Metric:

The equation defining the space-time of Bianchi Type III is as follows

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 \tag{1}$$

In this context A, B, and C represent dependent functions solely dependent on time 't', while α remains a constant

We assume the coordinate to be commoving vectors so that

$$v = v^2 = v^3 = 0, v^4 = 1^1$$

When coupled minimally with a scalar field and accompanied by a scalar field potential V (φ), the gravitational field are demonstrates

$$L = \int \sqrt{-g}\left(R - V(\varphi) - \frac{1}{2} g_{ij}\partial_i \varphi \partial^j \varphi\right) d^4 x \tag{2}$$

Einstein Field Equation is given by

$$R_{ij} = \frac{1}{2} R g_{ij} = -T_{ij} \tag{3}$$

With

$$T_{ij} = (p+\rho)v_i v_j + p g_{ij} - \partial_i \varphi \partial_j \varphi - \left[\frac{1}{2}\partial_p \varphi \partial^p \varphi + V(\varphi)\right] g_{ij} \tag{4}$$

And

$$\frac{1}{\sqrt{-g}}\partial_i\left[\sqrt{-g}\,\partial^i \varphi\right] = -\frac{dV}{d\varphi} \tag{5}$$

Using matric (1) and field equation (3), we get

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -\frac{1}{2}\varphi_4^2 + V(\varphi) - P \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -\frac{1}{2}\varphi_4^2 + V(\varphi) - P \tag{7}$$

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} - \frac{\alpha^2}{A^2} = -\frac{1}{2}\varphi_4^2 + V(\varphi) - P \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{\alpha^2}{A^2} = \frac{1}{2}\varphi_4^2 + V(\varphi) + \rho \tag{9}$$

$$\alpha\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0 \tag{10}$$

Where indices 4 represent differential with respect to t

3. Field equation with solution:

Using equation (10)

$$A = KB \tag{11}$$

Where K is constant. Taking K=1

$$A = B \tag{12}$$

Using equation (12) the field equation
(6)- (9) becomes

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-\frac{1}{2}\varphi_4^2+V(\varphi)-P \tag{13}$$

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{\alpha^2}{B^2}-=-\frac{1}{2}\varphi_4^2+V(\varphi)-P \tag{14}$$

$$\frac{B_4^2}{B^2}+2\frac{B_4C_4}{BC}-\frac{\alpha^2}{B^2}=\frac{1}{2}\varphi_4^2+V(\varphi)+\rho \tag{15}$$

We are taking flat potential so

$$\frac{dV(\varphi)}{d\varphi}=0 \tag{16}$$

Using equation (5) and (16) we obtain

$$\varphi_{44}+\varphi_4\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \tag{17}$$

Applying stiff fluid condition $p=\rho$ we have

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{B_4}{B}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)-\frac{\alpha^2}{B^2}=0 \tag{18}$$

We are taking

$$C=B^n \tag{19}$$

$$\frac{B_{44}}{B}+(1+n)\frac{B_4^2}{B^2}-\frac{1}{1-n}\frac{\alpha^2}{B^2}=0 \tag{20}$$

We consider $B_4=f(B)$ then $B_{44}=ff'$

Where $f'=\frac{df}{dB}$

$$f^2=\left(\frac{dB}{dt}\right)^2=\frac{\alpha^2}{(1-n)^2}+\frac{l}{B^{2(1+n)}} \tag{21}$$

Metric (1) leads to

$$ds^2=-\left(\frac{\alpha^2}{(1-n)^2}+\frac{l}{B^{2(1+n)}}\right)dT^2+T^2dX^2+T^2e^{-2\alpha X}dY^2+T^{2n}dZ^2 \tag{22}$$

Taking suitable transformation

S x=X, y=Y, z=Z ,B=T

4. Physical and Geometrical Aspects

$$\frac{\varphi_{44}}{\varphi_4} = -\left(2\frac{B_{44}}{B} + \frac{C_4}{C}\right) \tag{23}$$

$$\varphi_4 = \frac{\gamma}{B^2 C} \quad \gamma \text{ Is a constant} \tag{24}$$

$$\varphi_4 = \frac{\gamma}{B^{n+2}} \tag{25}$$

$$d\varphi = \frac{\gamma}{T^{n+2}} dt \tag{26}$$

$$\varphi = -\gamma \frac{n+2}{2(n+1)} \sqrt{\left(\frac{\alpha^2}{(1-n)^2 T^{2(n+1)}} + \frac{l}{T^{4(1+n)}}\right)} + \gamma_1 \tag{27}$$

