

## Some games via $G_\alpha$ - separation axioms

Tabarak A. Ali \*

R. B. Esmaeel <sup>†</sup>

*Department of Mathematics*

*College of Education for Pure Science (Ibn Al-Haithem)*

*University of Baghdad*

*Baghdad*

*Iraq*

---

### Abstract

In this research, we study new types of games by applying the axioms of separation through the open set  $G_\alpha$ -open set, the formation of different relationships between different types that have been identified, and the losing and winning losing strategy of any players.

---

**Subject Classification:** Primary 54c05, Secondary 54c10

**Keywords:** Grill,  $G_\alpha$ - $T_0$ -space,  $G_\alpha$ - $T_1$ -space,  $G_\alpha$ - $T_2$ -space,  $G(T_0, X, \mathcal{G})$ ,  $G(T_1, X, \mathcal{G})$ ,  $G(T_2, X, \mathcal{G})$ .

### 1. Introduction

The concept of grills has garnered significant interest among scholars [1]. Notably, Choquet is recognized as the pioneer who laid the groundwork for understanding the grill concept and formulating a definition [2]. Subsequently, numerous efforts have been made to explore the properties associated with this concept and understand their interrelationships. Among the various kinds of grill topological spaces, examples include discrete and cofinite topologies [3]. Each game in this research is a type in which games are defined between  $player_1$  and  $player_2$  and each  $player$  has choices that can be expressed in the following form;  $K_1, K_2, K_3, \dots$ , and these options are called moves. If the  $player$  chooses a single move, that operation is called a single move. When the game contains.

---

\* E-mail: tabarakayad314@gmail.com (Corresponding Author)

<sup>†</sup> E-mail: Rana.b.i@ihcoedu.uobaghdad.edu.iq, ranamumosa@yahoo.com

A game is referred to as a repetitive game involving multiple moves. The number of repetitions in such games can vary, sometimes countable and sometimes uncountable. There is a prize that the *player* gets after every move. The player obtaining a prize is equivalent to losing the opponent player[4,5].

There are two main types of matches, simultaneous and rotational matches. If *player*<sub>1</sub> chooses one move, *player*<sub>2</sub> chooses the other move, given that *player* two knows how player one moved to the next move. But if two players choose simultaneously, provided that neither of them knows the choice of the other, then the game is called simultaneous [6,7].

## 2. Games Via $G_\alpha$ - Separation Axioms

**Definition 2.1:** Suppose  $A$  is a subset of a G.T.S  $(\wp, \tau, G)$ ,  $A$  is  $G_\alpha$  -open set if there exist  $\zeta \in \tau$ ;  $\zeta \subseteq A \subseteq \text{htcl}_{G_\alpha}(\zeta)$ . the complement of

$G_\alpha$  - open set is  $G_\alpha$  -closed set, the set of all  $G_\alpha$  -open set denoted by

$G_\alpha -(\wp)$  and the set of all  $G_\alpha$  -closed set denoted by  $G_\alpha -c(\wp)$  as shown in

Figure 1.

**Definition 2.2:** The G.T.S  $(X, \tau, G)$  is  $G_\alpha$  -To-space if and only if for each  $x, y \in X$ ;  $x \neq y \exists \mu \in \tau_{G_\alpha}$ ; then  $(x \in \mu \wedge y \notin \mu) \vee (x \notin \mu \wedge y \in \mu)$

If  $(X, \tau)$  is not  $G_\alpha$  -to space, we define  $X$  is not  $G_\alpha$  -To-space if and only if  $x, y \in X$ ;  $x \neq y \exists \mu \in \tau_{G_\alpha}$ ;  $(x, y \in \mu) \vee (x, y \notin \mu)$ .

**Definition 2.3:** The grill topological space  $(X, \tau, G)$  is said to be  $G_\alpha - T_1$ -space if and only if  $x \neq y \in X$ , there is either an  $G_\alpha$  -open set containing  $x$  but not containing  $y$  and an  $G_\alpha$  -open set containing  $y$  but not containing  $x$ , in other meaning,  $X$  is  $G_\alpha - T_1$ -space if and only if  $x, y \in X$ ;  $x \neq y \exists v, \mu \in \tau_{G_\alpha}$ ;

$(x \in \mu \wedge y \notin \mu) \vee (x \notin \mu \wedge y \in \mu)$  then  $X$  is not  $G_\alpha - T_1$ -space if and only if  $x, y \in X$ ;  $x \neq y \exists v, \mu \in \tau_{G_\alpha}$ ;  $(x, y \in \mu) \vee (x, y \notin \mu)$  or  $(x, y \in v) \vee (x, y \notin v)$

