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A study of different class of almost Para-Sasakian Riemannian manifold

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Abstract

In this work, the author thought about and looked into the Nijenhuis tensor, the Affine Connection, and the conformal curve tensor on Nearly Para Sasakian different class-Riemannian differential Manifold. We developed and proved the different results for above mentioned class. We also established new condition of Geodesic for different class-Riemannian differential Manifold.

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Keywords: *Almost Para Sasakian manifold, Nijenhuis tensor, Affine connection, Conformal curvature, Geodesic.*

1. Introduction

Sato [1] came up with the idea of the almost para-contact manifold and the para-contact metric manifold for the first time. Matsumoto et al. [2] described and studied unique cases of "Almost Para Contact-Riemannian Differential manifold." This kind of manifold is also called a para-sasakian manifold and a unique para-sasakian manifold. Adati et al. [3, 8], and Chuman [4] are among the more recent researchers who have made contributions to the study of a large number of classes of nearly

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para-contact multifarious equipped with a Riemannian measure.The idea of a quasi-Sasakian manifold was first put forward by D. E. Blair in 1967 [12], and S. Tanno made some important discoveries in 1971 [11]. Both of these are things that D. E. Blair has done. The plan for dealing with the problems and the ways that they are solved are fairly new. Tensors are shown by notes that don't have any indicators, and we've been thinking of a tensor as a vector-valued, multi-direct function all along.Sezin Aykurt Sepet and Mahmut Ergut [18] looked at the geometric properties of pointwise slant submersion and listed many requirements that must be met for a pointwise slant submersion to be called a Riemannian submersion. According to the results of the study, pointwise slant submersions could help us learn more about differentiable manifolds and give us clues about how Riemannian manifolds are put together.

Affine connections are objects of geometry on smooth manifolds that connect tangent spaces that are close to each other. Because of this, tangent vector fields can stand out as if they were skills at the manifold, and their values are contained in a set vector space. But the idea of affine connection wasn't presented until the early 1920s by Élie Cartan (as part of his well-known connection theory) and Hermann Weyl (who used this idea as part of the inspiration for his well-known theory of relativity). In mathematics and tensor calculus, the idea of a "affine connection" was first presented in the late 1800s. A lot of people, like R. S. Mishra [7, 8, 9], Lai, K. F., and Mok, K. P. [15], and Murat, A., and Aydin. G. [16], added to the way affine relationships could be found later on. Porwal S. K. & S. K. MishraS. K. [19] have also come to the same conclusion: geodesic semi local E-convex functions could be used to solve planning problems and could help with further research.

The conformal curvature tensor is a subtype of the Weyl tensor's curvature tensor that has its own features. When the metric g is changed in a conformal way, that is, when g becomes less than g, for certain positive scalar functions. This work looked at the type (1, 3) [3] and talked about how it helped people.

Definition 1.1: Assume that M is an n-dimensional Riemannian differential manifold defined by a tensor field (λ), a vector field (ζ) , and a 1-form (μ) that is appropriate for any arbitrary vector field A, B, or C.

(a) $\lambda^2 A = A - \mu(A)\zeta$, (1.1)

(b) $\lambda(\zeta) = 0$

(c) $\mu(\lambda A) = 0$

(d) $\mu(\zeta) = 1$.

Then (λ, ζ, μ) is known as almost para-contact structure& (M, λ, ζ, μ) be an almost para -contact manifold [5], [6], [12].

We can now introduce a metric on the considered manifold. From [5], [6], [12] we know that the manifold M admits a Riemannian metric g which is compatible with the structure of the manifold as follows:

(e) $g(\lambda A, \lambda B) = -g(A, B) + \mu(A)\,\mu(B)$.

Then (λ, ζ, μ, g) is known as almost para- contact metric structure [5], [6], [12] and $(M, \lambda, \zeta, \mu, g)$ will be called "Almost para-contact Riemannian differential manifold"[5], [6], [12], [13], [14], [15].

In[1], [5], [6] it is considered Fundamental two form λ'(say) of type (0,2)for arbitrary vector field A,B, which is defined as,

(f) $\lambda'(A,B) = g(\overline{A}, B)$, where $\overline{A} = \lambda A$ & g is metric tensor.

we can verify that Fundamental two form λ' is skew- symmetric for vector field A and B. i.e. $\lambda'(A, B) + \lambda'(B, A) = 0$, we can check from(1.1) (e),after replacing arbitrary vector field A & B by a vector field ζ & an arbitrary vector field A respectively, we get the following result

(g) $g(\zeta, A) = \mu(A)$.

