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# On maps of Γ -period 2 on prime Γ- near-rings

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**Abstract**

We present the concept of maps Γ- periodi2 on Γ -near-ring S. Our main goal is to research and explore the presence and mapping traits such as h Γ –hom anti-Γ –hom, Γ –α-derivations of Γ -periodi2 on Γ- near-rings.

***Subject Classification:*** *14E05, 37E05.*

***Keywords:*** *Prime Γ -near-ring, semiprime Γ -near-ring, mapping of Γ -periodi2, Γ -hom, anti-Γ -hom, Γ-α- -derivations.*

## 1. Introduction

In this paper, A Γ-near-ring was submitted in [3-5]. A Γ-near- ring S possesses the condition for $s,r \in S$, $s\Gamma S\Gamma r = 0$ gives $s = 0$ or $r=0$ is said prime Γ -near-ring and called semiprime if S possesses the property $t\Gamma S\Gamma t = 0$ gives $t = 0$ for every $t \in S$ [4,5]. The additional commutators are as follows; $[u, v]_\rho = u\rho v - v\rho u$ and $(u, v) = u+v-v-u$ designate the commutator for additive groups [4, 6]. If (S, +) is abelian, S is said to be commutative and 2-torsion-free if $2s = 0$ gives $s = 0$[5,6]. K is a non-empty subset of the S semi-group that is left-ideal (resp. right-ideal) if $S\Gamma K \subset K$ (resp. $K\Gamma S \subset K$) and K is a semi-group ideal if both a semi-group left ideal and a semi-group right ideal [4,5]. The notion of map period 2 is given in the ring by Bell and Daif in [2]. In this work, we create a map of $\Gamma$-period 2 on $\Gamma$-near -ring. A map of S into itself is of $\Gamma$ –period 2 on S if $hh(s\gamma s) = s\gamma s$, $\forall$ $s \in S$, $\gamma \in \Gamma$. Let K be a subset of S be non-empty, a map $h:S \to S$ is of Γ-period 2 on K if $hh(s\gamma s) = s\gamma s$, $\forall s \in K$, $\gamma \in \Gamma$. A map that is additive $h:S \to S$ is called Γ- hom (anti-Γ -hom) in Γ-near-ring if $h(r + s)=h(r) + h(s)$ and

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$h(s\gamma r)=h(s)\gamma h(r)$, $((h(s\gamma r) = h(r)\gamma h(s))$, $\forall$ s, $r\in S, \gamma\in \Gamma$) [4]. An-additive endomorphism h: S→ S called an $\Gamma$ -derivation of S if $h(s\gamma r) = (s)\gamma h(r) + h(s)\gamma r$, or equivalently that $h(s\gamma r) = h(s)\gamma r + (s)\gamma h(r)$ $\forall$ $s,r \in S, \gamma\in \Gamma$ [3]. Let α be an auto-morphism of S. An-additive endomorphism h: S→ S is called an $\Gamma$-α-derivation of S if $h(s\gamma r) = \alpha(s)\gamma h(r) + h(s)\gamma r$, or equivalently, that $h(s\gamma r) = h(s)\gamma r + \alpha(s)\gamma h(r)$ $\forall s,r \in S, \gamma\in \Gamma$. [7]. Bell and Daif in [1,2] intended the subsistence and characteristic of derivations of period 2 on prime and semiprime rings. In this work, we have extensive noting of $\Gamma$ -α-derivations on prime and semiprime Γ-near-ring when they are of Γ-period two. We recall in this paper Γ- near—ring by $\Gamma$-(n-r) and $\Gamma$ –derivations by $\Gamma$ –der.

## 2. Preliminaries

**Lemma 2.1: [7]** *When S be Γ-(n-r) and d be Γ-α-der. of S, then ∀ s, r, z ϵ S, γ,β ϵ Γ. (i)* $(\alpha(s)\gamma h(r) + h(s)\gamma r)\beta z = \alpha(s)\gamma h(r)\beta z + h(s)\gamma r\beta z$.

(ii) $(h(s)\gamma r + \alpha(s)\gamma h(r))\beta z = h(s)\gamma r\beta z + \alpha(s)\gamma h(r)\beta z$

**Lemma 2.2:** [7] *When S be prime Γ-(n-r), h be Γ –α-der. of S, ∀ a ϵ S, if* $s\gamma h(S) = 0$, *thus* $s =0$.

**Lemma 2.3:** [4,5] *When S be a prime Γ-(n-r). If K is a nonzero semi-group and s ϵ S such that* $S\gamma s = \{0\}$*(resp,* $s\gamma S = \{0\}$ *), then* $s = 0$.

