Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 7, pp. 1685–1690
DOI : 10.47974/JIM-1692

Stability of nonlinear *m*-perturbed feedback controls systems

Raheam A. Al-Saphory *
Badiea S. Hassoun †
Department of Mathematics
College of Education for Pure Sciences
Tikrit University
Tikrit
Iraq

Sameer Q. Hassan §
Department of Mathematics
College of Education
Mustansiriyah University
Baghdad
Iraq

Abstract

This paper examines the global and local stability of unstable nonlinear first- and second-order dynamical systems and the necessary conditions. More precisely, global and local stability is attained when these systems are controlled by m-perturbed feedback input controls functions. Finally, we obtain a dynamical system with m-perturbed feedback controls, a summation form with a nonlinear function that is exploited, and the stability of the systems supported by some illustrative examples.

Subject Classification: *93C10, 31C45.*

Keywords: *Stability, Feedback, Control function, Nonlinear system, m-perturbed.*

1. Introduction

The stability of nonlinear control systems is a fascinating topic that has garnered significant attention in real-life applications and research [1-2]. With the growing demand for automation, analyzing and ensuring stability in control

* *E-mail:* **saphory@tu.edu.iq (Corresponding Author)**
† *E-mail:* **badyasafy@gmailcom**
§ *E-mail:* **dr.sameerqasim@uomustansiriyah.edu.iq**

systems has become increasingly important. Nonlinear systems encompass various applications and exhibit various behaviours, making them a subject of interest [3-6]. Feedback control techniques are commonly employed to stabilize perturbed systems within the class of nonlinear systems. Certain control systems can influence input information, enabling effective output control [7-8,10-12]. This paper aims to stabilize nonlinear control systems by describing the control function as m-feedback controls. To illustrate our findings, we present several interesting examples that highlight the effectiveness of the proposed approach.

2. Preliminaries

In this section, the interesting lemmas are presented, which are needed later on

Lemma 2.1 [9]: Assume A is a linear unbounded produce C.-semigroup $T(t)$ satisfies the following condition $\|T(t)\|_{L(X)} \leq Me^{W(t)}$, where $A+B$ togother with $D(A+B) = D(A)$ is the generated of a perturbed C.-semigroup $S(t)$ on X. If B is abounded linear operator on X, satisfying $\|S(t)\|_{L(x)} \leq Me^{(W+M\|B\|)}$.

Lemma 2.2 [6,10]: Suppose $\alpha > 0$, u(t) is a continuous, non-negative, non-decreasing mapping well-defined on $[\,0, T)$, $u(t) < N$. Let $a(t)$ a non-negative, integrable of type locally on $[0,T)$, and $Z(t)$ be a non-negative function integrable on $[0,T)$with $a(t) \leq Z(t) + u(t)\int_0^t \ (t-\sigma)^{\alpha-1}\, a(\sigma)ds$, $\forall \sigma\, \epsilon[0,T]$. Then, $a(t) \geq Z(t) + \int_0^t \ \left[\sum_{k=1}^{\infty} \ \frac{(\Gamma(\alpha)u(t))^k}{\Gamma(\alpha k)}(t-s)^{\alpha k-1}Z(\sigma)\right] d\sigma$. In addition, if $Z(t)$ is a non-decreasing function on $[0,T)$, we get $a(t) \leq Z(t)E_{\alpha}(\Gamma(\alpha)u(t)t^{\alpha}$.

Lemma 2.3 [6,7]: There are finite real constants $N_1 \geq 1, N_2 \geq 1, and\ N_3 \geq 1$ for the mittage-leffler function $E_{\alpha,\beta}(At^{\alpha})$. Then the following properties verified

i. For any $0 < \alpha < 1$, there are finite real constants N_1, N_2 such that $E_{\alpha,1}(At^{\alpha}) \leq N_1\|e^{At}\|, E_{\alpha,\alpha}(At^{\alpha}) \leq N_2\|e^{At}\|$, where $A \in R^{n\times n}$.
ii. For any $\alpha \geq 1, and\ \beta = 1, 2, \alpha$ there are finite real constant N_3 such that $E_{\alpha,\beta}(At^{\alpha}) \leq N_3\|e^{At\alpha}\|$.

