Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 7, pp. 1523–1531
DOI : 10.47974/JIM-1576

Using an ADI finite difference method in the solution of heat transfer problem on a horizontal porous plate

Ahmed Muhammed Juma'a
Department of Mathematics
College of Computer Science and Mathematics
University of Mosul
Mosul
Iraq

Abstract

The paper is devoted to finding an approximation solution to the boundary value problem obtained from heat transfer by convection of a Newtonian incompressible fluid flowing on a flat plate. The partial differential equations have been treated numerically to get the approximate solution using the ADI finite difference technique, considered a good approximate mathematical method with accepted accuracy. The results indicated that the steady state of the governing equations can be reached for some parameters.

Subject Classification: *35Q79, 80Axx.*

Keywords: *Mathematical model, Porous plate, Free convective, Finite difference, Heat transfer.*

1. Introduction

A numerical solving of boundary value problems is one the favorite techniques, especially in applied sciences, which is used to solve more complicated of heat transfer samples. Baraga E. J. and Marcelo J. S. [1] present an approximate solution to the tranquil and disorganized flow of natural convection inside a porous cylindrical annuls laid horizontally. They used upwind differencing method and performed their solutions by using the SIMPLE algorithm. They found that a laminar case for non Darcian fluid and heat transfer for porous cavity depend upon Rayleigh together with Darcy number, while other parameters such as porosity, Prandtl, solid ratio and thermal conductivity of fluid are fixed. Mahmud H. Ali, Khalaf H., Ahmed Al- Sammarraie [2]

E-mail: **ahmed.m.j.jassim@uomosul.edu.iq**

investigated the natural convection heat transfer for non-Darcian fluid in a rectangular hollow filled with porous matter, modified Brinkman and Forchheimer extended Darcy flow has been employed with no slip at the boundaries. The problem was solved by using the ADI numerical method, and they achieved their solution for aspect ratio $(2 \leq H/L \leq 30)$. Results indicate that Nasselt number is directly proportional to moderate, and porosity is reversely proportional to Darcy number. Omar R. Alomar, I. A. Mohamed [3]. They investigated a non-Darcian steady flow of natural convection with heat transfer problems inside square and rectangular cavities for two parallel walls with constantly different temperatures and isolated other walls; they found that the Darcy number has an effect and plays an important role and aspect ratio. Sadeq Z. and Sayyed A. Fanaee [4] examined thermal radiation influence on the steady Magneto hydro dynamics of heat transfer by convection over a vertical plate submerged with porous media, where the chemical reaction is also considered. Chaoli Z., Liancun Z., and Xinxin Z. [5] discussed the flow of magnetohydrodynamics with radiation heat transfer in nanofluids in the presence of a flat plate in a porous medium. They reduced the nonlinear equations of their problem to the system of ordinary differential equations type and used transformation base functions to obtain an approximation solution. Farooq A., Sajjad H., Shan E. and Roman U. [6] attempted to reach a numerical solution using an approximation analysis of the major flow in tiny polar fluids. They deduced that a decrease in flow speed, temperature, distribution and shell function coefficients enclose the heat transfer rate. They found the real nature of fluid motion and thermal transportation. Zaynab A. Almishlih, and Ala'a A. Hammodat [7-12] studied a mathematical model of dissipated fluid and expressed the controlled equations by a nonlinear partial differential equations system in two dimensions. The outcome results indicated that some parameters like Prandtl, Grashof and radiation were affected in both steady and unsteady states.

2. The Model of the problem

Let a viscous Newtonian fluid is flowing above a constantly heated permeable surface accompanied by initial degree T_0, on the surface, T_1 is degree of air around fluid; assume also that the perpendicular component of the initial velocity is fixed with constant value w_0 as it is shown in the diagram below. Other components u v are changing with x and y, such that increasing vertical velocity cause absorption of heat from the fluid by permeable surface as shown in Figure 1.

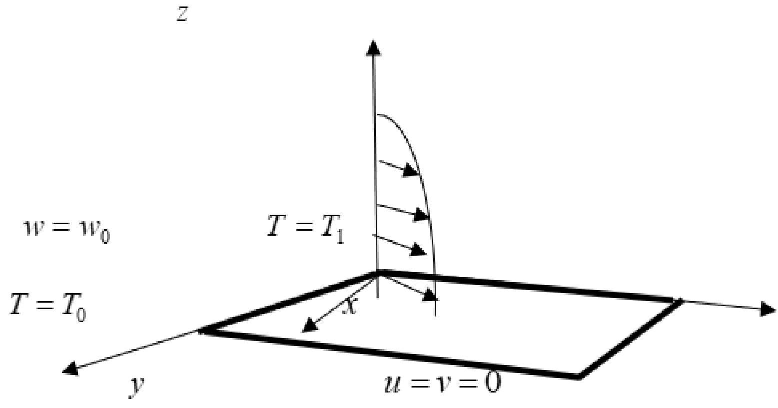

Figure 1
The graph of the Model

This paper aims to find an approximate solution to the boundary-value problem resulting from fluid deformation by using an approximate difference method under the same circumstances with analogous boundary conditions.