Definition 1.2: Assume $(M, \lambda, \zeta, \mu, g)$ be an almost para contact Riemannian differential manifold whose fundamental 2 – form λ' satisfies the condition.

$$2\lambda' = d\mu. \tag{1.2}$$

If this is the case, it can be called a nearly para-sasakian different class Riemannian differential manifold or a contact Riemannian differential manifold. [5], [6], [7], [12]

We considered the following results of an almost para-sasakian type Riemannian differential manifold [6].

$$2\lambda'(A, B) = (D_A\mu)(B) - (D_B\mu)(A). \tag{1.3}$$

$$(d\lambda') = 0 \Leftrightarrow (D_A\lambda')(B,C) + (D_B\lambda')(C,A) + (D_C\lambda')(A,B) = 0. \tag{1.4}$$

where D is the Riemannian connection.

An almost para-sasakian kind Riemannian differential manifold , on which (DxA)(Y) + (DyA)(X) = 0 , then it's known as para-K-contact type Riemannian manifold [6]. Now we taken into consideration the subsequent consequences of para-K-contact type Riemannian differential manifold [6],

$$\lambda' (A,B) = (D_A \mu)(B) = -(D_B\mu)(A). \tag{1.5}$$

$$(D_C\lambda')(A,B) = \mu(K(A,B,C)). \tag{1.6}$$

$$(D_A \zeta) = -\bar{A}. \tag{1.7}$$

An almost para-contact type Riemannian differential manifold is also called PKCT Riemannian differential manifold if(1.7) is hold [6].

$$(D_A \lambda')(B, \zeta) = g(\bar{A}, \bar{B}). \tag{1.8}$$

$$(D_C\lambda')(\bar{A}, \bar{B}) - (D_C\lambda')(A,B) - g(C,A)\,\mu(B) + g(C,B)\mu(A) = 0. \tag{1.9}$$

$$(D_C\lambda')(A,B) = \mu(B)(D_C\lambda')(A,\zeta) - \mu(A)(D_C\lambda')(B,\zeta). \tag{1.10}$$

$$\begin{aligned}(D_{\bar{A}} \lambda)(B) - (D_{\bar{B}} \lambda)(A) + (D_A \lambda)(\bar{B}) - (B_A \lambda)(\bar{A}) \\ -\mu(B)(\bar{A}) + \mu(A)(\bar{B}) + 4\lambda'(A,B)\zeta = N(A,B).\end{aligned} \tag{1.11}$$

$$\begin{aligned}(D_C\lambda')(\bar{A}, B) + (D_C\lambda')(A, \bar{B}) = -\mu(B)\lambda'(A,C) + \mu(A) \\ \lambda'(B,C) - 4\lambda'(A,B)\mu(C).\end{aligned} \tag{1.12}$$

Using the above results from (1.1- 1.12), we study two problems, firstly we obtain some results for Nijenhuis Tensor. Secondly we obtain some useful results for Affine Connection

2. Some results for Nijenhuis Tensor on PKCT- Riemannian differential manifold

Theorems 2.1: *Nijenhuis tensor is skew symmetric for arbitrary vector field A, B iff Structure (M , λ , ζ, μ, g) is Para K-contact type Riemannian differential manifold i.e.*

$$N(A, B) = L(A, B) - L(B, A) = -N(B, A) \tag{2.1}$$

Proof: Consider a tensor of type (0, 2) with vector field A & B, Riemannian connection D, tensor field λ of type (1, 1) and 1-form μ

$$L(A, B) \overset{\text{def}}{=} (D_{\bar{A}}\lambda)(B) - \overline{(D_A \lambda)(B)} + (D_A \mu)(B)\zeta$$

$$= (D_{\bar{A}}\lambda)(B) + (D_A\lambda)(\bar{B}) - \mu(B)(\bar{A}) + 2(D_A\mu)(B)\zeta.$$

Now , we obtain $L(A, B) - L(B, A) = (D_{\bar{A}}\lambda)(B) - (D_{\bar{B}}\lambda)(A) + (D_A\lambda)(\bar{B}) - (D_B\lambda)(\bar{A}) - \mu(B)(\bar{A}) + \mu(A)(\bar{B}) + 4\lambda'(A,B)\zeta$,and using (1.11), the result (2.1) hold.

