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Score for the groups SL(2,121) and SL(2,169)

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Abstract

A factor group is a mathematical group obtained by aggregating similar elements of a larger group using an equivalence relation that preserves some of the group structure. In this paper, the factor groups $K(SL(2,121))$ and $K(SL(2,169))$ computed for each group from the character table of rational representations.

Subject Classification: *06F05, 06F15.*

Keywords: *Factor group, Rational representation, Character table, Rational character table, Cyclic group.*

1. Introduction

For any finite group G, let N be a normal subgroup of G. Then, the set of all left (or right) cosets of N in G is also a group itself, known as the factor group of G by N (or the quotient group of G by N) [1] [2].

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Definition 1.1: For the finite n-dimensional vector space V (n, F), the special linear group, written $\mathcal{SL}$(n, F), is the subgroup of GL(n, F) consisting of all unimodular transformations [3] [4]. SL(n, F) is the kernel of the homomorphism det : GL(n, F) → F*. Also, the group GL(n, F) is not simple in general [5][6]. The group of all matrices of determinant 1 is $\mathcal{SL}(n$, F), [7] and [8]. In [9], a researcher defined complexified adjoint representations of the complexification Lie algebras associated with the special orthogonal group SO(3) and special linear group $\mathcal{SL}(2,\mathbb{C})$ have been obtained. A new representation of their tensor product is naturally arisen and computed in detail. By employing the ideas in [10-13], we get $\mathcal{K}(\mathcal{SL}(2,121))$ and $\mathcal{K}(\mathcal{SL}(2,169))$.

2. Basic Concepts

Theorem 2.1: [6] $\mathcal{K}(G) = \mathbb{Z}_{\wp}$ *And* $\mathcal{K}(G) = \bigoplus_{i=1}^{n} \mathbb{Z}_{p^i}$ *. for the* $\wp$ *–group G.*

Theorem 2.2: [1] $\mid \mathcal{SL}(2, q^n) \mid = q^n \, (q^{2n} - 1)$.

3. Main Results

The researchers in [10] study the character table of rational representations of $\mathcal{SL}(2, \wp^k)$. We employ that for $\mathcal{SL}(2,121)$ and $\mathcal{SL}(2,169)$.

3.2 *Results for* $\mathcal{SL}(2,121)$

$|\mathcal{SL}(2,121)| = 1771440$. The character table of rational representations is

C_g	**1**	z	$c = d$	$zc = zd$	a	a^2	a^3	a^4	a^5	a^6
$\mid C_g \mid$	**1**	**1**	**7320**	**7320**	**14762**	**14762**	**14762**	**14762**	**14762**	**14762**
$\mid C_G(g) \mid$	**1771440**	**1771440**	**242**	**242**	**120**	**120**	**120**	**120**	**120**	**120**
1_G	1	1	1	1	1	1	1	1	1	1
Ψ	121	121	0	0	1	1	1	1	1	1
χ_1	3904	- 3904	32	- 32	0	0	0	- 128	0	0
χ_2	976	976	8	8	0	- 16	0	16	0	32
χ_3	1952	- 1952	16	- 16	0	0	0	64	0	128
χ_4	488	488	4	4	- 4	4	8	0	16	- 8
χ_5	976	- 976	8	- 8	0	0	0	16	- 16	0
χ_6	488	488	4	4	0	8	0	- 16	0	- 16
χ_8	488	488	4	4	4	32	- 8	0	- 16	- 16
χ_{10}	244	244	2	2	0	4	0	- 4	- 8	- 8
χ_{12}	244	244	2	2	2	- 2	- 4	- 4	- 8	- 8
χ_{15}	488	- 488	4	- 4	0	0	- 8	- 16	- 16	16
χ_{20}	122	122	1	1	1	- 1	- 2	- 2	2	2
χ_{24}	244	244	2	2	- 2	- 4	- 8	8	8	8
χ_{30}	122	122	1	1	0	- 2	2	2	2	2
χ_{40}	122	122	1	1	- 1	- 2	2	2	2	2
θ_1	7200	- 7200	- 60	60	0	0	0	0	0	0
θ_2	3600	3600	- 30	- 30	0	0	0	0	0	0
ξ	122	122	1	1	- 2	2	- 2	2	- 2	2
η	240	- 240	- 1	1	0	0	0	0	0	0