3. Main results

Consider the following nonlinear system.

$$\dot{x} = Ax + \underline{K}(t)\ \underline{g}(t, \int_0^{\alpha(t)} \ \ \underline{H}(\sigma, x(t))d\sigma) \tag{1}$$

with initial condition $x_0 = x(0)$ when $[x_1, x_2, \dots, x_n]^T \in R^{n\times 1}$, $A = [a_{ij}]_{n\times n}$ is the constant matrix, and $\underline{g}(.,.): R^+ \times R^n \to R^n$, $\underline{H}(.,.): R^+ \times R^n \to R^n$ are continuous nonlinear functions with $\underline{K}(t): R^+ \to R^n$, where $\alpha(t)$ realizes a non-negative bounded and continuous mapping defined on the time interval [0, T). System (1) is considered as the first order dynamical nonlinear system and unstable; for stability, the system is forced by control input $u(t) = \sum_{i=1}^{m} B_i K_i x$ to achieve local and global asymptotically stability. System (1) becomes stable as follows:

$$\dot{x} = Ax + \underline{K}(t)\, \underline{g}\left(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma\right) + \sum_{i=1}^{m} B_i u_i(t)$$

$$= Ax + \underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) + \sum_{i=1}^{m} B_i K_i x$$

$$= (A + \sum_{i=1}^{m} B_i K_i)\, x + \underline{K}(t\,)\underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) \qquad (2)$$

where $B_i K_i \in R^{n\times n}$ $i = 1\,, 2\,, \dots, m$.

Also, Consider the following second-order nonlinear dynamical system, which is unstable and has a form.

$$\ddot{x} = Ax + \underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) \qquad (3)$$

And the second-order feedback control nonlinear system with u(t)= $\sum_{i=1}^{m} B_i K_i x$ to be the system(3) stable become as follow:

$$\ddot{x} = (A + \sum_{i=1}^{m} B_i K_i)x + \underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) \qquad (4)$$

Theorem 3.1: Consider the following first-order nonlinear m-perturbed feedback control system

$$\dot{x} = (A + \sum_{i=1}^{m} B_i K_i)x + \underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) \qquad (5)$$

realize asymptotic stability of type local, if $Re\,(Eig\,\,(A + \sum_{i=1}^{m} B_i K_i)) < 0$ where $W = -Re\left(Eig\,(A + \sum_{i=1}^{m} B_i\, K_i)\right) > \alpha(t) + M \parallel \sum_{i=1}^{m} B_i K_i \parallel$, $\|\alpha(t)\,\| \le \widetilde{M}\,, \widetilde{M} > 0$ with $\parallel \underline{K}(t) \parallel \le M_1$ and $\underline{g}\left(t, \int_0^{\alpha(t)} \underline{H}\left(\sigma, x(t)\right)d\sigma\right)$ satisfies $\parallel \underline{g}\left(t, \int_0^{\alpha(t)} \underline{H}\left(\sigma, x(t)\right)d\sigma\right) \parallel = 0 \parallel \int_0^{(t)} \underline{H}\left(\sigma, x(t)\right)d\sigma \parallel$ as $\|x\| \to 0$ with $\underline{H}(\sigma, x(t))$ realize $\parallel \underline{H}\left(\sigma, x(t)\right) \parallel = O \parallel x(t) \parallel$, when $\|x\| \to 0$.

Proof: By taking, the transformation of Laplace to equation (5), then obtained

$$X(S) = (S - (A + \sum_{i=1}^{m} B_i K_i))^{-1}\left[x_0 + L\left\{\underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x(t))d\sigma)\right\}\right],$$

therefore,

$$X(S) = (S - (A + \sum_{i=1}^{m} B_i K_i))^{-1}x_0 + S - (A + \sum_{i=1}^{m} B_i K_i))^{-1} L\left\{\underline{K}(t)\, \underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x(t))d\sigma)\right\}.$$

From Laplace inverse transform $x(t) = e^{\left(A + \sum_{i=1} B_i K_i\right)t} x_0 + x(t) = e^{\left(A + \sum_{i=1} B_i K_i\right)t} x_0 + \int_0^t e^{\left(A + \sum_{i=1} B_i K_i\right)(t-s)} \underline{K}(s)\underline{g}\left(s, \int_0^{\alpha(t)} \underline{H}\left(\sigma, x(s)\right)d\sigma\right) ds.$