Theorem 2.2: *In Para K-contact type Riemannian differential manifold* $(M, \lambda, \zeta, \mu, g)$

If, $'Y(A, B, C) \overset{\text{def}}{=} g(L(A, B), C)$, Then

$$'Y(A, B, C) + 'Y(A, C, B) = 2[\lambda'(A,B)\mu(C) + \lambda'(A,C)\mu(B)]. \tag{2.2}$$

Proof: Consider a tensor of type (0, 3) with vector field A, B & C, Riemannian metric g & Nijenhuis tensor N

a) $'A(A,B,C) \overset{\text{def}}{=} g(N(A, B), C)$ b) $'Y(A, B, C) \overset{\text{def}}{=} g(L(A, B), C)$.

Take the result

(b) $'Y(A, B, C) \overset{\text{def}}{=} g(L(A, B), C)$.

$= g((D_{\bar{A}}\lambda)B,C) + g((D_{\bar{A}}\lambda)\bar{B}, C) - \mu(B)g(\bar{A}, C) + 2\lambda'(A,B)g(\zeta)$.

we obtain, $'Y(A, B, C) + 'Y(A, C, B) = (D_{\bar{A}}\lambda')(B,C) + (D_{\bar{A}}\lambda')(\bar{B}, C) - \mu(B)\lambda'(A, C) + 2\lambda'(A,B)\mu(C) + (D_{\bar{A}}\lambda')(C,B) + (D_{\bar{A}}\lambda')\bar{C}, B) - \mu(C)\lambda'(A,B) + 2\lambda'(A,C)\mu(B)$.

It can also be written as

$$'Y(A, B, C) + 'Y(A, C, B) = (D_{\bar{A}}\lambda')(\bar{B}, C) + (D_{\bar{A}}\lambda')(\bar{C}, B) + \lambda'(A, C)\mu(B) + \lambda'(A, B)\mu(C).$$

Using (1.10), the result (2.2) hold.

Theorem 2.3: *InPara K-contact type Riemannian differential manifold* $(M, \lambda, \zeta, \mu, g)$*,The Nijenhuis tensor is vanishes iff (2.3)* $'Y(A,B) = 2\{\mu(B)(\bar{A}) + \mu(A)(\bar{B})\}$.

Proof: Consider a tensor of type (0, 3) with vector field A, B & C, Riemannian metric g & Nijenhuis tensor N

a) $\mathbb{A}(A,B,C) \overset{\text{def}}{=} g\,(N\,(A,B),C)$

b) $\Upsilon\,(A,B,C) \overset{\text{def}}{=} g\,(L\,(A,B),C)$.

Take (a) $\mathbb{A}\,(A,B,C) \overset{\text{def}}{=} g\,(N\,(A,B),C)$

$\mathbb{A}(A,B,C) = (D_{\bar{A}}\lambda')\,(B,C) - (D_{\bar{B}}\lambda')\,(A,C) + (D_{\bar{A}}\lambda')\,(\bar{B},C) - (D_B\lambda')\,(\bar{A},C) - \mu(B)\,\lambda'\,(A,C) + \mu\,(A)\,\lambda'\,(B,C) + 4\lambda'\,(A,B)\,\mu\,(C)$, now we obtain

$\mathbb{A}(A,B,C) - \mathbb{A}(B,C,A) + \mathbb{A}(C,A,B) = (D_{\bar{A}}\lambda')\,(B,C) - (D_{\bar{B}}\lambda')\,(A,C) + (D_{\bar{A}}\lambda')\,(\bar{B},C) - (D_B\lambda')\,(\bar{A},C) - \mu\,(B)\,\lambda'\,(A,C) + \mu\,(A)\,\lambda'\,(B,C) + 4\lambda'\,(A,B)\,\mu\,(C) - (D_{\bar{B}}\lambda')\,(C,A) + (D_{\bar{C}}\lambda')\,(B,A) - (D_B\lambda')\,(\bar{C},A) + (D_{\bar{C}}\lambda')\,(\bar{C},A) + \mu\,(C)\,\lambda'\,(B,A) - \mu\,(B)\,\lambda'\,(C,A) - 4\lambda'\,(B,C)\,\mu\,(A) + (D_{\bar{C}}\lambda')\,(A,B) - (D_{\bar{C}}\lambda')\,(C,B) + (D_{\bar{C}}\lambda')\,(\bar{A},,B) - (D_A\lambda')\,(\bar{C},B) - \mu\,(A)\,\lambda'\,(C,B) + \mu\,(C)\,\lambda'\,(A,B) + 4\lambda'\,(C,A)\,\mu\,(B)$, using (1.10), we get $\mathbb{A}(A,B,C) = 2\,\Upsilon\,(A,B,C) - 4\,\{\lambda'\,(A,C)\,\mu\,(B) + \lambda'\,(B,C)\,\mu\,(A)\}$.

