

Some covering properties and connectedness via G_α -open sets

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Abstract

The novel ideas in grill topological spaces are explored by applying them to the setting of specified sets, where the G_α -open sets are defined. The set's attributes are investigated, and examples and properties related to this set are provided. The set includes a variety of functions, such as open, closed, and continuous functions. This is the first step in a series of analyses of various topological features of this collection. By studying the new concept G_α -open set and its properties we created a new concept can be named as connectedness via G_α -open sets and some covering properties.

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1. Introduction

Many scholars were interested in studying the grill concept [1], such as the scholar Choquet, who is considered the first to lay the foundation stone in forming an idea about the grill concept and formulating a definition for this concept [2]. After that, many attempts were made to study the properties associated with this concept and to know the relationship of these properties with each other [3]. Several kinds of grill topological spaces exist, including discrete and cofinite topologies [4]. Let A be a subset of a grill topological space $(\mathcal{G}, \tau, G)$. A is a G_α -open set if there exist $\zeta \in \mathcal{G}$; $\zeta \subseteq A \subseteq \text{Int}cl_G(\zeta)$. A space $\mathcal{W}$ is compact if every open cover has a finite subcover for $\mathcal{W}$. A topological space

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(Y, τ) is called Lindelöf if every open cover has a countable subcover. A topological space (Y, τ) is disconnected if there exist two open disjoint nonempty sets A and B such that $A \cup B = Y$. The topological Space (Y, τ) is connected if it is not disconnected. The complement of α - open set is

α -closed set, the set of all α -open set denoted by $\alpha - (\varphi)$ and the set of all α -closed set denoted by $\alpha - c(\varphi)$ [5].

A grill is a collection of nonempty subsets of (φ, τ) . If i. $\mu \in G$ and $\mu \subseteq v$ so $v \in G$, ii. $\mu, v \subseteq \varphi$ and $\mu \cup v \in G$, then $\mu \in G$ or $v \in G$. a grill topological space denoted by (φ, τ, G) [6]. let (φ, τ) is a topological space, If $G = P(\varphi) \setminus \{\varphi\}$, then $\tau_G = \tau$ [7].

2. Studying some properties of the concept G_α -open sets

Definition 2.1: Suppose (φ, τ, G) is a grill topological space and $u \in \tau$. A closure grill alpha of u which is shortly $\zeta l_{G_\alpha}(u)$ is defined by;

$$\zeta l_{G_\alpha}(u) = \bigcap \{V \subseteq \varphi; V \text{ is a closed set in } \tau_{G_\alpha} \text{ whenever } u \subseteq V\}.$$

Definition 2.2: let A be a sub-set of a grill topological space (φ, τ, G) , A is a G_α -open set if there exist $\zeta \in G$; $\zeta \subseteq A \subseteq \zeta l_{G_\alpha}(\zeta)$. the complement of G_α - open set is G_α -closed set, the set of all G_α -open set denoted by $G_\alpha - (\varphi)$ and the set of all G_α -closed set denoted by $G_\alpha - c(\varphi)$.

Proposition 2.3: If K is an open set, it is also G_α^* -open set. And for all closed set K^c then it is also G_α^* -closed set.

Proposition 2.4: Whenever set A is G_α - o it is also G_α -o set.

Theorem 2.5: The union of any family of G_α -o sets is G_α -o set.

Definition 2.6: Let $f: (\varphi, \tau, G) \rightarrow (\vartheta, \tau', G')$ be a function then :

1. f is a G_α - continuous function, denoted by " G_α -cf" if $f^{-1}(u) \in G_\alpha - o(\varphi)$ for each $u \in \tau'$.
2. f is Strongly G_α -continuous function, denoted by " $s G_\alpha$ -cf" if $f^{-1}(u) \in \tau$, for each $u \in G_\alpha - o(Y)$.
3. f is G_α -irresolute function, denoted by " G_α -irr f " if $f^{-1}(u) \in G_\alpha - o(\varphi)$, for each $u \in G_\alpha - o(Y)$.

3. G_α -open sets in grill compact space

Definition 3.1: Suppose that (ϑ, τ, G) is a grill topological space. By a " G_α -open cover" of X , we mean a sub-family of $G_\alpha - O(\vartheta)$, which covers ϑ .