3. The governing equations

The governing equations for incompressible fluids, which represent this problem under consideration, are given by:

$$\nabla \cdot \bar{V} = 0 \tag{1}$$

$$\frac{D\bar{V}}{Dt^*} = -\frac{1}{\rho}\nabla\bar{p} + \mu\nabla^2\bar{V} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + \bar{V}\cdot gradT^* = \bar{\alpha}\left\{\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right\} - \frac{1}{\rho c_p}div\bar{q}_r + \frac{1}{\rho c_p}\bar{\Phi} \tag{3}$$

$$\nabla \times \bar{V} = 2\bar{\Omega} \tag{4}$$

$$\bar{\Phi} = 2\mu\left\{\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2 + \frac{1}{2}\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2 + \frac{1}{2}\left(\frac{\partial v^*}{\partial z^*} + \frac{\partial w^*}{\partial y^*}\right)^2 + \right.$$
$$\left. + \frac{1}{2}\left(\frac{\partial w^*}{\partial x^*} + \frac{\partial u^*}{\partial z^*}\right)^2 + \lambda\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*}\right)^2\right\} \tag{5}$$

Whereas, $\bar{\Omega}$, $\bar{V}$, μ, ρ, T^*, $\bar{p}$, C_p, $\bar{q}_r$, $\bar{\Phi}$, t^*, are the vorticity, principal velocity vector, fluid viscosity, thickness, temperature, main pressure, specific heat, vector of radiation heat flux, dispersed function, period, and thermal conductivity constantly according to the present Model, the governing equations can be changed into a set of new equations as follows

$$\frac{\partial u^*}{\partial t^*} - 2v^*\bar{\Omega} - w_0\frac{\partial u^*}{\partial z^*} = \vartheta\frac{\partial^2 u^*}{\partial z^{*2}} \tag{6}$$

$$\frac{\partial v^*}{\partial t^*} - 2(u^* - U)\bar{\Omega} - w_0\frac{\partial v^*}{\partial z^*} = \vartheta\frac{\partial^2 v^*}{\partial z^{*2}} \tag{7}$$

$$\frac{\partial T^*}{\partial t^*} - w_0\frac{\partial T^*}{\partial z^*} = \bar{\alpha}\frac{\partial^2 T^*}{\partial z^{*2}} + \left(\frac{\mu}{\rho c_p}\right)\left[\left(\frac{\partial u^*}{\partial z^*}\right)^2 + \left(\frac{\partial v^*}{\partial z^*}\right)^2\right] \tag{8}$$

ϑ fluid kinematic viscosity, $u^*, v^*, w^*$are velocity components at x, y, z, respectively, define the non-dimensional variables as:

$$T^* = T_1\theta, y = \frac{U}{\vartheta}z^*, v^* = Uv, t = \frac{t^*U}{L}, u = \frac{u^*}{U} \quad (9)$$

Implying these non-dimensional quantities in equations (1)–(7), the governing equations become:

$$\frac{\partial u}{\partial t} = Re\frac{\partial^2 u}{\partial y^2} + S\,Re\frac{\partial u}{\partial y} + E\,Re\,v \quad (10)$$

$$\frac{\partial v}{\partial t} = E\,Re(u-1) + S\,Re\frac{\partial v}{\partial y} + Re\frac{\partial^2 v}{\partial y^2} \quad (11)$$

$$\frac{\partial \theta}{\partial t} = S\,Re\frac{\partial \theta}{\partial y} + \frac{Re}{pr\frac{\partial^2\theta}{\partial y^2}Re\,E\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right\}} \quad (12)$$

$S = \frac{w_0}{U}$ suction number, $E = \frac{2\vartheta\bar{\Omega}}{U^2}$ rotation number, $pr = \frac{\rho c_p \vartheta}{k}$ Prandtl number, $Ec = \frac{U^2}{c_p T_1}$ Eckert number, $Re = \frac{LU}{\vartheta}$ Reynolds number.