**Definition 2.4:** The grill topological space  $(X, \tau, G)$  is  $G_\alpha - T_2$  -space if and only if for each pair of distinct points  $x, y \in X$ , there is either an open set containing  $x$  and an open set containing  $y$ , in other meaning,  $X$  is  $G_\alpha - T_2$ -space if and only if  $x, y \in X$ ;  $x \neq y \exists v, \mu \in \tau_{G_\alpha}$ ;

$(x \in \mu \wedge y \in v) \text{ and } (\mu \cap v) = \emptyset$

A link has been made between the axioms of separation and the axioms of separation presented above:

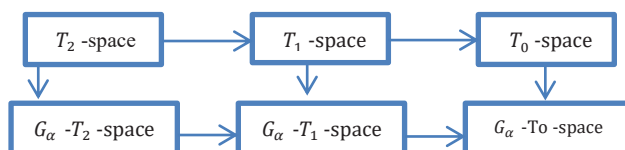


**Figure 1**

The relations among the different types of separation axioms

**Definition 2.5:** For a grill topological space  $(X, \tau, \mathbb{G})$ , determine

$\mathcal{G}(T0, X, \mathbb{G})$  in the following form:

$player_1$  and  $player_2$  take turns for every positive integer in the row n-th

In this game: the  $player_1$  : there exist  $\mu_n \neq v_n$  such that  $\mu_n, v_n \in X$

In the second step,  $player_2$  : let  $\mathcal{U}_r$  be a  $G_\alpha$  -open set consist of one of elements  $\mu_r, v_r$ . a  $player_2$  wins the game  $\mathcal{G}(T0, X, \mathbb{G})$  if  $\beta = \{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \dots, \mathcal{B}_r, \dots\}$  is a family of  $G_\alpha$  -open for all  $\mu_r \neq v_r \in X$ ,  $\exists \mathcal{B}_r \in \beta$  consist of one of the element  $\mu_r, v_r$ . Unlike that,  $player_1$  wins.

**Remark 2.6:** In a G.T.S  $(K, \tau, \mathbb{G})$ :

- i- If  $player_2 \nearrow \mathcal{G}(T_0, K)$ , then  $player_2 \nearrow \mathcal{G}(T_0, K, \mathbb{G})$  then  $player_2 \searrow \mathcal{G}(T_0, K)$ , then  $player_2 \searrow \mathcal{G}(T_0, K, \mathbb{G})$ .
- ii- If  $player_1 \nearrow \mathcal{G}(T_0, K, \mathbb{G})$ , then  $player_1 \nearrow \mathcal{G}(T_0, K)$ .
- iii- If  $player_1 \searrow \mathcal{G}(T_0, K)$  then  $player_1 \searrow \mathcal{G}(T_0, K, \mathbb{G})$ .

**Theorem 2.7:** The G.T.S  $(X, \tau, \mathbb{G})$  is  $G_\alpha$  -To space if and only if,  $player_2 \nearrow \mathcal{G}(T_0, X, \mathbb{G})$ .

**Proof:** Let  $(X, \tau, \mathbb{G})$  be a  $G_\alpha$  -To- space. For the first player  $\mu_r \neq v_r \in X$ , a  $player_2$  find  $G_\alpha$  -open set  $\mathcal{U}$  containing either  $\mu_r$  or  $v_r$  then,  $player_2 \nearrow \mathcal{G}(T_0, X, \mathbb{G})$ .

When  $player_2 \nearrow \mathcal{G}(T_0, X, \mathbb{G})$ , then if there exists  $\mu_1 \neq \mu_2$  in  $X$  of  $player_1$ , then  $player_2$  can be chosen  $\exists$  it is an open set, and it includes one of the elements .then the G.T.S is  $G_\alpha$  -To-space.

**Corollary 2.8:** In a G.T.S  $(X, \tau, \mathbb{G})$  a  $player_2 \nearrow \mathcal{G}(T0, X, \mathbb{G})$  if and only if  $\forall \mu_1 \neq \mu_2 \in X$  there exist  $\beta$  a  $G_\alpha$  -closed set whenever  $\mu_1 \in \beta$ ,  $\mu_2 \notin \beta$ .

**Proof:** Clearly.

**Corollary 2.9:** A grill topological space  $(X, \tau, \mathbb{G})$  is  $G_\alpha$  -To space if and only if,  $player_1 \nearrow \mathcal{G}(T0, X, \mathbb{G})$ .

**Proof:** When  $(X, \tau, \mathbb{G})$  is  $G_\alpha$  -To- space. If  $player_1 \nearrow \mathcal{G}(T_0, X, \mathbb{G})$ , there exist  $\mu_1 \neq \mu_2 \in X$  no exist  $G_\alpha$  -o set  $\mathcal{U}$  includes only one of the elements. (This contradiction).

