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On semi-open sets in grill nano topological spaces

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Abstract

Open sets may be viewed as an extension of semi-open sets by applying the notions of semi-open sets and grill nano to nGs-open sets, with the following four goals in mind: The objective is to characterize nGs-open sets by examining and proving numerous of its attributes and comments. And investigate and define new kinds of functions based on the concept of nGs-open sets.

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1. Introduction

The concept for grill topological spaces rests on using two operators: and. The pioneer of this concept was Choquet [1]. Some parallels have been discovered between the Choquat concept and ideas, nets, and filters. Several hypotheses and characteristics have been discussed in [2–5]. It allows for the growth of the topological assembly utilized to account for intangibles like love, intelligence, beauty, instructional quality, etc.

Additionally, it broadens the frontiers of Nano topological spaces by employing the concept of grill modifications in the lower approximation, the upper approximation, and the boundary region. First proposed in 1970 by Levine [6], enlarging closed sets is widely credited as a breakthrough in the field. Lower, higher, and boundary estimates of a subset of a cosmic set with an

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important basis are the foundation on which the concept of Nano topological assemblage rests.

Also, the concept of Nano is used to introduce the definitions of the closed set, the interior set, and the closure set. Lellis [7] first developed this concept in 2013. The primary goal of this study is to embed a grill into a space consisting of a generalized closed Nano topology. We've established contact with some very important people.

2. Preliminaries

Definition 2.1:[4] Ꜹ Grill is a nonempty collection of nonempty subsets of a topological space χ.

1. $Ꜹ \in G$ *and* $Ꜹ \subseteq Ɓ \subseteq \chi$ *then* $Ɓ \in G$
2. $Ꜹ, Ɓ \subseteq \chi$ *and* $Ꜹ \cup Ɓ \in G$ *then* $Ꜹ \in G$ *or* $Ɓ \in G$. [3], [8]

Assuming that χ is a nonempty set, the following sets are grills on χ.

- $\acute{\emptyset}$ & $Ꝑ(\chi)\backslash\{\acute{\emptyset}\}$ are examples of trivial grills on χ.
- G_∞ is the grill of all infinite subset of χ.
- $G_{ÇÒ}$ is the grill of all uncountable subsets of χ.
- $G_{Ꝑ} = \{Ꜹ : Ꜹ \in Ꝑ(\chi), Ꝑ \in Ꜹ\}$ is a certain point grill on χ.
- $G_{Ꜹ} = \{Ɓ : Ɓ \in Ꝑ(\chi), Ɓ \cap Ꜹ^{Ç} \neq \acute{\emptyset}\}$.

* If $(\chi, ţ)$ If a topological space, and so the set of the all dense subset that does not already exist here is known as $G = \{Ꜹ : inţ(Çl(Ꜹ)) \neq \acute{\emptyset}\}$ is one kind of grill on χ [4].

* Suppose that G a grill on $(\chi, ţ)$. A mapping $∯ : Ꝑ(\chi) \rightarrow Ꝑ(\chi)$ is also known as $∯(Ꜹ) = \{\chi \in \chi : Ꜹ \cap û \in G$ for every $û \in ţ; \chi \in û\}$ for every $Ꜹ \in Ꝑ(\chi)$. A mapping $\psi : Ꝑ(\chi) \rightarrow Ꝑ(\chi)$ is also known as $\psi(Ꜹ) = Ꜹ \cup ∯(Ꜹ)$ for every $Ꜹ \in Ꝑ(\chi)$.[9]

Kuratowski's Axioms of Closure for the map ψ are verified: [9], [3], [10]

1. $\psi(\acute{\emptyset}) = \acute{\emptyset}$,
2. when $Ꜹ \subseteq Ɓ$, then $\psi(Ꜹ) \subseteq \psi(Ɓ)$,
3. when $Ꜹ \subseteq \chi$, then $\psi(\psi(Ꜹ)) = \psi(Ꜹ)$,
4. when $Ꜹ, Ɓ \subseteq \chi$, then $\psi(Ꜹ \cup Ɓ) = \psi(Ꜹ) \cup \psi(Ɓ)$.

Definition 2.2: [9] There exists a special topology $ţ_G = \{û \subseteq \chi : \psi(\chi - û) = (\chi - û)\}$, when for any $Ꜹ \subseteq \chi$ that corresponds inside the topological space, to a grill G $(\chi, ţ)$.

$$\psi(Ꜹ) = Ꜹ \cup ∯(Ꜹ) = ţ_G\text{-}Çl(Ꜹ) \text{ and } ţ \subseteq ţ_G.$$

Remark 2.3: [4] If $G = Ꝑ(\chi)/\{\acute{\emptyset}\}$, then $ţ_G = ţ$.