The non-dimensional boundary conditions:

$u = v = 0$ at $y = 0$ $v = 0$ $u = U$ at $y = \infty$

$\theta = consant$ at $y = 0$ $\theta = \frac{T_1}{\bar{T}_0}$ at $y = \infty$

4. Method of solution

Under this assumption, the resulting non-dimensional equations convert into a system of multiple dependent partial differential equations for which an ADI finite difference scheme is suitable. Equations (10)–(12) have been solved numerically to get the approximate solution; each of the equations has been converted into a tri-diagonal system, which is used twice: firstly in the x-direction to find the intermediate values, followed by the y-direction to find the wanted values in the advanced time step. So the set of differential equations is reduced into a system of linear algebraic equations of the form $LX = D$at each step, where L represent the coefficients of the diagonal matrix and $X$the variable vector, D is the given variables. This assumption leads to the following stage:

1- For$X$ direction, u component of motion in equation (10) becomes:

$$A_1(I)u^\bullet_{i-1,j} + B_1(I)u^\bullet_{i,j} + C_1(I)u^\bullet_{i+1} = D_1(I)$$

whereas $u^\bullet$ is an intermediate value of the u component of velocity.

For the Y direction, the equation becomes:

$$A_2(I)u^{n+1}_{i-1,j} + B_2(I)u^{n+1}_{i,j} + C_2(I)u^{n+1}_{i+1} = D_2(J)$$

whereas u^{n+1}represents the advanced value of the u component of velocity as shown in Figure 2.

The same procedure was used to find the v component of motion equation (11) and the temperature θ in equation (12), respective as shown in Figure 3, 4, 5 and 6.

