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# A mathematical analysis of the PAPR in the NOMA waveform using SLM algorithm for 64-QAM

Arun Kumar [‡]
*Department of Electronics and Communication Engineering*
*New Horizon College of Engineering*
*Bengaluru*
*Karnataka*
*India*

Anish Pon Yamini [†]
*Department of Electronics and Communication Engineering*
*Vel Tech Rangarajan Dr Sagunthala R & D Institute of Science and Technology*
*Tamil Nadu*
*India*

Nishant Gaur [§]
*Department of Physics*
*JECRC University, Jaipur*
*Jaipur*
*Rajasthan*
*India*

Aziz Nanthaamornphong*
*College of Computing*
*Prince of Songkla University*
*Phuket*
*Thailand*

**Abstract**

In Non-Orthogonal Multiple Access (NOMA) technology, a paradigm shift in wireless communication is observed, as it allows numerous users to simultaneously transmit data over

[‡] *E-mail:* dr.arunk.nhce@newhorizonindia.edu
[†] *E-mail:* dranishponyaminik@veltech.edu.in
[§] *E-mail:* nishant.gaur@jecrcu.edu.in
* *E-mail:* aziz.n@phuket.psu.ac.th (Corresponding Author)

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the same spectral resources in a non-orthogonal manner. Despite the advancements it brings to the table, NOMA is faced with a notable technical obstacle, that being the elevated Peak-to-Average Power Ratio (PAPR). Elevated PAPR levels can lead to less efficient use of power and a deterioration in the fidelity of the signal. We proposed a Selective Mapping (SLM) as a powerful solution to address this issue. SLM is a PAPR reduction technique that applies controlled phase rotations to the symbols of different users before transmission, aiming to minimize the PAPR of the combined signal. In the context of NOMA, the application of SLM offers several notable advantages. SLM modifies the Power Spectrum Density (PSD), leading to a more uniform and spectrally efficient power distribution over the frequency spectrum. This enhancement in spectral efficiency concurrently lowers the risk of disrupting nearby frequency channels. Moreover, SLM exerts a favorable influence on the Bit Error Rate (BER) in NOMA systems, which contributes to improved transmission accuracy. Utilizing SLM to achieve lower PAPR values significantly diminishes the probability of symbol errors, thus enhancing the dependability. Additionally, it has been observed that the method in question delivers an improvement in PAPR, BER, and PSD with gains of 2.2 dB, 4 dB, and -620 respectively, in comparison to traditional SLM techniques.

***Subject Classification:*** *94A05.*

***Keywords:*** *5G, NOMA, PAPR, SLM.*

## 1. Introduction

NOMA represents a cutting-edge technique in wireless communications that has been the focus of substantial interest lately. This is largely because of its capability to enhance spectral efficiency and accommodate an extensive array of devices, a feature that is particularly relevant as we progress into the 5G era and beyond. One of the key challenges in NOMA is managing the PAPR of transmitted signals, which can lead to inefficient power amplification and increased interference. In this article, we will explore the concept of PAPR reduction in the context of NOMA and its importance in enhancing the performance of modern wireless networks [1]. NOMA marks a significant shift from conventional orthogonal access methods such as Time-Division Multiple Access (TDMA) and Frequency-Division Multiple Access (FDMA), which allocate distinct, non-overlapping time slots or frequency bands to each user for their communication needs [2]. In contrast, NOMA permits several users to concurrently utilize identical time-frequency resources, segregating them by distinct power levels and employing the code domain for differentiation. This non-orthogonal sharing of resources enables higher spectral efficiency, as more users can simultaneously access the network. However, it also introduces challenges related to signal separation and power control. Dynamic power allocation is another key approach to

PAPR reduction in NOMA. This technique adjusts the power allocation for users in real-time, aiming to balance the PAPR across different users and time-frequency resources [3]. By dynamically allocating power, NOMA systems can achieve better PAPR control while maintaining efficient power utilization. Furthermore, PAPR reduction techniques in NOMA can facilitate the coexistence of a massive number of devices in a shared spectrum, a crucial aspect in the era of the Internet of Things (IoT) and smart cities [4].