Remark 2.4: [3] We can find $ţ_G$ by using the base as follows

$$\mathcal{B}(ţ_G, ţ) = \{v - Ꜹ ; v \in ţ, Ꜹ \notin G\}$$

Definition 2.5:[9] Let $\chi \neq \acute{\emptyset}$ and Ɽ be an equivalence relation on χ, $\mathbb{A} \subseteq \chi$.

1. The upper approximation of $\mathbb{A}$ for Ɽ is denoted by $\overline{Ɽ_w}(\mathbb{A})$, where
$$\overline{Ɽ_w}(\mathbb{A}) = \cup_{\chi \in \chi}\{Ɽ(\chi): Ɽ(\chi) \cap \mathbb{A} \neq \acute{\emptyset}\}.$$
2. The lower approximation of $\mathbb{A}$ for Ɽ is denoted by $\underline{Ɽ_w}(\mathbb{A})$, where
$$\underline{Ɽ_w}(\mathbb{A}) = \cup_{\chi \in \chi}\{Ɽ(\chi): Ɽ(\chi) \subseteq \mathbb{A}\}.$$
3. The boundary region of $\mathbb{A}$ for Ɽ is denoted by $B_w(\mathbb{A})$, where
$$B_w(\mathbb{A}) = \overline{Ɽ_w}(\mathbb{A}) - \underline{Ɽ_w}(\mathbb{A}).$$

Definition 2.6:[7] Let $\chi \neq \acute{\emptyset}$ and Ɽ be an equivalence relation on χ and $ƫ_w(\mathbb{A}) = \left\{\chi, \acute{\emptyset}, \overline{Ɽ_w}(\mathbb{A}), \underline{Ɽ_w}(\mathbb{A}), B_w(\mathbb{A})\right\}$, where $\mathbb{A} \subseteq \chi$. Then $ƫ_w(\mathbb{A})$ is a topology on χ named nano topology for $\mathbb{A}$ $(\chi, ƫ_w(\mathbb{A})$ space is known as nano topological. The components of $ƫ_w(\mathbb{A})$ are called nano-open sets denoted by $\eta - open\ sets$. The supplement of a $\eta - open\ sets$ is named a nano-closed set denoted by $\eta - closed\ sets$.

Definition 2.7:[7], [11] Let $(\chi, ƫ_w)$ be N.T.S (nano topological space) and $\mathbb{A} \subseteq \chi$. The nano closure (respectively, nano interior) of $\mathbb{A}$ which is short $\eta Çl_w(\mathbb{A})$ (respectively, $\eta int_w(\mathbb{A})$) is defined by; $\eta Çl_w(\mathbb{A}) = \cap\{\mathcal{F}, \mathcal{F}^{Ç} \in ƫ_w, \mathbb{A} \subseteq \mathcal{F}\}$, (resp., $\eta int_w(\mathbb{A})) = \eta int_w(\mathbb{A}) = \{û; û \in ƫ_w, û \in \mathbb{A}\}$.

Remark 2.8: We said the triple $(\chi, ƫ_w, G)$ G.N.T.S (Grill nano topological space).

Definition 2.9:[7] Let $(\chi, ƫ_w)$ be an N.T.S, a subset $\mathbb{A}$ of χ is named N.S.O (nano semi-open set) if $\mathbb{A} \subseteq \eta Çl_w(\eta int_w(\mathbb{A})) \Leftrightarrow \exists û \in ƫ_w; û \subseteq \mathbb{A} \subseteq \eta Çl_w(û)$. $\mathbb{A}$ subset Ɓ of χ is called N.S.C(nano semi-closed) if (χ-Ɓ) is N.S.O set The collection of all N.S.O (respectively,N.S.C) sets in a nano topological space $(\chi, ƫ_w)$. will be symbolized by $ŞȎ(\chi)$ (respectively,$\eta G_Ş Ç(\chi)$).

Definition 2.10: There exists a special topology $\eta ƫ_{w G} = \{û \subseteq \chi: \psi(\chi - û) = (\chi - û)\}$, when for any $\mathbb{A} \subseteq \chi$ that corresponds to a nano grill G on the topological space $(\chi, ƫ_w)$.
$$\psi(\mathbb{A}) = \mathbb{A} \cup \oiint(\mathbb{A}) = \eta ƫ_{wG}\text{-} Çl(\mathbb{A}) \text{ and } ƫ_w \subseteq \eta ƫ_{wG}.$$

Definition 2.11: Let $(\chi, ƫ_w)$ be N.T.S and $\mathbb{A} \subseteq \chi$. The nano closure (respectively, nano interior) of $\mathbb{A}$ which is short $\eta Çl_w(\mathbb{A})$ (respectively, $\eta int_w(\mathbb{A})$) is defined by; $\eta Çl_{wG}(\mathbb{A}) = \cap\{\mathcal{F}, \mathcal{F}^{Ç} \in ƫ_w, \mathbb{A} \subseteq \mathcal{F}\}$, (resp., $\eta int_{wG}(\mathbb{A})) = \eta int_{wG}(\mathbb{A}) = \{û; û \in ƫ_w, û \in \mathbb{A}\}$.