4.1 *Figures and Diagrams*

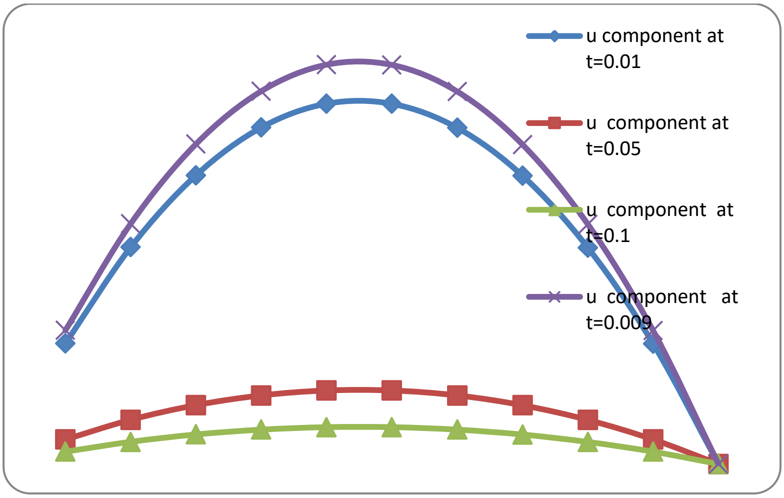

Figure 2
Horizonltal velocity copmponent u, $Pr = 50.0, Ra = 1000.0, S = 0.01$

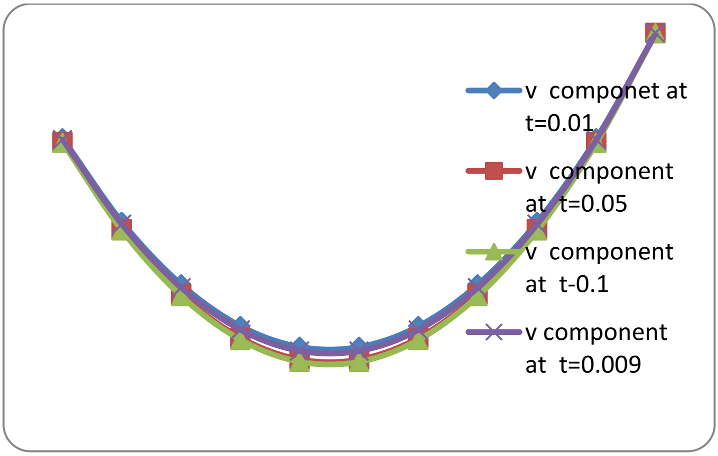

Figure 3
Vertical velocity copmponent v, $Pr = 50.0, Ra = 1000.0, S = 0.01$

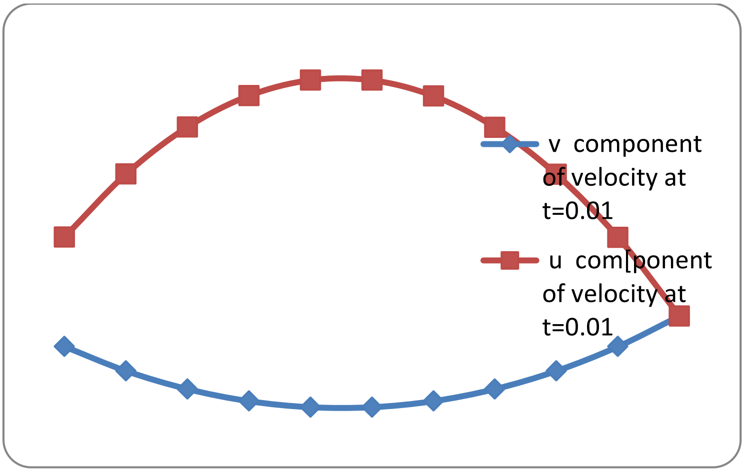

Figure 4
**Velocity distribution on x and y, $Pr = 50.0, Ra = 1000.0, S = 0.01,$
$E = 0.04$**

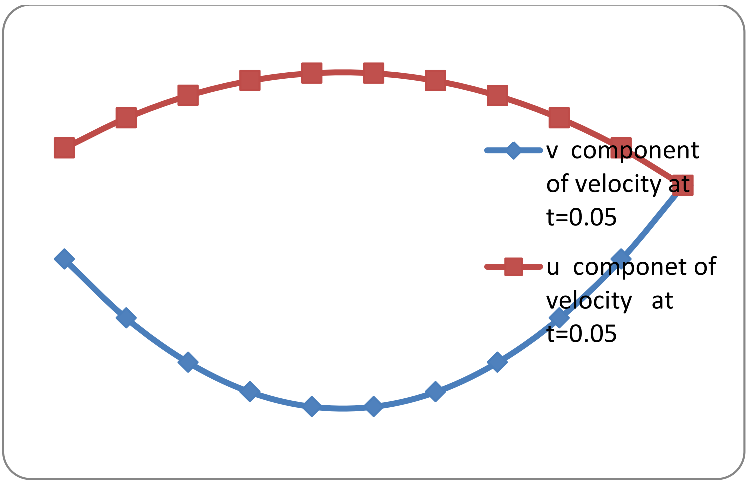

Figure 5
**Velocity distribution on x and y axis, $Pr = 50.0, Ra = 1000.0, S = 0.01,$
$E = 0.04$**

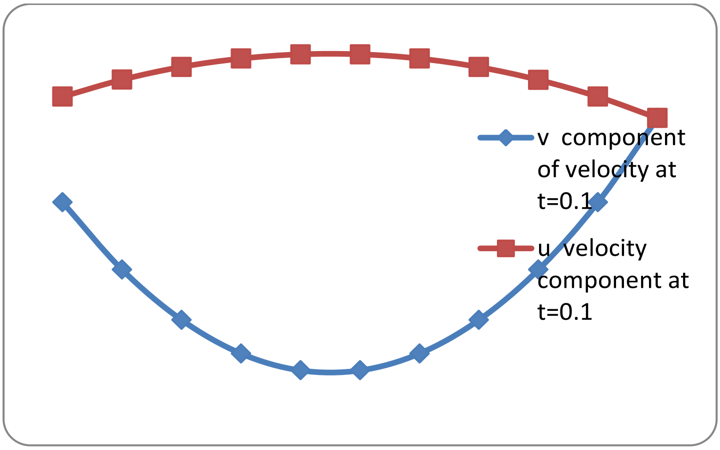

Figure 5
Velocity distributionin x and y axis, $\mathbf{Pr}=50.0$, $\boldsymbol{Ra}=1500.0$, $\boldsymbol{S}=0.01$, $\boldsymbol{E}=0.04$

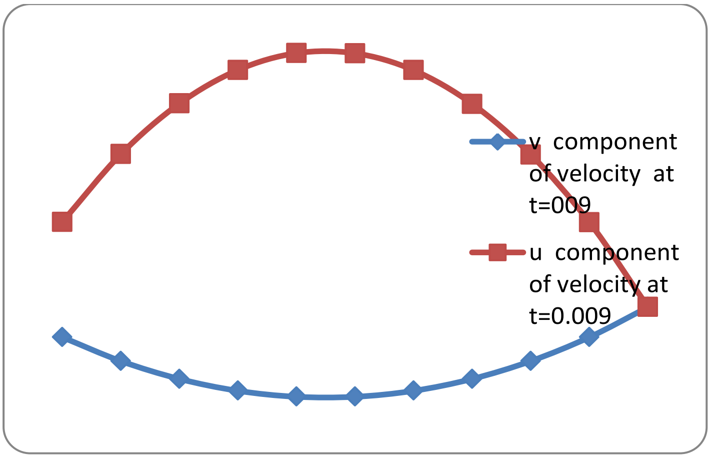

Figure 6
Velocity distribution on x and y axis, $\boldsymbol{Pr}=\mathbf{50.0}, \boldsymbol{Ra}=\mathbf{1500.0}, \boldsymbol{S}=\mathbf{0.01}$, $\boldsymbol{E}=\mathbf{0.04}$