**Lemma 2.4:** *When S is Γ-(n-r), h is Γ-α-der. of S thus* $2(h(s)\gamma r + \alpha(s)\gamma h(r))\beta z = 2h(s)\gamma r\beta z + 2\alpha(s)\gamma h(r)\beta z$

***Proof:*** Using Lemma 2.1 gives

$$2(h(s)\gamma r + \alpha(s)\gamma h(r))\beta z = 2 (h(s)\gamma r\beta z + \alpha(s)\gamma h(r)\beta z )$$
$$= h(s)\gamma r\beta z + \alpha(s)\gamma h(r)\beta z + h(s)\gamma r\beta z + \alpha(s)\gamma h(r)\beta z$$
$$= h(s)\gamma r\beta z + ( h(s)\gamma r + \alpha(s)\gamma h(r))\beta z + \alpha(s)\gamma h(r)\beta z$$
$$= h(s)\gamma r\beta z + h(s)\gamma r\beta z + \alpha(s)\gamma h(r)\beta z + \alpha(s)\gamma h(r)\beta z$$
$$= 2h(s)\gamma r\beta z + 2\alpha(s)\gamma h(r)\beta z, \forall\ s , r ,z \in S, \gamma,\beta \in \Gamma$$

## 3. Main-Result

In this part we survey $\Gamma$ -hom, anti Γ- hom, $\Gamma$-α-der. where there of $\Gamma$ -period 2.

**Theorem 3.1:** *When S be a 2-torsionfree prime Γ-(n-r), K be nonzero semi-group ideal of S, thus S confess no nonzero Γ-α-der. h, hα = αh and h be Γ-period 2 on K.*

***Proof:*** Suppose there is a nonzero Γ-α-der. h on S so $hh(xs\gamma s) = s\gamma s$, $\forall$ s ϵ K. Thus $h(s)\gamma r\in$ K and the condition

$$h(s)\gamma r = hh(h(s)\gamma r) = h(hh(s)\gamma r + \alpha(h(s))\gamma h(r)) = h(s\gamma r + h(\alpha(s))\gamma h(r))$$
$$= h(s\gamma r) + h(h(\alpha(s))\gamma h(r))= h(s)\gamma r + \alpha(s)\gamma h(r) + hh(\alpha(s))\gamma h(r) + \alpha(h(\alpha(s))\ \gamma hh(r)$$

$= h(s)\gamma r + 2\alpha(s)\gamma h(r) + \alpha^2(h(s))\gamma r.$

Which means that $2\alpha(s)\gamma h(r) + \alpha^2(h(s))\gamma y = 0$ ,$\forall$ s, r $\epsilon$ K. (1)

Also we have

$$s\gamma r = hh(s\gamma r) = h(h(s)\gamma r + \alpha(s)\gamma h(r)) = hh(s)\gamma r + \alpha(h(s))\gamma h(r) + h(\alpha(s))\gamma h(r) + \alpha^2(s)\, \gamma hh(r) = s\gamma r + 2\alpha(h(s))\gamma h(r) + \alpha^2(s)\gamma r, \forall\ s, r \in K, \gamma \in \Gamma.$$

Thus we get $2\alpha(h(s))\ \gamma h(r) + \alpha^2(s)\gamma r = 0$, $\forall$ s , r $\epsilon$K, $\gamma \in \Gamma$. (2)

Putting h(s) instead of s in eq (2), we get

$$2\alpha(s)\gamma h(r) + \alpha^2(h(s))\gamma r= 0, \forall\ s,r \in K, \gamma \in \Gamma. \ (3)$$

Taking s instead of uγs in (2), where u$\epsilon$ S, and using Lemma 2.4 gives $0 = 2\alpha(h(u\gamma s))\gamma h(r) + \alpha^2(u\gamma s)\gamma r$

$$= 2h(\alpha(u\gamma s))\gamma h(r) + \alpha^2(u)\gamma\alpha^2(s)\gamma r = 2h(\alpha(u)\gamma\alpha(s))\ \gamma h(r) + \alpha^2(u)\gamma\alpha^2(s)\ \gamma\ r$$

$$= 2h(\alpha(u))\gamma\alpha(s)\gamma h(r) + 2\alpha^2(u)\gamma\ h(\alpha(s))\gamma h(r) + \alpha^2(u)\gamma\alpha^2(s)\ \gamma r$$

$$= 2h(\alpha(u))\gamma\alpha(s)\gamma\ h(r) + \alpha^2(u)\ \gamma\ (2h(\alpha(s))\gamma h(r) +\alpha^2(s)\gamma r), \forall\ s, r\in K \text{ and } u \in S, \gamma \in \Gamma.$$

Using eq (2) in previous relation implies

$$2h(\alpha(u))\gamma\alpha(s)\gamma h(r) = 0\ ), \forall\ s, r\in K \text{ and } u \in S, \gamma \in \Gamma. \ (4)$$