This holds the result (2.3)

Theorem 2.4: *OnPara K-contact type Riemannian differential manifold (M, λ, ζ, μ, g) , (2.4) ʻL (A, ζ , C)= g* $(\bar{A},\bar{C})$

Proof: Let us consider a tensor of type (0,2) ([3]

$$L\,(A,B) = (D_A\lambda)\,B + (D_B\lambda)A - \mu\,(A)B - \mu\,(B)A + 2g\,(A,B)\,\zeta$$

& $$'L\,(A,B,C) = g\,(L\,(A,B),C)$$

we easily seen that L(A, B) – L (B, A) =0.

i.e. L (A, B) is symmetric for the arbitrary vector field A & B

Now $'L\,(A,B,C) = g\,(L\,(A,B),C)$

or $$'L\,(A,B,C) = ((D_A\lambda')\,B,C) + ((D_B\lambda')\,A,C) - \mu\,(A)\,g\,(B,C) - \mu\,(B)\,g\,(A,C) + 2g\,(A,B)\,\mu\,(C)$$

Replace B by ζ, we obtain $'L\,(A,\zeta,C) = -((D_A\lambda')\,C,\zeta) - ((D_\zeta\lambda')\,C,A) - \mu\,(A)\,\mu\,(C) - g\,(A,C) + 2\mu\,(A)\,\mu\,(C)$ and using (1.1)(c), (f), (g),we obtain $\lambda'\,(C,\zeta) = g\,(\bar{C},\zeta) = \mu\,(\bar{C}) = 0$, then $'L\,(A,\zeta,C) = -((D_\zeta\lambda')\,C,A) - \mu\,(A)\,\mu\,(C) - g\,(A,C)$, again using (1.8), we obtain $(D_\zeta\lambda')\,(C,A) = g\,(\bar{\zeta},\bar{C}) = 0$, then $'L\,(A,\zeta,C) = \mu\,(A)\,\mu\,(C) - g\,(A,C)$, again using (1.1) (e), we hold result (2.4)

Theorem 2.5: *On Para K-contact type Riemannian differential manifold(M, λ, ζ, μ, g) tensor of type (0, 3) is cyclic i.e. (2.5) ʻL (A, B, C) + ʻL (B, C, A) + ʻL (C, A, B) = 0.*

Proof: Let us consider a tensor of type (0,2) ([3])

$$L(A,B) = (D_A \lambda)\; B + (D_B \lambda) A - \mu(A)B - \mu(B)A + 2g(A,B)\zeta$$

& $$'L(A,B,C) = g(L(A,B), C)$$

i.e. $'L(A,B,C) = ((D_A \lambda')\; B, C) + ((D_B \lambda')\; A, C) - \mu(A)\, g(B,C) - \mu(B)\, g(A, C) + 2\, g(A,B)\, \mu(C)$

use above result we easily we easily hold (2.5)

Theorem 2.6: *On Almost para nearly Sasakian manifold. We have L (A, B) = 0, then*

$$N(A,B) = -(D_{\bar{B}} \lambda)\;(A) + 2\,\overline{(D_B \lambda)(A)} - \overline{(D_A \lambda)(B)} - \mu(A)\,(\bar{B})$$

$$+\mu(B)(\bar{A}) - 3\lambda'\;(A,B)\zeta$$

And if almost para Sasakian Type Riemannian differential Manifold is integrable, then N (A, B) =0 i.e $(D_{\bar{B}} \lambda)\,(A) - 2\,\overline{(D_B \lambda)(A)} + \overline{(D_A \lambda)(B)} + \mu(A)\,(\bar{B}) - \mu(B)\,(\bar{A}) + 3\lambda'\,(A,B)\zeta = 0$

Proof: Let us consider a tensor of type (0,2) ([3])

$$L(A,B) = (D_A \lambda)\; B + (D_B \lambda) A - \mu(A)B - \mu(B)A + 2g(A,B)\zeta$$