5. Discussion and Conclusion

In the context of this research, mathematical models were constructed to predict market share in a duopoly. One Model considered the effects of quantity demand and research and development. In contrast, the other Model emphasizes quantity demand more while improving its relationship with price. Both models were taken into consideration. Case studies were used to apply these models, which revealed the significance of beginning market share and the sensitivity of relative price increase rate on market share. The findings are consistent with well-known economic models, which indicates that the results will be positive.

However, it is essential to remember that the study has a few restrictions. The insufficient availability of statistical data for the validation of the Model presents a difficulty, and it is possible that gathering the necessary data will be challenging. Despite this, additional research can be carried out to investigate the connection between selling price and market share, provided the right data is collected. In addition, the current study only looked at a small sample of the characteristics that influence market share. It didn't take into account things like marketing strategy or distribution. In the next research, consideration will be given to these aspects to achieve a more holistic comprehension of the workings of the market.

References

[1] Braga E. J. and Marcelo J. S. de Lemos, "Simulation of turbulent natural convection in porous cylindrical annulus using a macroscopic two- equation model", *International Journal of Heat and Mass transfer* 49, 4340-4351 (2006).

[2] Mahmud H. Ali, Khalaf H., Ahmed Al- Sammarraie, "Numerical study of Non- Darcian natural convection heat transfer in a rectangular enclosure filled with porous medium saturated with viscous fluid", *Tikrit Journal of Engineering Sciences*, (2008).

[3] Omar R. Alomar, Ibrahim A. Mohamed, "Numerical investigating of boundary surface effects on natural convection in a horizontal porous cavity by using Non- Darcian flow", *Alteqani Journal* vol. 26, No. 5 (2013).

[4] Sadeq Z. and Sayyed A. Fanaee, "MHD mixed convective flow past vertical plate embedded in a porous medium with radiation effects and convective boundary condition considering chemical reaction", Cankaya university *Journal of Science and Engineering*, vol. No. 1, pp: 123-136 (2013).

[5] Chaoli Z. , Liancun Z., Xinxin Z., " MHD flow and radiation heat transfer of nano fluids in porus media with variable surface heat flux and chemical reaction", *Applied Mathematical Modeling*, vol. 39, pp: 165-181 (2015).

[6] Farooq A., Sajjad H., Shan E. and Roman U., " Numerical solution of nonlinear differential equation for heat transfer in macro polar fluids over stretching domain", *Mathematics,* 8, 854 (2020); doi:10.3390/math8050854.

[7] Zaynab A. almishlih, Ala'a A. Hammodat, "Numerical solution of fluid flow in a horizontal tube under effects of radiation field", *Open access Library Journal*, vol. 7, e6283 (2020).

[8] Ismail F., Mohamed B. and Abdelkahlek A., "Analytical and numerical study of Soret and Dufour effects on thermosolatal convection in a horizontal Brikman porous layer with a stress-free upper boundar", Hindawi, mathematical problems in Engineering, vol. 2020, ID 4046570, 17 pages, https://doi.org/10.1155/2020/4046570.

[9] Zahraa L. Allyas and A. Hammodat, "Heat transfer and thermal radiation in two dimensional vertical glass cavities", AIP conference proceedings 2386, 080024 (2022). https://doi.org/10.1063/5.0068026.

[10] Ala'a. A. Hammodat, H. D. saleem, "Numerical solution of electromagnetic problem in a horizontal porous medium", *Italian Journal of Pure and Applied Mathematics*- N 47- 2022 (1118-1135) (2022).

[11] Abd Ali, D. M., Chalob, D. F., & Khudhair, A. B. : Networks Data Transfer Classification Based On Neural Networks. *Wasit Journal of Computer and Mathematics Sciences*, 1(4), 207-225 (2022).

[12] Shubham Sharma & Ahmed J. Obaid : Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.

[13] Li Li. Partial differential equation calculation and visualization, *Journal of Discrete Mathematical Sciences and Cryptography*, 20:1, 217-229 (2017), DOI: 10.1080/09720529.2016.1178918.