Definition 3.2: Suppose (ϑ, τ, G) is a grill topological space. By a " G_α -open cover" of ϑ we mean a sub-family of $G_\alpha - o(\vartheta)$, which covers for ϑ .

Definition 3.3: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is said to be G_α -compact Space if every G_α -open cover for ϑ has a finite G_α -subcover.

Theorem 3.4: Every G_α -compact Space is compact Space.

Proof: Suppose $\mathcal{L} = \{ \mathcal{L}_i; i \in I; \mathcal{L}_i \in \tau \forall i \}$ is an open cover for ϑ such that $\vartheta = \bigcup_{i \in I} \mathcal{L}_i$ and since every open set is G_α -open set. So, $\mathcal{L}$ is a G_α -open cover for ϑ , and since ϑ is a G_α -compact set. So, there exists a finite subcover, say $\mathcal{L}_i = \{ \mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_n \}$ such that $X = \bigcup_{i=1}^n \mathcal{L}_i$. Therefore, ϑ is compact.

Theorem 3.5: The G_α -irresolute function image of G_α -compact Space is a G_α -compact Space.

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ be an G_α -irresolute function such that ϑ is a G_α -compact, and let

$\mathcal{L} = \{ \mathcal{L}_i; \mathcal{L}_i \in G_\alpha\text{-}o(Y) \}$ be a G_α -open cover for $f(\vartheta)$, then $f(\vartheta) \subseteq \bigcup_{i \in I} \mathcal{L}_i; \mathcal{L}_i \in G_\alpha\text{-}o(Y)$, hence $f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup_{i \in I} \mathcal{L}_i)$. So, $\vartheta \subseteq f^{-1}(\bigcup_{i \in I} \mathcal{L}_i)$. Then $\vartheta \subseteq \bigcup_{i \in I} f^{-1}(\mathcal{L}_i)$. Now, since f is an G_α -irresolute function. Therefore, $f^{-1}(\mathcal{L}_i) \in G_\alpha\text{-}o(\vartheta)$, for each $i \in I$. So, $\{f^{-1}(\mathcal{L}_i); i \in I\}$ is a G_α -open cover for ϑ , but ϑ is a G_α -compact, then there exist $\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_n$ such that $X \subseteq \bigcup_{i=1}^n f^{-1}(\mathcal{L}_i)$. So, $f(\vartheta) \subseteq f(\bigcup_{i=1}^n f^{-1}(\mathcal{L}_i))$ hence $f(\vartheta) \subseteq \bigcup_{i=1}^n \mathcal{L}_i$.

Therefore, $f(\vartheta)$ is a G_α -compact Space.

Theorem 3.6: The strongly G_α -the continuous image of compact Space is a G_α -compact Space.

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ be a strongly G_α -continuous such that ϑ is a compact, and $\mathcal{L} = \{ \mathcal{L}_i; \mathcal{L}_i \in G_\alpha\text{-}o(Y); i \in I \}$ such that $f(\vartheta) = \bigcup_{i \in I} \mathcal{L}_i$ so $f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup_{i \in I} \mathcal{L}_i)$, $\mathcal{L}_i \in G_\alpha\text{-}o(Y); i \in I$ then $\vartheta = \bigcup \{ f^{-1}(\mathcal{L}_i), f^{-1}(\mathcal{L}_i) \in \tau \}$. Now since ϑ is compact there exist $i=1, \dots, n$. $\vartheta = \bigcup \{ f^{-1}(\mathcal{L}_i), f^{-1}(\mathcal{L}_i) \in \tau \} i=1, \dots, n$. So, $f(\vartheta) = \bigcup \{ f f^{-1}(\mathcal{L}_i), f f^{-1}(\mathcal{L}_i) \in \tau \} i=1, \dots, n \subseteq \bigcup \{ \mathcal{L}_i; \mathcal{L}_i \in \tau \} i=1, \dots, n$. Therefore, $f(\vartheta)$ is a G_α -O-compact space.

Theorem 3.7: The G_α -continuous image of G_α -compact is compact.