The Volume scale factor is given by

$$R^3 = V = B^2 C = B^{n+2} = T^{n+2} \tag{28}$$

The expansion scalar θ can be obtain form

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{29}$$

$$\theta = (n+2)\sqrt{\frac{\alpha^2}{1-n)^2 T^2} + \frac{l}{T^{4(1+n)}}} \tag{30}$$

And shear scalar σ can be obtain form

$$\sigma^2 = \frac{1}{2}\sigma_{\mu v}\sigma^{\mu v} \tag{31}$$

$$\sigma^2 = (2 - 4n + 4n^2)\left[\frac{\alpha^2}{(1-n^2)T^2} + \frac{l}{T^{4(1+n)}}\right] \tag{32}$$

And Hubble's parameter is

$$H = \sum_{i=1}^{3} H_i \tag{33}$$

Where $H_1 = \frac{A_4}{A}$, $H_2 = \frac{B_4}{B}$, $H_3 = \frac{C_4}{C}$

$$H = \frac{(n+2)}{3}\sqrt{\frac{\alpha^2}{(1-n^2)T^2} + \frac{l}{T^{4(1+n)}}} \tag{34}$$

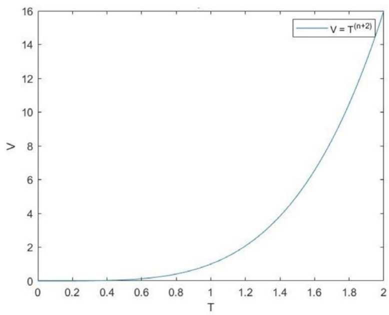

Figure 1

The plot of proper volume V vs. T

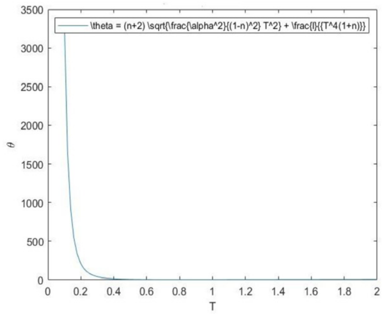

Figure 2

The expansion scalar θ vs. T

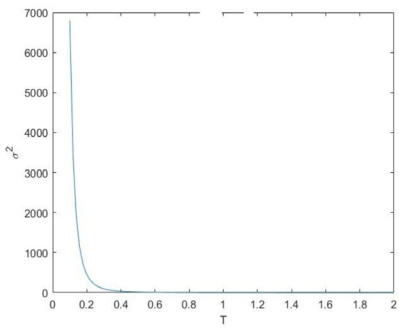

Figure 3

The shear scalar σ vs. T

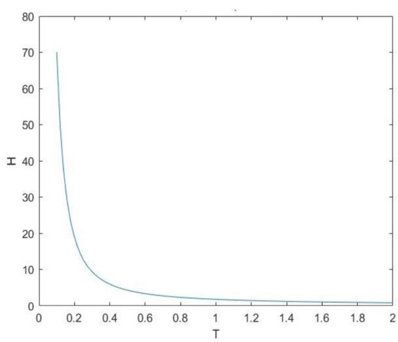

Figure 4

The Hubble parameter H vs. T

Figure 1: Represents Eq. 28. The spatial volume is zero at T = 0 and it increases with the increase of T

Figure 2: Represents Eq. 30. The values of theta increase, the values of T also tend to increase.

Figure 3: Represents Eq. 32. The values of sigma squared increase, the values of T also tend to increase.

Figure 4: Represents Eq. 34. The value of Hubble Parameter H increases, the value of T also increases.

4. Conclusion

Based on the presented results, it is evident that the spatial volume (V) exhibits a value of zero at T=0 and progressively increase with the time progresses (T). This finding highlights the underscores the dynamic nature of the universe's development and evolution of the universe, which initiates with a zero volume at T=0 and subsequently undergoes expansion over cosmic time (T). Additionally, it is noteworthy that the shear scalar (σ) approaches zero as T approaches infinity $(T \to \infty)$, as evidenced by equation (34). Furthermore, the Hubble parameter, which is also derived from the same equation, exhibits a similar trend. It shows a tendency to move towards a zero value as the time variable (T) approaches an infinite value. This further complements our understanding of the evolving cosmic parameters over an extended period towards infinity.

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