Let  $player_1 \nearrow \mathcal{G}(T_0, X, \mathbb{G})$ . If  $(X, \tau, \mathbb{G})$  is not  $G_\alpha$  -To -space, therefore there exist  $\mu_1 \neq \mu_2$  in  $X$  and not exist  $G_\alpha$  - open set  $\mathcal{U}$  includes only one of the items  $player_1$  can win the game by choice of these two elements  $\mu_1$  and  $\mu_2$  (contradiction).

**Theorem 2.10:** A grill topological space is not  $G_\alpha$  -To-space if and only if  $player_1 \nearrow \mathcal{G}(T0, X, \mathbb{G})$ .

**Proof:** Let  $(X, \tau, \mathbb{G})$  is not  $G_\alpha$ -To-space, and suppose that  $\mu_1 \neq \mu_2 \in X$  then does not exist  $G_\alpha$ -open set  $\mathcal{U}$  includes only one of the items. and  $\mu_1 \neq \mu_2$  will be the choice of  $player_1$ , and a  $player_2$  not find an  $G_\alpha$ -open set  $\mathcal{U}$  includes only one of the items, then,  $player_1 \nearrow G(T_0, X, \mathbb{G})$ .

Reverse proof: If  $player_1 \nearrow G(T_0, X, \mathbb{G})$ , this mean that there is a choice  $\mu_1 \neq \mu_2$  and  $player_2$  cannot be taken an  $G_\alpha$ -open set includes only one of the items, then  $(X, \tau, \mathbb{G})$  is not  $G_\alpha$ -To-space.

**Corollary 2.11:** A grill topological space is not  $G_\alpha$ -To space if and only if  $player_2 \nearrow G(T_0, X, \mathbb{G})$ .

**Proof:** Suppose that  $(X, \tau, \mathbb{G})$  is not  $G_\alpha$ -To-space If  $player_2 \nearrow G(T_0, X, \mathbb{G})$ , then if  $\mu_1 \neq \mu_2$  exist in  $X$ , let  $\mathcal{U}$  is  $G_\alpha$ -open set includes either  $\mu_1$  or  $\mu_2$ . This contradicts the hypothesis

Let  $player_2 \nearrow G(T_0, X, \mathbb{G})$ . If G.T.S  $G_\alpha$ -To space,  $\mu_1 \neq \mu_2 \in X$ , let  $\mathcal{U}$  be a  $G_\alpha$ -open set includes only one of the items, then  $player_2$  win This contradicts the hypothesis.

**Definition 2.12:** For a G.T.S  $(X, \tau, \mathbb{G})$ , determine a game  $G(T_1, X, \mathbb{G})$  In the following form: The two players take turns with each natural number in this game in the game n-th:

Firstly,  $player_1$ : take  $v_1 \neq v_2$  where  $v_1, v_2 \in X$

Secondly,  $player_2$ : take  $T_r, K_r$  are  $G_\alpha$ -open sets such that  $v_1 \in T_r$  and  $v_1 \notin K_r$ . and  $v_2 \notin T_r$  and  $v_2 \in K_r$ .

Then  $player_2$  wins in the game  $G(T_1, X, \mathbb{G})$  if

$$\beta = \{\{T_1, K_1\}, \{T_2, K_2\}, \{T_3, K_3\}, \dots, \{T_r, K_r\}, \dots\}$$

It is a family of  $G_\alpha$ -open sets since  $\forall v_1 \neq v_2$  in  $X$ ,  $\exists \{T_r, K_r\} \in \beta$  It contains one element, but not both  $v_1, v_2$ . irrespective of,  $player_1$  wins in  $G(T_1, X, \mathbb{G})$ .

**Remark 2.13:** In the grill topological space :

- i- If  $player_2 \nearrow G(T_1, X)$ , then  $player_2 \nearrow G(T_1, X, \mathbb{G})$ . Therefore,  $player_2 \searrow G(T_1, X)$  then  $player_2 \searrow G(T_1, X, \mathbb{G})$ .
- ii- If  $Player_1 \nearrow G(T_1, X, \mathbb{G})$ , then  $Player_1 \nearrow G(T_1, X)$
- iii- If  $Player_2 \searrow G(T_1, X)$ , then  $Player_2 \searrow G(T_1, X, \mathbb{G})$ .

**Theorem 2.14:** AG.T.S is  $G_\alpha$ - $T_1$ -space if and only if  $Player_2 \nearrow G(T_1, X, \mathbb{G})$ .

**Proof:** Let  $(X, \tau, \mathbb{G})$  is  $G_\alpha$ - $T_1$ -space. And let  $v_1 \neq v_2 \in X$  of  $Player_1$ , and  $player_2$  there exist  $T$  and  $K$  are  $G_\alpha$ -open sets  $v_1 \in T-K$  and  $v_2 \in K-T$ . so  $player_2 \nearrow G(T_1, X, \mathbb{G})$ .