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ G_α -continuous function such that ϑ is a G_α -compact, and $\mathcal{L} = \bigcup_{i \in I} \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \}$ such that $f(\vartheta) = \bigcup_{i \in I} \mathcal{L}_i$, so

$f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup_{i \in I} \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \})$ then $\vartheta = \bigcup \{ f^{-1}(\mathcal{L}_i), f^{-1}(\mathcal{L}_i) \in G_\alpha\text{-}o(\vartheta) \}$.

Now since ϑ is G_α -o-compact there exist $i = 1, \dots, n$.

$\vartheta = \bigcup \{ f^{-1}(\mathcal{L}_i), f^{-1}(\mathcal{L}_i) \in \tau \} i=1, \dots, n$. therefore there exist $i=1, \dots, n$ such that $f(\vartheta) \subseteq \bigcup \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \} i=1, \dots, n$. Therefore $f(\vartheta)$ is a compact space.

Theorem 3.8: *If A and B are G_α -compact, then $A \cup B$ is a G_α -compact.*

Proof: Let $\mathcal{L} = \{ \mathcal{L}_i, i \in I ; \mathcal{L}_i \in \tau \forall i \}$ is an G_α -open cover for $A \cup B$; $A \cup B = \bigcup_{i \in I} \mathcal{L}_i$, hence $A \subseteq \bigcup_{i \in I} \mathcal{L}_i$ and $B \subseteq \bigcup_{i \in I} \mathcal{L}_i$, but A and B are G_α -compact, then there exist $\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_n$ such that $A \subseteq \bigcup_{i=1}^n \mathcal{L}_i$; and $B \subseteq \bigcup_{i=1}^m \mathcal{L}_i$. So, $A \cup B \subseteq \bigcup_{s=1}^{n+m} \mathcal{L}_s$. That mean $\mathcal{L}_{i,i=1,2,\dots,s}$ is a finite sub-cover for $\mathcal{L}$ then $A \cup B$ which is a G_α -compact set.

4. G_α -lindelöff Space

Definition 4.1: A grill topological space $(\vartheta, \tau, \mathbb{G})$ is called G_α -lindelöff Space if every G_α -open cover for ϑ has a G_α -countable subcover of ϑ .

Theorem 4.2: *Every G_α -lindelöff space is a lindelöff space.*

Proof: Let $\mathcal{L} = \{ \mathcal{L}_i, i \in I ; \mathcal{L}_i \in \tau \forall i \}$ is an open cover for ϑ such that $\vartheta = \bigcup_{i \in I} \mathcal{L}_i$ and since every open set is a G_α -open set. So, $\mathcal{L}$ is a G_α -"open cover" for ϑ , and since ϑ is a G_α -lindelöff set. So, there exist a finite subcover say $\mathcal{L}_i = \{ \mathcal{L}_1, \mathcal{L}_2, \dots, \}$ such that $\vartheta = \bigcup_{i=1}^\infty \mathcal{L}_i = 1$. Therefore, ϑ is a lindelöff space.

Theorem 4.3: *The G_α -irresolute continuous function image of G_α -lindelöff Space is a G_α -lindelöff Space.*

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ G_α -irresolute continuous function such that ϑ is a G_α -lindelöff, and $\mathcal{L} = \{ \mathcal{L}_i \}$ be a G_α -open cover for $f(\vartheta)$ then $f(\vartheta) \subseteq \bigcup_{i \in I} \mathcal{L}_i$; $\mathcal{L}_i \in G_\alpha o(Y)$, hence $f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup_{i \in I} \mathcal{L}_i)$ So, $\vartheta \subseteq f^{-1}(\bigcup_{i \in I} \mathcal{L}_i)$, then $\vartheta \subseteq \bigcup_{i \in I} (f^{-1}(\mathcal{L}_i))$ Now, since f is an G_α -irresolute continuous function. Therefore, $f^{-1}(\mathcal{L}_i) \in G_\alpha o(X)$, for each $i \in I$ So $\{ f^{-1}(\mathcal{L}_i) \}_{i \in I}$ is a G_α -open cover for ϑ , but ϑ is a G_α -lindelöff, then there exist $\mathcal{L}_1, \mathcal{L}_2, \dots, \bigcup_{i=1}^\infty \mathcal{L}_i = 1$. So, $f(\vartheta) \subseteq f(\bigcup_{i=1}^\infty f^{-1}(\mathcal{L}_i) = 1)$, hence $f(\vartheta) \subseteq \bigcup_{i=1}^\infty \mathcal{L}_i = 1$ Therefore, $f(\vartheta)$ is a G_α -lindelöff Space.