Substituting rγu for u in (4), and using Lemma 2.4 we get

$$0 = 2h(\alpha(r\gamma u))\gamma\alpha(s)\gamma h(r)= 2h(\alpha(r)\gamma\alpha(u))\gamma\alpha(s)\gamma h(r)$$

$$= 2h(\alpha(r))\gamma\alpha(u)\gamma\alpha(s)\gamma h(r) + 2\alpha^2(r)\gamma h(\alpha(u))\gamma\alpha(s)\gamma h(r)\ ), \forall\ s, r\in K \text{ and } u \in S, \gamma \in \Gamma.$$

Using eq (4) again implies $2h(\alpha(r))\gamma\alpha(u)\gamma\alpha(s)\gamma h(r)=0$. Since α is an automorphism, then we get $h(\alpha(r))\Gamma S\Gamma 2\alpha(s)\ \gamma h(r) = \{0\}$ . By 2-torsion freeness and primeness of S take, $\forall$ r$\epsilon$ K. either $h(\alpha(r)) = 0$ or $\alpha(s)\gamma h(r) = 0$ , $\forall$ s$\epsilon$ K, $\gamma \in \Gamma$.

If $h(\alpha(r)) = \alpha(h(r)) = 0$, since α is an automorphism, then $h(r) = 0$, $\forall$r$\epsilon$ K. Hence $0 = h(u\gamma r) = h(u)\gamma r$, $\forall$ r$\epsilon$ K and u$\epsilon$ S, By Lemma 2.2 gives h = 0; as a result, we have a contradiction. Hence there exists r$\epsilon$ K such that $h(r) \neq 0$ and $\alpha(s)\gamma h(r) = 0$ $\forall$ s,r$\epsilon$ K. By eq (1) we get $0 = \alpha(s\gamma a)\gamma h(r) = \alpha(s)\gamma\alpha(a)\gamma h(r)$. Therefore $\alpha(s)\Gamma S\Gamma h(r) = \{0\}$, $\forall$ s$\epsilon$ K. Since $h(r) \neq 0$, primeness of S implies that $\alpha(s) = 0$ for all x$\epsilon$ K, so we have K = 0, a contradiction as K is a nonzero semi-group ideal of S.

**Theorem 3.2:** *When S be prime Γ-(n-r), K be the nonzero semi-group ideal of S, when h endomorphism on K, where h be Γ -period 2, h(sγr) = h(rγs), ∀ s, r∈ K, γ ∈ Γ; thus S is commutative-ring.*

***Proof:*** By supposition. $h(h(s\gamma r)) = h(h(r\gamma s))$, $\forall$ s, r $\epsilon$ K, $\gamma \in \Gamma$. Where h is of Γ-period 2, obtains $s\gamma r = r\gamma s$ . Taking s by sγu, where u $\epsilon$ S, in above eq. and taking it again, gives $s\gamma[r,u]_\gamma = 0$ , $\forall$ s, r $\epsilon$ K, u $\epsilon$S, $\gamma \in \Gamma$ .Hence, get $K\Gamma[r, u]_\gamma = 0$.

By using Lemma 2.3 once more, we arrive at our conclusion that

$r\gamma u = u\gamma r$ $\forall$ r $\epsilon$K, u $\epsilon$ S, $\gamma \in \Gamma$, we obtain $(s+r)\gamma z = z\gamma(s+r) = z\gamma s + z\gamma r = s\gamma z + r\gamma z$, $\forall$ s ,r ,z$\epsilon$ S, $\gamma \in \Gamma$ (4) By applying (4) we obtain

$$(z+z)\gamma(s+r) = z\gamma\ (s+r)+z\gamma\ (s+r) = z\gamma s + z\gamma r + z\gamma s + z\gamma r, \forall\ s, r,z \in S, \gamma \in \Gamma.$$

On the other side, there's $(z+ z)\ \gamma\ (s+ r) = (z+ z)\ \gamma s + (z +z)\ \gamma r = z\gamma s + z\gamma s + z\gamma r + z\gamma r$. By comparing both expressions, we obtain $z\gamma s + z\gamma r = z\gamma r + z\gamma s$.

Gives to $z\gamma$ (s+ r – s - r) = 0, ∀ s ,r ,z∊ S , $\gamma \in \Gamma$ , get $S\Gamma$ (s + r – s - r) = 0, Lemma 2.3 get (s + r – s – r) = 0, ∀ s ,r ∊ S, we get that S be commutative-ring.