## 2. Literature Review

The research presented focuses on reducing the PAPR in NOMA systems through an innovative SLM technique. This novel SLM approach is applied to the transmitter side of F-NOMA, in contrast to conventional SLM techniques that tend to increase system complexity. The effectiveness of SLM in enhancing F-NOMA is evaluated for transmission systems utilizing 16, 64, and 256-Quadrature Amplitude Modulation (QAM) schemes. A variety of transmission patterns are used to estimate and compare parameters, including BER, PAPR, PSD, and complexity [5]. The authors in this paper figured out the best way to use SLM, PTS, and CT together for O-NOMA waveforms that are not orthogonal. When applied to O-NOMA waveforms, the SLM-PTS-CT hybrid technique effectively reduces PAPR [6]. This research introduces an effective strategy for diminishing PAPR that leverages both precoding techniques and the insertion of dummy sequences (DSI). Furthermore, a novel methodology for the creation of dummy sequences has been formulated for this proposed strategy. The results demonstrate that the presented scheme surpasses traditional precoding and PTS methods in reducing PAPR and improving BER performance. In addition, it is far less computationally demanding than PTS approaches [7]. The study presents a distinctive SLM method tailored for OFDM-NOMA systems to lower PAPR. This proposed SLM technique delivers performance on par with the traditional SLM approach while reducing computational complexity by exploiting the inherent design of the OFDM-NOMA transmitter [8]. SAG performs better than the active constellation extension in the first two iterations when compared to the gradient project (SGP) and projection onto convex sets (ACE-POCS) approaches. Consequently, the method proposed is more effective at lowering the PAPR while maintaining an acceptable probability of error and is characterized by a substantially reduced computational complexity [9]. For a higher spectrum efficiency, OFDM can be paired

with NOMA. With the OFDM-based NOMA (OFDM-NOMA) scheme, the highest PAPR is its primary disadvantage. Consequently, a novel discrete-cosine transform is presented in this study. Uplink OFDM-NOMA technique for PAPR reduction based on matrix precoding [10]. The contributions of the proposed article are given below:

- The proposed SLM effectively reduces the PAPR of transmitted signals in NOMA, mitigating distortion and spectral regrowth, which can interfere with system performance.
- SLM retains the BER performance in NOMA systems by reducing the likelihood of signal clipping and improving the SNR at the receiver.

## 3. Mathematical Model of NOMA-SLM waveform

The mathematical equations for NOMA waveform transmissions and reception can be quite complex and depend on various factors, including the modulation scheme, coding, user prioritization, and channel conditions. Assuming that we have two users (User 1 and User 2) with different power allocation and data symbols, the transmit signals for each user can be represented as follows [11-14]:

For User 1:

$$x(t) = \sqrt{P} * s(t) * \cos(2\pi f_c t + \varphi_1) \tag{1}$$

For User 2:

$$x(t) = \sqrt{P} * s(t) * \cos(2\pi f_c t + \varphi_2) \tag{2}$$

Where $x_1(t)$ and $x_2(t)$ are the transmitted signals for User 1 and User 2, respectively, $\sqrt{P_1}$ and $\sqrt{P_2}$ are the square root of the power allocated to User 1 and User 2, $s_1(t)$ and $s_2(t)$ represent the data symbols for User 1 and User 2. $f_c$ is the carrier frequency, $\varphi_1$ and $\varphi_2$ are the phase terms. At the receiver, the received signal y(t) can be expressed as the superposition of the signals from both users, taking into account the channel effects:

$$y(t) = h_1 * x_1(t) + h_2 * x_2(t) + n(t) \tag{3}$$

Where: $h_1$ and $h_2$ represent the complex channel gains for User 1 and User 2, $n(t)$ is the noise at the receiver. Let x(t) be the NOMA waveform:

$$P_{peak} = \max(|x(t)|^2), \text{for all } t \tag{4}$$

The average power is determined by integrating the square of the signal's magnitude over its duration (T) and then dividing by T:

$$P_{avg} = (1/T)\int[0\ \text{to}\ T]\,|x(t)|^2\,dt \tag{5}$$

The PAPR can be calculated as the ratio:

$$PAPR_{dB} = 20\,l0g_{10}\frac{P_{peak}}{P_{avg}} \tag{6}$$

SLM is a sophisticated technique employed to mitigate the detrimental effects of high PAPR in NOMA waveforms. For each user in the NOMA system, the transmitted signal x(t) can be represented as follows:

$$x(t) = \sqrt{P} * s(t) * \exp(j\theta) \tag{7}$$

P is the average power allocated to the user, s(t) is the modulated signal waveform and θ is the phase rotation applied to the signal, which is selected from a predefined set of possible phase values. Let $\theta_i$ denote the $i_{th}$ possible phase rotation:

$$x_i(t) = \sqrt{P} * s(t) * \exp(j\theta_i) \tag{8}$$

Where i = 1, 2, ..., N represents the N candidate signals created for each user. To determine the PAPR for each potential signal, the peak power is divided by the average power. For a given signal $x_i(t)$, $PAPR_i$ can be calculated as:

$$PAPR_{i\,dB} = 20\ \log_{10}\frac{(|x_i(t)|^2)}{E[|x_i(t)|^2]} \tag{9}$$

Where $E[|x_i(t)|^2]$ is the expected value of the squared magnitude of $x_i(t)$ over time. The candidate signal with the lowest PAPR, denoted as $x_\min(t)$, is selected for transmission:

$$x_{\min}(t) = \arg\min(PAPR_i),\ \text{where}\ i = 1, 2, ..., N \tag{10}$$

The calculation of the BER for a NOMA waveform involves various factors, including modulation scheme, channel model, and specific system parameters. The BER $(P_b)$ for a NOMA waveform can be expressed as:

$$P_b = 1/2 * erfc(sqrt(SNR)) \tag{11}$$

Where: $P_b$ is the bit error rate, *erfc* is the complementary error function. The CCDF for a NOMA waveform can be expressed mathematically as:

$$P(X > x) = 1 - F_X(x) \tag{12}$$

Where $P(X > x)$ is the probability that the power of the NOMA waveform exceeds the threshold x and $F_X(x)$ is the CDF of the power of the NOMA waveform.

## 4. Simulation Results Analysis

In the proposed work, we have estimated the performance of the SLM approach on NOMA waveform. The simulation is performed on Matlab-2016. The simulation parameters considered such as 10000 symbols, 64-sub-carriers, 64-QAM, 64-FFT under a Rayleigh channel. The PAPR analysis of the multi carrier advanced waveforms is shown in Figure 1. At a complementary cumulative distribution function (CCDF) threshold of $10^{-3}$, the PAPR for NOMA stands at 8.7 dB, for FBMC it is 9.1 dB, for UFMC it is 10.6 dB, and for OFDM it reaches 12.5 dB, in scenarios where PAPR reduction techniques are not applied. OFDM exhibits a relatively high PAPR. This is because it divides the signal into orthogonal subcarriers, and their amplitudes can add constructively, causing high peaks in the composite signal. FBMC is designed to have a lower PAPR compared to OFDM. It achieves this by using filter banks to shape the signal in both the time and frequency domains, reducing peak amplitudes. UFMC shares a similar characteristic with FBMC in terms of having a lower PAPR compared to OFDM. Its use of filtering and overlapping subcarriers helps reduce PAPR, making it more power-efficient. NOMA, as a multiple access technique, does not have its own inherent waveform. The PAPR in NOMA depends on the modulation and access schemes used within the NOMA framework. It can vary, but when multiple users share the same resources, it can potentially lead to PAPR levels comparable to or lower than those in OFDM. In Figure 2, SLM is used to reduce the PAPR in NOMA systems. The original NOMA signal has a PAPR of 9.1 dB at a CCDF of $10^{-3}$. This PAPR is reduced to 7 dB by the Standard Selected Mapping (SLM) technique, to 5.3 dB by the PTS technique, and further down to 3.8 dB by the proposed SLM algorithm. The proposed SLM algorithm demonstrates an improvement, achieving a gain of 2.2 dB over the conventional SLM method. In NOMA, SLM introduces controlled phase rotations to the symbols of different users before transmission. By adjusting the phases, SLM redistributes the power more evenly across the symbols, reducing peak amplitudes. This results in a lower PAPR, which is desirable for practical communication systems as it allows power amplifiers to operate

more efficiently and with reduced distortion. Figure 3 depicts the BER performance for the NOMA system. A BER of $10^{-3}$ is achieved at SNRs of 3 dB, 4.5 dB, 5.8 dB, and 8 dB by the proposed SLM, PTS, conventional SLM, and the standard NOMA waveform, respectively. This demonstrates that the proposed SLM method effectively maintains the BER performance. SLM is a PAPR reduction technique in NOMA, and it can have a positive impact on BER performance. By reducing the PAPR in NOMA signals, SLM mitigates signal distortion and improves the linearity of power amplifiers. This, in turn, can lead to a reduction in signal clipping and nonlinearities, resulting in a lower BER. SLM can enhance the BER performance in NOMA by reducing the likelihood of symbol errors, especially in high PAPR scenarios, ultimately contributing to more reliable and efficient communication in multi-user systems. However, the extent of BER improvement depends on the specific SLM implementation and system parameters. PSD describes how the power of a signal is distributed across various frequency components. Figure 4 presents the PSD for NOMA utilizing SLM and PTS techniques. The PSD values achieved are -880, -1050, -1100, and -1500 for NOMA, SLM, PTS, and the proposed SLM methods, respectively. In NOMA systems, the PSD is impacted by PAPR reduction techniques such as SLM and PTS. SLM works by redistributing the signal's power, which contributes to a lower PAPR, and this redistribution inherently modifies the PSD of the signal. SLM typically results in a flatter and more evenly distributed PSD, reducing spectral regrowth and out-of-band emissions. PTS is another PAPR reduction method. It selectively combines subsets of signal constellations to minimize the PAPR.