Theorem 4.4: *The Strongly G_α -continuous function image of lindelöff Space is a G_α -lindelöff Space.*

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ Strongly G_α -continuous function such that ϑ is compact, and $\mathcal{L} = \bigcup \{ \mathcal{L}_i ; \mathcal{L}_i \in G_\alpha o(Y) \}_{i \in I}$ be an G_α -open cover for $f(\vartheta)$ such that $f(\vartheta) = \bigcup (\mathcal{L}_i)$ so, $f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup \{ \mathcal{L}_i ; \mathcal{L}_i \in G_\alpha o(Y) \}_{i \in I})$ then $\vartheta = f^{-1}(\mathcal{L}) = \bigcup \{ f^{-1}(\mathcal{L}_i) \}$, $f^{-1}(\mathcal{L}_i) \in \tau$. Now since ϑ is a Lindelöff space, then there exists $i=1,2,\dots$ such that $\vartheta = \bigcup_{i=1}^\infty f^{-1}(\mathcal{L}_i)$, $f^{-1}(\mathcal{L}_i) \in \tau$. So $f(\vartheta) = \bigcup_{i=1}^\infty f(f^{-1}(\mathcal{L}_i)) \subseteq \bigcup_{i=1}^\infty \{ \mathcal{L}_i ; \mathcal{L}_i \in \tau \}$. Therefore, $f(\vartheta)$ is a G_α -lindelöff Space as shown in Figure 1.

Theorem 4.5: The G_α - continuous function image of G_α -lindelöff is lindelöff.

Proof: Let $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G}')$ be G_α - continuous function such that ϑ is a G_α o-lindelöff, and $\mathcal{L} = \bigcup \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \}; i \in I$ be an open cover of $f(\vartheta)$ such that $f(\vartheta) \subseteq \bigcup_{i \in I} (\mathcal{L}_i)$ so, $f^{-1}f(\vartheta) \subseteq f^{-1}(\bigcup \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \}; i \in I)$ then $\vartheta = \bigcup_{i \in I} (\mathcal{L}_i) = \bigcup \{ f^{-1}(\mathcal{L}_i); f^{-1}(\mathcal{L}_i) \in G_\alpha o(\vartheta) \}$. Now since ϑ is a G_α o-lindelöff there exist $i=1, 2, \dots$, $\vartheta = \bigcup_{i=1}^{\infty} \{ f^{-1}(\mathcal{L}_i); f^{-1}(\mathcal{L}_i) \in \tau \}; i=1, 2, \dots$. So $f(\vartheta) = \bigcup_{i=1}^{\infty} \{ f f^{-1}(\mathcal{L}_i); f f^{-1}(\mathcal{L}_i) \in \tau' \}; i=1, 2, \dots \subseteq \bigcup_{i=1}^{\infty} \{ \mathcal{L}_i; \mathcal{L}_i \in \tau' \}$. Therefore, ϑ is a lindelöff space.

Proposition 4.6:

1. Every G_α -compact is G_α - lindelöff.
2. Every G_α -compact is compact.
3. Every G_α -Lindelöff is Lindelöff.

The inverse of (proposition 4.6) this is not always the case see (Example 4.7) and (Example 4.8).

Example 4.7: Let $\vartheta = \mathbb{N}$, $\tau = \mathbb{D}$ with any grill, it's clear that $(\vartheta, \tau, \mathbb{G})$ is a G_α -lindelöff since every open cover has a countable subcover, but it is not G_α -compact since $\mathcal{L} = \{ \{n\}; n \in \mathbb{N} \}$ is a G_α -open cover has no finite subcover.

Example 4.8: Let $\vartheta = \mathbb{R}$, $\tau = \mathbb{I}$, $\mathbb{G} = \emptyset$, $G_\alpha o(\vartheta) = \mathbb{P}(\vartheta)$, it's clear that $(\vartheta, \tau, \mathbb{G})$ is a compact and lindelöff, but it is not G_α -compact and not G_α - lindelöff, since there exists a G_α -open cover $\mathcal{L} = \{ \{r\}; r \in \mathbb{R} \}$ has no finite subcover.