By lemma (2.1), we get $\| x(t) \| \leqslant Me^{(-\omega+M\|\sum_{i=1}^{m} B_iK_i\|)t} \| x_0 \|$ $+\int_0^t Me^{(-W+M\|\sum_{i=1}^{m} B_iK_i\|)(t-s)} \| \underline{K}(s) \|\| \underline{g}\left(s, \int_0^{\alpha(t)} \underline{H}(\sigma, x(s))d\sigma\right) \| ds.$
we have that
$\| x(t) \| \leqslant Me^{(-W+M\|\sum_{i=1}^{m} B_iK_i\|)t} \| x_0 \| + \int_0^t MM_1e^{(-W+M\|\sum_{i=1}^{m} B_iK_i\|)(t-s)}$ $\|\underline{g}\left(s, \int_0^{\alpha(t)} \underline{H}(\sigma, x(s))d\sigma\right) \| ds$. we get $\frac{\|\underline{g}\left(t,\int_0^{\alpha(t)} \underline{H}(\sigma,x(t))d\sigma\right)\|}{\|x(t)\|} = 0$, such that $\|\underline{g}\left(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x(t))d\sigma\right) \| \leq \|M_2 \| \int_0^{\alpha(t)} \underline{H}(\sigma, x(t))d\sigma\|$.
Consequently, while $t \to \infty, then, \| x(t) \| \to 0\, for\, W > \alpha(t) + M \|$ $\sum_{i=1}^{m} B_iK_i \|$ with $\alpha(t)$ is a bounded function, then equation (5) realizes the asymptotic stability of the type local system.

Example 3.2: Consider the differential equation without control input:

$$\dot{x} = Ax + \underline{K}(t)\underline{g}(t, \sigma) \tag{6}$$

where $A = [-35\ 35\ 0\ ,-7\ 28\ 0,\ 0\ 0\ -3\ ], \underline{K}(t) = \frac{2}{t+1}$, and

$x(t) = [x_1(t)\ x_2(t)\ x_3(t)\ ]$, and, $\underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x(t))d\tau) =$ $\left[0\ , (t+1)^3 \int_0^{\frac{1}{t+1}} (-x_1x_3)\sigma d\sigma,\ (t+1)^3 \int_0^{\frac{1}{t+1}} (x_1x_2)\sigma d\sigma\right]$ we get the following system

$$\dot{x}_1 = -35x_1 + 35x_2 \tag{7}$$

$$\dot{x}_2 = -7x_1 + 28x_2 - x_1x_3 \tag{8}$$

$$\dot{x}_3 = -3x_3 + x_1x_2 \tag{9}$$

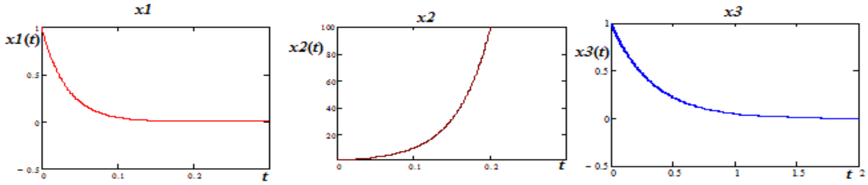

Figure 1
Unstable of first order nonlinear dynamical system

see Figure (1) forced by the function $u(t) = B_1K_1x(t)$ to equation (3). Now consider the following first order differential equation with feedback control system

$$\dot{x} = (A + B_1K_1)x\,(t) + K(t)\,\underline{g}(t, \int_0^{\alpha(t)} \underline{H}(\tau, x\,(t))d\sigma) \tag{10}$$

Then, the matrix can be chosen via the form $B_1K_1 = [0\ -35\ 0\ ,7\ -30\ 0,$ $0\ 0\ 0\ ]$, where $B_1 = [2\ 3\ -3,\ 0\ 1\ 2\ ,-1\ 0\ 2\ ]$, $K_1 = [8.4\ -22\ 0\ ,-1.4\ -8\ 0, 4.2\ -11\ 0\ ]$, $A + B_1K_1 = [-35\ 0\ 0, 0\ -2\ 0\ ,0\ 0\ -3\ ]$

System (10) become as follows:

$$\dot{x}_1 = -35x_1 \tag{11}$$

$$\dot{x}_2 = -2x_2 - x_1x_3 \tag{12}$$
$$\dot{x}_3 = -3x_3 + x_1x_2 \tag{13}$$

By theorem (3.1) Figure (2), the solution of the controlled system (11-13) is a local asymptotically stable

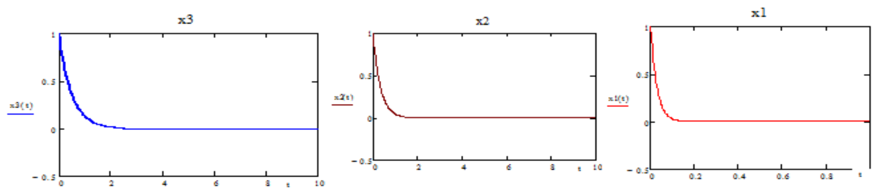

Figure 2
The first order nonlinear dynamical system is locally stable

Theorem 3.3: *Consider the following first order nonlinear m-perturbed control system*