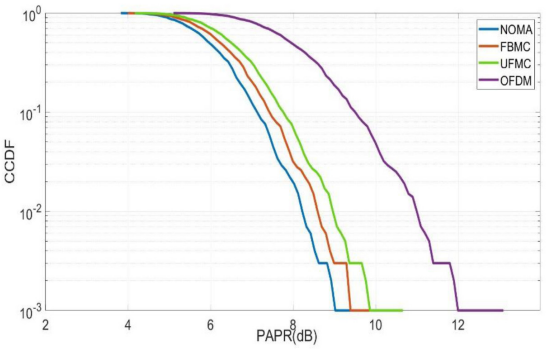



**Figure 1**

**PAPR of Advanced waveforms**

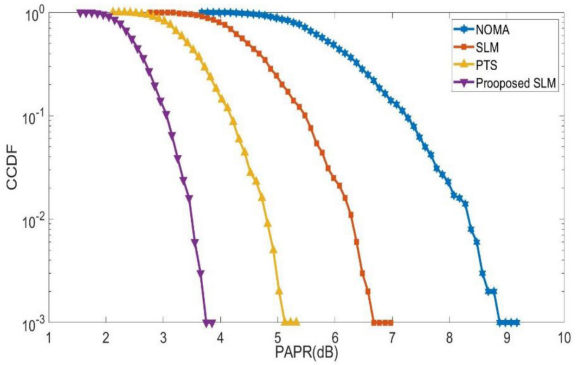



**Figure 2**

**PAPR analysis of NOMA with SLM and PTS**

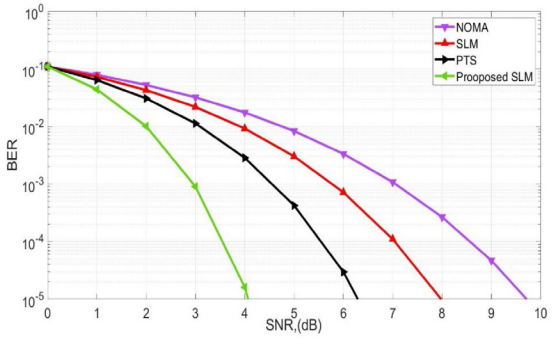



**Figure 3**

**BER analysis of NOMA with SLM and PTS**

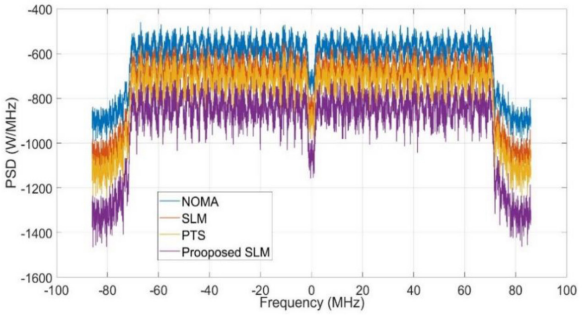



**Figure 4**

**PSD analysis of NOMA with SLM and PTS**

## 4. Conclusion

The utilization of SLM for PAPR reduction in NOMA systems is highlighted in this paper as a promising advancement for the efficacy and dependability of contemporary communication networks. The benefits that the proposed SLM technique introduces to NOMA are significant. By lowering PAPR, it facilitates a more straightforward design of power amplifiers and decreases the likelihood of signal distortion and spectral regrowth. This leads to an improved overall system performance and a decrease in interference with adjacent frequency bands, which is vital for achieving a communication system that is both spectrally efficient and resilient to interference. Additionally, SLM's impact on the BER is equally important. The reduced PAPR through SLM contributes to lowering symbol errors, thereby ensuring a more dependable and error-resistant communication environment, particularly in multi-user NOMA contexts.

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