Replace A by $\bar{A}$, we obtain

$$L(\bar{A}, B) = (D_{\bar{A}}\lambda)\;(B) + (D_B \lambda)\;(\bar{A}) - \mu(\bar{A})(B) - \mu(B)(\bar{A}) + 2g(\bar{A}, B)\zeta$$

$$L(\bar{A}, B) = D_{\bar{A}}\bar{B} - \overline{D_{\bar{A}}B} + D_B \bar{\bar{A}} - \overline{D_B \bar{A}} - \mu(B)\,(\bar{A}) + 2g(\bar{A}, B)\zeta$$

The Nijenhuis tensor is given by [7],

$$N(A,B) = [\bar{A}, \bar{B}] + \overline{\overline{[A,B]}} - \overline{[\bar{A},B]} - \overline{[A,\bar{B}]}$$

$$= D_{\bar{A}}\,\bar{B} - D_{\bar{B}}\,\bar{A} + [A,B] - \mu[A,B]\zeta - \overline{D_{\bar{A}}B} + \overline{D_B \bar{A}} - \overline{D_A \bar{B}} + \overline{D_{\bar{B}}A}$$

$$\text{Or } N(A,B) = D_{\bar{A}}\bar{B} - D_{\bar{B}}\bar{A} + D_A B - D_B A - \mu(D_A B)\zeta + \mu\,(D_B A)$$

$$\zeta - \overline{D_{\bar{A}}\, B} + \overline{D_B\, \bar{A}} - \overline{D_A\, \bar{B}} + \overline{D_{\bar{B}}\, A}$$

Now we obtain $N\ (A\ ,B) - L(\bar{A}, B) = -D_{\bar{B}}\, \bar{A} + D_A B - 2D_B A -$ $\mu(D_A B)\, \zeta + 2\mu(D_B A)\, \zeta + 2\overline{D_B\, \bar{\bar{A}}} - \overline{D_A \bar{\bar{\bar{B}}}} + \overline{D_{\bar{B}}\, A} + (D_B \mu)\ (A)\ \zeta - \mu\ (A)$ $(\bar{B}) + \mu(B)\ (\bar{A}) - 2\lambda'(A, B)\zeta$ and using (1.5), we hold the theorem.

3. Affine Connexion

From what we can see, an affine connection could be a geometric object on a smooth manifold that connects two tangent spaces next to each other. Because of this, an affine connection is a manifold whose values are all in a fixed vector space.

Theorem 3.1: *On Para K-contact type Riemannian differential manifold(M, λ, ζ, μ, g), affine connexionis hold (3.1)* $\lambda(\nabla_{\bar{A}}\zeta) = -\bar{A} + \psi\ (H(\bar{A},\ \zeta))$.

Proof: Consider in para-K-contact type Riemannian differential manifold (M, λ, ζ, μ, g), let Riemannian connexion D and affine connexion ∇ is connected as [7], [8], [9] $\nabla_A B = D_A B + H\ (A, B)$, replace an arbitrary vector field B by a vector field ζ ,then ,we obtain $\nabla_A \zeta = D_A \zeta + H\ (A, \zeta)$,now using the result (1.7), and apply tensor field λ for affine connexion ∇ and tensor field ψ for Riemannian connexion D, we obtain the result $\lambda(\nabla_A \zeta)$ $= -A + \mu\ (A)\ \zeta + \psi\ (H(A, \zeta))$, again by replacing A by $\bar{A}$ and use result (1.1) (c) ,we hold result (3.1).

Theorem 3.2: *On Para K-contact type Riemannian differential manifold(M, λ, ζ, μ, g), affine connexion on tensor of type (0,3)* $(\nabla_A \lambda')$ *(B,C) = μ(K(B,C,A)) if ʻH (A,* $\bar{B}$ *,C) +ʼH(A,B,* $\bar{C}$) *=0.*

Proof : Consider in para-K-contact type Riemannian differential manifold (M, λ, ζ, μ, g), let Riemannian connexion D and affine connexion ∇ is connected as [7],[8],[9]

$\nabla_A B = D_A B + H\ (A, B)$, replace an arbitrary vector field B by $\bar{B}$ then we obtain $\nabla_A \bar{B} = D_A \bar{B},\ + H\ (A,\ \bar{B})$, differentiate $\bar{B} = \lambda B$ on both sides , using the both affine connecxions ∇ and D then, we obtain $(\nabla_A \lambda)$ (B) = $(D_A\ \lambda)$ (B) + H (A, $\bar{B}) - \overline{H(A, B)}$, now introduce the Riemannian metric g which is compatible with the structure (M, λ, ζ, μ, g),then again we obtain $(\nabla_A \lambda')$ (B,C) = (DA λ') (B,C) + ʻH (A, $\bar{B}$, C) + ʻH (A, B, $\bar{C}$), now using the result (1.6), we hold theorem.