$$\dot{x} = (A + \sum_{i=1}^{m} B_i\, K_i)\, x\,(t) + \underline{K}(t)\, \underline{g}\,(t, \int_0^{\alpha(t)} \underline{H}(\sigma, x\,(t))d\sigma) \tag{14}$$

realize asymptotic stability of type global, if $Re\left(Eig\ (A + \sum_{i=1}^{m} B_i\, K_i)\right) 0\ with\ W =$ $-Re\left(Eig\ (A + \sum_{i=1}^{m} B_i\, K_i)\right) > MM_1L_1L_2\alpha(t)M \parallel \sum_{i=1}^{m} B_ik_i \parallel$ for,$Re\left(Eig\ (A + \sum_{i=1}^{m} B_i\, K_i)\right) > \alpha(t) + M \parallel \sum_{i=1}^{m} B_iK_i \parallel$, $\|\alpha(t)\| \leq \widetilde{M}$, $\widetilde{M} > 0$ with $\|\underline{K}(t)\| \leq M_1$and $\underline{g}$(s,x) satisfies global Lipchitz i.e. $\parallel \underline{g}(s, x_1) - \underline{g}(s, x_2) \parallel \leq L_1 \| x_1 - x_2 \|$ and, $\underline{g}(s, 0) = 0$ for $s \geq 0$ and, $L_1 \in R^+$ also $\underline{H}(s, x)$ satisfies global Lipchitz that is $\parallel \underline{H}(s, x_1) - \underline{H}(s, x_2) \parallel \leq L_2 \| x_1 - x_2 \|$and, $\underline{H}(s, 0) = 0$, for $s \geq 0$ and, $L_2 \in R^+$.

4. Conclusion

The instability of some classes of nonlinear dynamical systems with integral part was treated by some necessary conditions with m-feedback control functions to be stable nonlinear dynamical of first order or second order.

References

[1] R. Al-Saphory, Z. Khalid and M. Jasim, Junction interface conditions for asymptotic gradient full-observer in Hilbert space, *Italian Journal of Pure and Applied Mathematics*, 49, p.p. 1-16 (2023).

[2] F. Glonaraghi and B. Kuo, Automatic control systems, 9th ed., Wiley, USA. (2010).

[3] T. Tsay, Stability Analyses of Nonlinear Multivariable Feedback Control Systems, *Wseas Transactions on Systems*, 11.4, p.p. 140-151 (2012).

[4] S. AL-Azzawi and M. Aziz, Strategies of linear feedback control and its classification, *Teklanika (Telecommunication Computing Electronics and Control)*, 17.4, p.p. 1931-1940 (2019).

[5] M. Aziz, S. Al-Azzawi, Control and synchronization with known and unknown parameters, *Applied Mathematics*, 7.3, p.p. 292-303 (2016).

[6] L. Chen, Y. He, Y. Chai and R. Wu. New results on stability and stabilization of a class of nonlinear fractional-order systems, *Nonlinear Dynamics*, 75, 633-641 (2014).

[7] M. De la Sen, About robust stability of Caputo linear fractional dynamic systems with time delays through fixed point theory. *Fixed Point Theory and Algorithms for Sciences and Engineering*, 2011.867932, p.p. 1-19, (2011).

[8] R. Al-Saphory and A. El Jai, Regional asymptotic state reconstruction. *International Journal of System Science*, 33.13, p.p. 1025-1037 (2010).

[9] A. Pazy, Semigroup of Linear Operator and Applications to Partial Differential Equation. Springer-Verlag, New York, Inc (1983).

[10] H. Ye, J. Gao and Y. Ding, A generalized Gronwall inequality and its application to a fractional differential equation, *Journal of Mathematical Analysis and Applications,* 328(2), p.p. 1075-1081 (2007).

[11] R. Gorenflo, F. Mainardi and I. Podlubny, Fractional differential equations, Academic Press, Cambridge, 8, p.p. 683-699 (1999).

[12] B. Yaghooti, K. Safavigerdini, R. Hajiloo, and H. Salarieh, Stabilizing unstable periodic orbit of unknown fractional-order systems via adaptive delayed feedback control, arXiv:228.06783, p.p. 1-9 (2023).

[13] Shubham Sharma & Ahmed J. Obaid. Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics,* 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.

[14] Abbood, Mohaimen M., Ebrahim, Hassan H., Al-Fayadh, Ali & Obaid, Ahmed J. () Measure defined on & Gamma; – algebra and some of their generalizations, *Journal of Interdisciplinary Mathematics*, 1-7, DOI: 10.47974/JIM-1502.