3. On semi-open in grill nano topological spaces

Definition 3.1: For any G.T.S (Grill topological space) $(\chi, ƫ_G)$ and $\mathbb{A} \sqsubseteq \chi$; $\mathbb{A}$ nano Grill is considered semi-open if it exists.$û \in ƫ_w$; $û - \mathbb{A} \notin G$ and $\mathbb{A} - \eta Çl_{wG}(û) \notin G$ .And $\mathbb{A}$ denoted by $\eta G_Ş$-open. $\chi - \mathbb{A}$ is a nano Grill semi-closed and denoted by ηG̶Ş-closed the collection of all ηG̶Ş-open presently by ηG̶ŞȮ $(\chi)$the collection of all ηG̶Ş -closed presently by ηG̶ŞÇ(χ)

Example 3.2: Let (χ, ȶ$_w$, G̶) be an N.G.T.S to be a nano grill

$$\chi = \{\chi_1, \chi_2, \chi_3, \chi_4\},$$

$$G = \{\hat{u} \sqsubseteq \chi; \chi_2 \in \hat{u}\}$$

$$G = \{\{\chi_2\}, \{\chi_1, \chi_2\}, \{\chi_3, \chi_2\}, \{\chi_4, \chi_2\}, \{\chi_1, \chi_2, \chi_3\}, \{\chi_1, \chi_2, \chi_4\}, \{\chi_2, \chi_3, \chi_4\}, \chi\}$$

$$R = \{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_4, \chi_4), (\chi_2, \chi_4), (\chi_4, \chi_2)\}$$

$$R \setminus [\chi] = \{\{\chi_1\}, \{\chi_2, \chi_4\}, \{\chi_3\}$$

$$w \sqsubseteq \chi, w = \{2,3\},$$

$$t_w = \{\chi, \emptyset, \{3\}, \{2,4\}, \{2,3,4\}\}$$

$$\mathcal{B} = \{v - A; v \in t_w \wedge A \notin G\}$$

$$\mathcal{B} = \{\chi, \emptyset, \{\chi_1, \chi_2, \chi_4\}, \{\chi_1, \chi_2, \chi_3\}, \{\chi_2, \chi_3, \chi_4\}, \{\chi_2, \chi_3\}, \{\chi_2, \chi_4\}, \{\chi_1, \chi_2\}, \{\chi_2\}, \{\chi_3\}\}$$

$$= t_{wG}$$

$$\therefore \eta G_{\c{S}} \dot{O}(\chi) = \Psi(\chi)$$

Proposition 3.3:
1. Every nano open set is a ηG̶Ş -open sets.
2. Every nano closed set is a ηG̶Ş -closed**.**

Proposition 3.4: Let A be a ηG̶Ş $-$ *open sets* and Ɗ ⊆ χ for each û ∈ ȶ$_w$. A ⊆ Ɗ ⊆ ηÇl_wG̶ (û) then Ɗ ∈ ηG̶Ş $-$ *open sets*, whenever û $\neq \emptyset \neq$ Ɗ

Remark 3.5: The concepts ηG̶Ş $-$ *open sets* set and nano semi-open set are independent See [**Example (3.6)**and **Example (3.7)**].

Example 3.6: Let (χ, ȶ$_w$, G̶) any nano grill topology and $\chi = \{\chi_1, \chi_2, \chi_3\}$,

$$G = \{\hat{u}; \chi_1 \in \hat{u}\}, G = \{\{\chi_1\}, \{\chi_1, \chi_2\}, \{\chi_1, \chi_3\}, \chi\}$$

$$R = \{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_2, \chi_3), (\chi_3, \chi_2),$$

$$R \setminus [\chi] = \{\{\chi_2, \chi_3\}, \{\chi_1\}\}, \textit{let } w = \{\chi_1, \chi_3\}$$

$$t_w = \{\chi, \emptyset, \{\chi_1\}, \{\chi_2, \chi_3\}\}$$

$$\eta G_{\c{S}} \dot{O}(\chi) = \Psi(\chi), \eta \c{S} \dot{O}(\chi) \in \hat{u}$$

Hence, it's clear that $\{\chi_2\} \in$ ηG̶ŞȮ(χ) but $\{\chi_2\} \notin$ ηŞ Ȯ(χ).