4. Geodesic

Definition (4.1) A Curve of extremum length between any two points in a Riemannian manifold is called geodesic; its equation is ([3])

$$D_{\zeta}\zeta = 0. \tag{4.1}$$

Theorem 4.1: *On Para K-contact type Riemannian differential manifold (M, λ, ζ, μ, g), if a curve is geodesic between any two points and* $D_{\zeta}\lambda = 0$ *then (4.1) K (A) =* $-\overline{\overline{A}}$.

Proof: Let us consider Curvature tensor ([3]) K (A) = R (A,ζ ,ζ) & R (A, B, C) = $D_A D_B C - D_B D_A C - D_{[A,B]} C$ [3], replace arbitrary vector field B & C by ζ, we obtain R (A, ζ, ζ) = $D_A D_{\zeta} \zeta - D_{\zeta} D_A \zeta - D_{[A,\zeta]}\zeta$ and using the result (4. 1) & Riemannian connexion D rules, we obtain, K (A) = $(D_{\zeta}\ \lambda\ )\ (A)) - \overline{\overline{A}}$ again using $D_{\zeta}\lambda = 0$ ([17]), we hold the result (4.1).

Theorem 4.2: *If a curve is geodesic between any two points and the scalar curvature r is n (1-n), then the conformal curvature tensor of type (1,3) Ć (A, ζ , ζ) disappears in an Einstein Para K-contact type Riemannian differential manifold M.*

Proof: Consider conformal curvature tensor of type (1,3) is ([3])

Ć (A, B, C) = K (A, B, C) - $\frac{1}{n-2}$ {Ric (B, C) A – Ric (A, C) B - g (A, C) RB+ g (B,C) RA} + $\frac{r}{(n-1)(n-2)}$ {g (B,C) A – g (A,C) B}, we have in Einstein manifold ([3])

Ric (A,B) = $\frac{r}{n}\, g(A,B)$ & R(A) = $\frac{r}{n}\, A$, using the above results ,we obtain

Ć(A, B, C) = K (A, B, C) - $\frac{r}{n\,(n-1)}$ {g (B, C) A – g (A, C) B}, now replace arbitrary vector field B & C by ζ , we obtain Ć (A, ζ ,ζ) = K (A, ζ , ζ) - $\frac{r}{n\,(n-1)}$ {g (ζ, ζ) A – g (A, ζ) ζ }, again using the results K (A) = - A + μ (A)ζ , and g (ζ ,ζ) = 1, we hold theorem.

5. Conclusion

In this paper, we have studied the Nijenhuis Tensor, affine connecxion & conformal curvature tensor of type (0,3)on APST- Riemannian differential manifold (M, λ, ζ, μ, g). Obtained (i)the Nijenhuis Tensor is skew-symmetric for arbitrary vector fields A & B ,(ii) It is vanishes or Para

K-contact type Riemannian differential manifold structure (λ, ζ, μ,) is said to be normal iff $\Upsilon(A,B) = 2\,\{\mu(B)(\bar{A}) + \mu\ (A)(\bar{B})\ \}$(iii) It is also vanishes if on an almost para nearly Sasakian manifold, $(D_{\bar{B}}\lambda)(A) - 2\,(D_B\lambda)(A) + (D_A\ \lambda)(B) + \mu(A)(\bar{B}) - \mu(B)(\bar{A}) + 3\lambda'\ (A,B)\zeta = 0$ .Obtained the results(i) $\lambda(\nabla_{\bar{A}}\zeta) = -\bar{A} + \psi\ (H\ (\bar{A},\ \zeta))$ (ii) $(\nabla_A\lambda')\ (B,C) = \mu(K(B,C,A))$ If $'H\ (A,\ \bar{B},\ C) +' H(A,B,\ \bar{C}) = 0$, for the affine connecxion ∇ & Obtained in an Einstein Para K-contact type Riemannian differential manifold (M, λ, ζ, μ, g), if a curve is geodesic between any two points and scalar curvature r is n (1-n), then conformal curvature tensor of type (0,3) Ć (A, ζ , ζ) is vanishes.

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