C_g	a^7	a^8	a^{12}	a^{14}	a^{21}	a^{24}	a^{28}	a^{42}	a^{56}	b
$\vert C_g \vert$	**28730**	**28730**	**28730**	**28730**	**28730**	**28730**	**28730**	**28730**	**28730**	**28392**
$\vert C_G(g)\vert$	**168**	**168**	**168**	**168**	**168**	**168**	**168**	**168**	**168**	**170**
1_G	1	1	1	1	1	1	1	1	1	1
Ψ	1	1	1	1	1	1	1	1	1	- 1
χ_1	0	0	0	0	0	- 384	1152	0	- 1152	0
χ_2	0	72	- 48	0	0	0	- 144	- 288	- 288	0
χ_3	0	- 48	- 144	0	- 288	- 576	- 576	576	576	0
χ_4	36	- 18	0	- 36	- 72	- 72	- 72	72	72	0
χ_6	0	0	- 72	- 72	72	- 72	72	72	72	0
χ_7	- 32	- 32	- 64	- 64	64	64	64	64	64	0
χ_8	- 36	- 36	- 72	- 72	72	72	72	72	72	0
χ_{12}	- 18	- 18	18	18	18	18	18	18	18	0
χ_{14}	- 8	- 8	8	8	8	8	8	8	8	0
χ_{21}	16	16	16	16	16	16	16	16	16	0
χ_{24}	18	18	18	18	18	18	18	18	18	0
χ_{28}	2	2	2	2	2	2	2	2	2	0
χ_{42}	2	2	2	2	2	2	2	2	2	0
χ_{56}	2	2	2	2	2	2	2	2	2	0
θ_1	0	0	0	0	0	0	0	0	0	64
θ_2	0	0	0	0	0	0	0	0	0	- 32
θ_5	0	0	0	0	0	0	0	0	0	- 16
θ_{10}	0	0	0	0	0	0	0	0	0	- 8
θ_{17}	0	0	0	0	0	0	0	0	0	- 4
θ_{34}	0	0	0	0	0	0	0	0	0	2
ξ	- 2	2	2	2	- 2	2	2	2	2	0
η	0	0	0	0	0	0	0	0	0	4

The diametrical matrix of the previous table is

$$\begin{pmatrix}
1771440 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 885720 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 442860 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Thus

$$K(\mathcal{SL}(2,121)) = \oplus\mathbb{Z}_{\wp}\ ,$$

$\wp$ = 1771440, 885720, 442860 ,4, 4,4,4,2,2,2,2,2,2,3,3,3,5,5,5,1

3.2 *Results for $\mathcal{SL}(2,169)$*

$|\mathcal{SL}\ (2,169)|$ = 4826640. The character table of rational representations is

C_g	1	z	$c = d$	$zc=zd$	a	a^2	a^3	b^2	b^5
$\lvert C_g \rvert$	1	1	14280	14280	28730	28730	28730	28392	28392
$\lvert C_G(g)\rvert$	4826640	4826640	338	338	168	168	168	170	170
1_G	1	1	1	1	1	1	1	1	1
Ψ	169	169	0	0	1	1	1	- 1	- 1
χ_1	8160	- 8160	48	48	0	0	0	0	0
χ_2	2040	2040	12	12	0	0	0	0	0
χ_3	4080	- 4080	24	24	0	0	96	0	0
χ_4	1020	1020	6	6	- 6	6	12	0	0
χ_6	1020	1020	6	6	0	12	0	0	0
χ_7	1360	- 1360	8	- 8	0	0	0	0	0
χ_8	1020	1020	6	6	6	18	- 12	0	0
χ_{12}	510	510	3	3	3	- 3	- 9	0	0
χ_{14}	340	340	2	2	0	4	0	0	0
χ_{21}	680	- 680	4	4	0	0	- 8	0	0
χ_{24}	510	510	3	3	- 3	0	-18	0	0
χ_{28}	170	170	1	1	1	- 1	- 2	0	0
χ_{42}	170	170	1	1	0	- 2	2	0	0
χ_{56}	170	170	1	1	- 1	- 2	2	0	0
θ_1	10752	- 10752	- 64	64	0	0	0	- 64	- 256
θ_2	5376	5376	- 32	- 32	0	0	0	0	128
θ_5	2688	- 2688	- 16	16	0	0	0	16	- 128
θ_{10}	1344	1344	- 8	- 8	0	0	0	0	64
θ_{17}	672	- 672	- 4	4	0	0	0	4	8
θ_{34}	336	336	- 2	- 2	0	0	0	4	- 8
ξ	170	170	- 1	- 1	- 2	2	- 2	0	0
η	336	- 336	1	- 1	0	0	0	- 4	4

The diametrical matrix of the previous table is

$$\begin{pmatrix}
4826640 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2413320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1206660 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Thus, $K(\mathcal{SL}(2,169)) = \oplus\mathbb{Z}_{\wp}$,

$\wp$ =4826640 2413320 1206660,17,5,7,7,7,3,3,3,4,4,4,2,2,2,2,2,2,2,2,2,1

4. Conclusion

For any group G, the diagonal matrix numbers of the result matrix having the same digit of the conjugacy classes in the character table of irreducible rational representation as follows : (1) The first number = $|G|$, (2) The second number = $|G|/2$, (3) The third number = $|G|/4$, (4) The reminder numbers equal to the factors of the order of the centralizer of the contumacy classes except the order of the centralizer of the conjugacy class 1 after remove the factors of the order of the group divided by 4 and 2 with complete the remaining number by 1.

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