Now let  $player_2 \nearrow G(T_1, X, \mathbb{G})$ , since  $v_1 \neq v_2 \in X$  of  $player_1$ , then  $Player_2$  can be take  $\mathcal{U}$  and  $\mathcal{V}$ , are  $G_\alpha$ -open sets  $v_1 \in T-K$  and  $v_2 \in K-T$  so,  $(X, \tau, \mathbb{G})$  is  $G_\alpha$ - $T_1$ -space.

**Corollary 2.15:** In A grill topological space a Player  $_2 \nearrow G(T_1, X, \mathbb{E})$  if  $\forall v_1 \neq v_2$  in  $X$  then, let  $T, K$  is  $G_\alpha$  -closed set where  $v_1 \in T-K$  and  $v_2 \in K-T$ .

**Corollary 2.16:** A grill topological space is  $G_\alpha$  - $T_1$  space if and only if  $player_1 \nearrow G(T_1, X, \mathbb{E})$ .

**Theorem 2.17:** A grill topological space is not  $G_\alpha$  - $T_1$  space if and only if  $player_1 \nearrow G(T_1, X, \mathbb{E})$ .

**Proof:** Let  $(X, \tau, \mathbb{E})$  is not  $G_\alpha$  - $T_1$  space, there exist  $v_1 \neq v_2 \in X$  such that there not find  $G_\alpha$  -open sets Each of them contains one of the two points and does not contain the other. So the two elements  $v_1 \neq v_2$  ( the choice of  $player_1$ ),  $player_2$  cannot be found  $T$  and  $K$ , where  $v_1 \in T-K$  and  $v_2 \in K-T$ .

Hence,  $player_1 \nearrow G(T_1, X, \mathbb{E})$ , If  $player_1 \nearrow G(T_1, X, \mathbb{E})$ , this means that there is a choice  $v_1 \neq v_2$  in  $X$  of  $player_1$ , such that  $player_2$  can not find two  $G_\alpha$  -open sets such that  $v_1 \in T-K$  and  $v_2 \in K-T$ . Hence,  $(X, \tau, \mathbb{E})$  is not  $G_\alpha$  - $T_1$  space.

**Corollary 2.18:** A grill topological space is not  $G_\alpha$  - $T_1$  space if and only if  $player_1 \nearrow G(T_1, X, \mathbb{E})$ .

**Definition 2.19:** The game  $G(T_2, X, \mathbb{E})$  is such as the game  $G(T_1, X, \mathbb{E})$  except  $\mathcal{U}, \mathcal{V}$  are two disjoint  $G_\alpha$  -open sets let  $\mathcal{B}$  is a family of  $G_\alpha$  -open sets, are a disjoint in  $X$ .

## References

- [1] Al-Omari, A., & Noiri, T. : Decompositions of continuity via grills. *Jordan J. Math. Stat*, 4(1), 33-46 (2011).
- [2] Hatir, E., & Jafari, S. : On some new classes of sets and a new decomposition of continuity via grills. *Journal of Advanced Mathematical Studies*, 3(1), 33-41 (2010).
- [3] Navaneethakrishnan, M., & Paulraj Joseph, J. : g-Closed sets in ideal topological spaces. *Acta Mathematica Hungarica*, 119(4), 365-371 (2008).
- [4] Bowen, K. : Games, theory and applications: Ellis Horwood, Chichester, 279 pages (1984).
- [5] Jabbar, N. A., & Nasir, A. I. : Some Types of Compactness in Bitopological Spaces. *Ibn AL-Haitham Journal For Pure and Applied Science*, 23(1), 321-328 (2017).

- [6] Shubik, M. : Games, Theory and Applications (LC Thomas). *SIAM Review*, 27(4), 608 (1985).
- [7] Császár, Á. : Separation axioms for generalized topologies. *Acta Mathematica Hungarica*, 104 (2004).
- [8] Shubham Sharma & Ahmed J. Obaid. Mathematical modeling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.
- [9] Sanjeev Kumar Punia, Manoj Kumar, Jyoti Aggarwal & Kuldeep Malik. Object based learning using multi-dimensional games, *Journal of Discrete Mathematical Sciences and Cryptography*, 23:2, 509-524 (2020), DOI: 10.1080/09720529.2020.1728904.
- [10] Shuwaie, J. S., & Hussain, A. K. : Topological Spaces  $F_1$  And  $F_2$ . *Wasit Journal of Computer and Mathematics Sciences*, 1(2), 62-70 (2022).