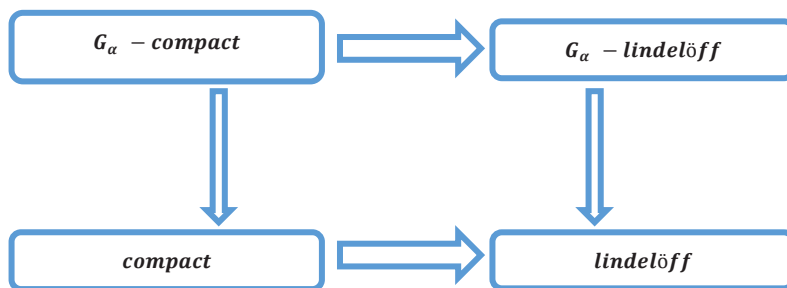


Figure 1
Compact via Lindelöff space

5. G_α o- connected set

Definition 5.1: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is said to be G_α o-disconnected if there exist two G_α -open disjoint nonempty sets A and B such that $A \cup B = \vartheta$. i.e., ϑ is G_α o- disconnected if and only if $\vartheta = A \cup B$;

$A, B \in G_\alpha(\vartheta)$, $A \cap B = \emptyset$. The sets A and B form a G_α α -separation of ϑ . and the grill topological space $(\vartheta, \tau, \mathbb{G})$ is a G_α α -connected if and only if it is not G_α α -disconnected. ϑ is a G_α α -connected if and only if $\vartheta \neq A \cup B$;

$$A, B \in G_\alpha(\vartheta), A \cap B = \emptyset, A, B \neq \emptyset.$$

Example 5.2: Let $(\vartheta, \tau, \mathbb{G})$ be a grill topological space and

$$\vartheta = \{\vartheta_1, \vartheta_2, \vartheta_3\}, \tau = \tau_G = \{\vartheta, \emptyset, \{\vartheta_2\}, \{\vartheta_3\}, \{\vartheta_2, \vartheta_3\}\}, \mathbb{G} = \mathbb{P}(\vartheta) \setminus \{\emptyset\} \text{ is } G_\alpha \text{ } \alpha\text{-connected Space.}$$

Proposition 5.3: Every disconnected is G_α α -disconnected Space.

Proof: Let (ϑ, τ) be disconnected Space, and then the exist $u \neq \emptyset$, $V \neq \emptyset$ and $u, V \in \tau; u \cap V = \emptyset$ and $u \cup V = X$ since every open set is a G_α -open set. Therefore, ϑ is G_α α -disconnected.

Proposition 5.4: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is G_α α -disconnected with any grill whenever $G_\alpha \alpha(\vartheta) = \mathbb{P}(\vartheta)$ if ϑ contained more than one element since there exists A and $A^c \in G_\alpha \alpha(\vartheta)$; $A \cup A^c = \vartheta$, $A \cap A^c = \emptyset$, $A, A^c \neq \emptyset$.

Proposition 5.5: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is a G_α α -connected where $\mathbb{G} = \mathbb{P}(\vartheta) \setminus \{\emptyset\}$ and $\tau = \tau_G = \{\vartheta, \emptyset\}$.

Proposition 5.6: If $\tau_G = \mathcal{F}_G$ and $\tau_G \neq$ Indiscrete topology that is, mean $(\vartheta, \tau, \mathbb{G})$, is a G_α α -disconnected.

Theorem 5.7: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is a G_α α -connected if and only ϑ can not be written as a union of two nonempty disjoint G_α -closed sets.