**Theorem 3.3:** *When S be prime $\Gamma$-(n-r), h: S→S additive mapping with a nonzero value thus $h(s\gamma r) = h(s)\gamma\alpha(r)$, ∀ s,r ∊ S, $\gamma \in \Gamma$, when α be anti-$\Gamma$-hom of $\Gamma$ -period 2 on S, thus S is a commutative-ring.*

***Proof:*** Let $h(s\gamma r)=h(s)\gamma\alpha(r)$, ∀ s,r ∊ S, $\gamma \in \Gamma$. replace r by $r\gamma t$ in above eq. ,when t ∊ S, gives $h(s\gamma r\gamma t) = h(s)\gamma\alpha(r\gamma t) = h(s)\gamma\alpha(t)\gamma\alpha(r)$.

On the other side, there's $h(s\gamma r\gamma t) = h(s\gamma r)\gamma\alpha(t) = h(s)\gamma\alpha(r)\gamma\alpha(t)$. Thus

$h(s)\gamma\alpha(r)\gamma\alpha(t) = h(s)\gamma\alpha(t)\gamma\alpha(r)$, ∀ s,r,t ∊ S, $\gamma \in \Gamma$. Gives

$h(s)\gamma(\alpha(r)\ \gamma\alpha(t)-\alpha(t)\gamma\alpha(r)) = 0$, where α bijective can take α(r) by u and α(t) by v, when u,v ∊ S, obtains $h(s)\gamma(u\gamma v-v\gamma u) = 0$, ∀ r,s,u,v ∊ S, $\gamma \in \Gamma$. Replace s by $s\gamma n$, where n ∊ S, in above eq. gives $h(s\gamma n)(u\gamma v - v\gamma u) = 0$, obtain $h(s)\gamma\alpha(n)\gamma(u\gamma v - v\gamma r u) = 0$, gets $h(s)\Gamma S\Gamma(u\gamma v - v\gamma u) = \{0\}$, thus S is a prime and h ≠ 0 then $u\gamma v = v\gamma u$ ,∀ u,v ∊ S, we conclude S is commutative ring.

**Theorem 3.4:** *When S be a prime $\Gamma$-(n-r). h :S →S additive endomorphism that is $h(s\gamma r) = h(s)\gamma r+ \alpha(s)\gamma h(r)$, ∀ s,r ∊ S,$\gamma \in \Gamma$ wherever α be an anti $\Gamma$ -hom of $\Gamma$ - period 2 thus S is a commutative ring.*

***Proof:*** ∀ s , r ,z ∊ S, $\gamma \in \Gamma$ , have

$$h((s\gamma r)\gamma z) = h(s\gamma r)\ \gamma z + \alpha(s\gamma r)\ \gamma h(z) = (h(s)\gamma r + \alpha(s)\gamma h(r))\gamma z + \alpha(r)\gamma\alpha(s)\gamma h(z).$$

On the other hand $h(s\gamma(r\gamma z)) = h(s)\gamma r\gamma z + \alpha(s)\gamma h(r\gamma z)$

$$= h(s)\gamma r\gamma z + \alpha(s)\gamma(h(r)\gamma z + \alpha(r)\gamma h(z))$$

$$= h(s)\gamma r\gamma z + \alpha(s)\gamma h(r)\gamma z+ \alpha(s)\gamma\alpha(r)\gamma h(z)$$

However, $(h(s)\gamma r + \alpha(s)\gamma h(r))\gamma z = h(s)\gamma r\gamma z + \alpha(s)\gamma h(r\gamma z)$, gives

$$\alpha(r)\gamma\alpha(s)\gamma h(z) = \alpha(s)\gamma\alpha(r)\gamma h(z),\text{ then } \alpha(\alpha(r)\gamma\alpha(s)\gamma h(z)) =\alpha(\alpha(s)\gamma\alpha(r)\gamma h(z))$$

$\alpha(h(z))\gamma s\gamma r = \alpha(h(z))\gamma r\gamma s$, taking s by $s\gamma t$ in above eq. and employ it get $\alpha(h(z))\gamma s\gamma t\gamma r = \alpha(h(z))\gamma s\gamma r\gamma t$, hold $\alpha(h(z))\gamma s\gamma(t\gamma r - r\gamma t) = 0$, that is $\alpha(h(z))\Gamma S\Gamma(t\gamma r - r\gamma t) = \{0\}$, since S prime gives

Either α(h(z)) = 0 or $(t\gamma r - r\gamma t) = 0$, when α is bijective get d = 0 take a contradiction then $t\gamma r= r\gamma t$, ∀ s, r ∊ S, $\gamma \in \Gamma$. we conclude that S is a commutative-ring.

## Conclusion

In the present work, we defined maps $\Gamma$- period 2 on prime and semiprime $\Gamma$-(n-r) in S and gave extensive results in the setting of $\Gamma$-α-der and of prime and semiprime Γ-(n-r) when they are of Γ-period two.

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