Example 3.7: Let (χ, ȶ$_w$,G̶) any nano grill topology and $\chi = \{\chi_1, \chi_2, \chi_3, \chi_4\}$

$$G = \Psi(\chi) \setminus \{\chi_3, \emptyset\}$$

$$R = \{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_4, \chi_4), (\chi_1, \chi_4), (\chi_4, \chi_1)\}$$

$$Ꞓ \setminus [\chi] = \{\{\chi_1, \chi_4\}, \{\chi_3\}, \{\chi_2\}\}$$
$$w = \{\chi_1, \chi_3\}$$
$$ʈ_w = \{\chi, \acute{\emptyset}, \{\chi_3\}, \{\chi_1, \chi_4\}, \{\chi_1, \chi_3, \chi_4\}\}$$
$$\mathcal{B} = \{\chi, \acute{\emptyset}, \{\chi_3\}, \{\chi_1, \chi_4\}, \{\chi_1, \chi_3, \chi_4\}, \{\chi_1, \chi_2, \chi_4\}\} = \eta ʈ_{wG}$$

Let $A = \{\chi_2, \chi_3\}, \exists \hat{u} \in ʈ_w ; \hat{u} - A \notin G \wedge A - \eta Ç l_{wG}(\hat{u}) \notin G$

$\{\chi_2, \chi_3\} \in \eta Ş\dot{O}(\chi)$, *since* $Çlint\{\chi_2, \chi_3\} = \{\chi_2, \chi_3\}$ *but* $\{\chi_2, \chi_3\} \notin \eta G_Ş \dot{O}(\chi)$

Lemma 3.8: If $G = \mathcal{P}(\chi) \setminus \{\acute{\emptyset}\}$, then $ʈ_w = ʈ_{wG}$

Proposition 3.9: Let $(\chi, ʈ_w, G)$ be an N.G.T. (nano grill topological space) and A is a subset of χ. If $G = \mathcal{P}(\chi) \setminus \{\acute{\emptyset}\}$. $\eta G_Ş - open\ sets \leftrightarrow A$ is Nano semi-open set. Then A is $\eta G_Ş - open\ sets$

Lemma 3.10: $\cup_{i\in\Lambda} (\eta Ç l_{wG}(A_i)) \subseteq (\eta Ç l_{wG}(\cup_{i\in\Lambda} A_i)$

Proof: $A_i \subseteq \cup_{i\in\Lambda} A_i, \eta Ç l_{wG}(A_i) \subseteq \eta Ç l_{wG}(\cup_{i\in\Lambda} A_i), (\eta Ç l_{wG}(A_i)) \subseteq (\eta Ç l_{wG}(\cup_{i\in\Lambda} A_i), \cup_{i\in\Lambda} (\eta Ç l_{wG}(A_i)) \subseteq (\eta Ç l_{wG}(\cup_{i\in\Lambda} A_i)$

Theorem 3.11: The union of any family of $\eta G_Ş - open\ sets$ is a $\eta G_Ş - open\ sets$.

Remark 3.12: The intersection of any two $\eta G_Ş - open\ sets$ need not be $\eta G_Ş - open\ sets$.

Example 3.13: Let $\chi = \{\chi_1, \chi_2, \chi_3, \chi_4\}$, $G = \mathcal{P}(\chi) \setminus \{\acute{\emptyset}\}$

$$Ꞓ = \{(\chi_1, \chi_1), (\chi_2, \chi_2), (\chi_3, \chi_3), (\chi_4, \chi_4), (\chi_1, \chi_4), (\chi_4, \chi_1)\}$$
$$Ꞓ \setminus [\chi] = \{\{\chi_1, \chi_4\}, \{\chi_2\}, \{\chi_3\}\} \text{ Let } w = \{\chi_1, \chi_3\}$$
$$ʈ_w = \{\chi, \acute{\emptyset}, \{\chi_1, \chi_4\}, \{\chi_3\}, \{\chi_1, \chi_3, \chi_4\}\}$$

Let $A = \{\chi_1, \chi_2, \chi_4\}$, *and let* $D = \{\chi_2, \chi_3\}$ $\{\chi_2, \chi_3\} \in \eta G_Ş \dot{O}(\chi)$, $\{\chi_1, \chi_2, \chi_4\} \in \eta G_Ş \dot{O}(\chi)$, but $A \cap D = \{\chi_1, \chi_2, \chi_4\} \cap \{\chi_2, \chi_3\} = \{\chi_2\} \notin \eta G_Ş \dot{O}(\chi)$

So, $\{\chi_2, \chi_4\} \in \eta Ş\dot{O}(\chi) = \eta G_Ş \dot{O}(\chi)$.

Remark 3.14: The family of all $\eta G_Ş - open\ sets$ is represented supra topology.

4. Conclusion

In this work, a new type of open set was studied using the concept of nano-topology and grill, called ηG_Ş-open sets. The properties of this set were studied. It was found that $\eta Ş\dot{O}(\chi)$ represents a supra-topology space. Applying this notion defined new forms of functionality, and the relationship between these functions was found.

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