Proof: Let ϑ is G_α α -connected if $\vartheta = A \cup B$ such that ; A and $B \in G_\alpha c(\vartheta)$, $A \cap B = \emptyset$, $A, B \neq \emptyset$, so. $A = B^c$ and $B = A^c$, then $\vartheta = A \cup B$; A and $B \in G_\alpha \alpha(\vartheta)$, $A \cap B = \emptyset$, $A, B \neq \emptyset$, that's means that ϑ is a G_α α -disconnected. That's a contradiction. Therefore, ϑ cannot be written as a union of two nonempty disjoint G_α -closed sets. Now, if ϑ is a G_α α -disconnected Space, so. $\vartheta = A \cup B$; A and $B \in G_\alpha(\vartheta)$, $A \cap B = \emptyset$, $A, B \neq \emptyset$, so $A = B^c$ and $B = A^c$, then A and $B \in G_\alpha c(\vartheta)$, but that is a contradiction. Therefore, ϑ is G_α α -connected Space.

Theorem 5.8: The grill topological space $(\vartheta, \tau, \mathbb{G})$ is a G_α α -connected if and only if the only subsets of the space ϑ which are G_α -open and G_α -closed are ϑ and $\emptyset$.

Proof: Let A and $A^c \in G_\alpha \alpha(x)$ and $A \neq \vartheta$, $A \neq \emptyset$, so $\vartheta = A \cup A^c$; $A \cap A^c = \emptyset$, $A, A^c \neq \emptyset$. Then ϑ is a G_α α -disconnected, and that is a contradiction. Let ϑ be a

G_α o -disconnected space that's mean that $\vartheta = U \cup V$; $u, V \in G_\alpha o(x)$ such that $u \cap V = \emptyset$ and $u, V \neq \emptyset$ implies that $u = v^c$ and $V = u^c$. So $u, V \in G_\alpha c(x)$ (which is a contradiction). Therefore, ϑ is G_α o -connected Space.

Remark 5.9: If $f: (\vartheta, \tau, \mathbb{G}) \rightarrow (Y, \tau', \mathbb{G})$ is a G_α o -irresolute and on to function, and Y is a G_α o -connected space, then ϑ is not necessary G_α o -connected space.

Example 5.10: Let $f: (\mathbb{R}, D, \mathbb{G}) \rightarrow (\mathbb{R}, I, \mathbb{G})$; $f(r) = r$ for all $r \in \mathbb{R}$ and $\mathbb{G} = \mathbb{P}(\vartheta) \setminus \{\emptyset\}$, so f is a G_α o -irresolute Now, onto function and $(\mathbb{R}, I, \mathbb{G})$ is a G_α o -connected Space and $(\mathbb{R}, D, \mathbb{G})$ is a G_α o -disconnected Space.

Theorem 5.11: If A and B are G_α o -connected sets of $(\vartheta, \tau, \mathbb{G})$ and $A \cap B \neq \emptyset$ then $A \cup B$ is G_α o -connected set.

Proof: Let $(\vartheta, \tau, \mathbb{G})$ be a grill topological space and $A, B \subseteq \vartheta$ such that A, B are G_α o -connected sets. Now, let $A \cup B$ be a G_α o -disconnected space, so $A \cup B = U \cup V$; $U, V \in G_\alpha o(X)$, $U \cap V = \emptyset$ and $U, V \neq \emptyset$, then $A \subseteq A \cup B$, $A \subseteq U \cup V$, $A \subseteq U$ or $A \subseteq V$. Now in a similar way on B that leads to either:

$A \subseteq U$ and $B \subseteq U$ then $A \cup B \subseteq U$ then $V = \emptyset$ which is a contradiction Or $A \subseteq V$ and $B \subseteq V$ then $A \cup B \subseteq V$ then $U = \emptyset$ which is a contradiction Or $A \subseteq U$ and $B \subseteq V$ then $A \cap B \subseteq U \cap V$ then $A \cap B = \emptyset$ which is a contradiction Or $A \subseteq V$ and $B \subseteq U$ then $A \cap B \subseteq U \cap V$ then $A \cap B = \emptyset$, that's contradiction!. So, $A \cup B$ is a G_α o -connected Space.

Remark 5.12: We can generalization **Theorem 5.11.** for a family of a G_α o -connected sets as follows let $\{A_\alpha\}, \alpha \in \Lambda$ be a family of a G_α o -connected subsets of a space $(\vartheta, \tau, \mathbb{G})$ and $\bigcap A_\alpha \neq \emptyset, \alpha \in \Lambda$, then $\bigcup A_\alpha, \alpha \in \Lambda$ is a G_α o -connected set.

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