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Numerical solution for solving fuzzy volterra integral 2nd –kind by using modified decomposition technique

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Abstract

We discuss methods for solving the second-class fuzzy nonlinear Volterra -integral formula using novel error analysis and convergence techniques. The modified decomposition method is utilized to investigate convergence in the solution of the second class of fuzzy nonlinear Volterra integral formula. Moreover, some numerical examples were solved using our fuzzy equation. The presented methods were discussed in detail, and we compared the existing methods through the illustrative example that has been presented and applied creation classes to all the programming in Maple version (22). The quantity demand and investment in research and development, while the other model focuses on a more realistic relationship between the quantity demand and the price.

Subject Classification: *03B52, 03E72.*

Keywords: *Fuzzy set, Modified decomposition method, Formula of Voltari -integral.*

1. Introduction

An integral equation is an equation under one or more integration signs unknown function appears; an integral equation is very important in applied and pure mathematics. An integral equation does not have analytical solutions. Numerical methods are used to solve them and approximate solutions. The fuzzy integral equation is necessary for solving topics related to applied mathematics, especially in medicine, geography, biology and other fields. The fuzzy number is the parameter instead of crisp was the first fuzzy integral presented by wu and Ma [4], those who studied the second kind of Fredholm integral equation (FFIE); thus, he was able to find a unique solution for (FFIE). Dubois and Prade

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presented the fuzzy integral [6]; also, there were alternative proposals for approaches by Goetschel and Foxman 1986 [2], and others specifically used kaleva[3] fuzzy function integral and chose the Lebasque type numerical algorithms can be developed to find out the fuzzy function when there are ways to combine the fuzzy function by solving the kernel instead of fuzzy

Formulation 1 (fuzzy nonlinear Volterra integral equation)

We will discuss our formulation**,**

$$\tilde{u}(x) = \tilde{f}(x) + \lambda \int_a^x \quad k(x,t,G(t,\tilde{u}(t))dt \tag{1.1}$$

Definition 1.1 [1]: A fuzzy set is one with the formula$\widetilde{M} = \{(x, \xi_{\widetilde{M}}(x)), x \in X\}$, $\xi_{\widetilde{M}}(x)$ seems to be the membership function of a fuzzy. The expression M is given as $\xi_{\widetilde{M}}(x): X \to [0,1]$. The value of $\xi_{\widetilde{M}}(x)$ is referred to as a degree of membership of x in X. This membership function, as just a function of crisp value, is shown in the instance below.

Definition 1.2 [2]: A fuzzy set $M$is called α-cut set M_α if composed of membership such that greater than or equal to a, $M_\alpha = \{x \in X : \xi_{\widetilde{M}}(x) \geq \alpha\}, \forall x \in X$

All instances of$\alpha \in [0,1]$ satisfy the ensuing conditions.

$i - (M \cup \aleph)_\alpha = M_\alpha \cup \aleph_\alpha$ $\quad ii - (M \cap \aleph)_\alpha = M_\alpha \cap \aleph_\alpha$

$v - M \subseteq \aleph \ gives \ M_\alpha \subseteq \aleph_\alpha$ $\quad iv - M = \aleph \ iff \ M_\alpha = \aleph_\alpha, \forall \alpha \in [0,1]$

$\alpha \leq \alpha' \ then \ M_\alpha \supseteq M_{\alpha'} \ if \ A \ is \ convex$.

Definition 1.3 [2]: Convexity exists for a fuzzy set $\widetilde{M}$ on R iff

$$\xi_{\widetilde{M}}(\lambda x_1 + (1-\lambda)x_2) \geq Min\{\xi_{\widetilde{M}}(x_1), \xi_{\widetilde{M}}(x_2)\}, \forall\ x_1, x_2 \in R, \forall\ \lambda \in [0,1]. \tag{1.2}$$

Definition 1.4 [3]: A fuzzy set or a map $\tilde{\xi}: R \to [\zeta, \varsigma]$ is a fuzzy number that fulfils

(1) Upper semi-continuous function equals ξ.

(2) Outside of a specific interval $[\zeta, \kappa]$ is $\xi(x) = 0$,

(3) Real valuesς and ι, like$\zeta \leq \varsigma \leq \iota \leq \kappa$, exist.

  (i) A monotonically growing function on $[\zeta, \varsigma]$is called $\xi(x)$.

  (ii) ξA monotonically decreasing function on $[\zeta, \varsigma]$is called $\xi(x)$.

  (iii) $\xi(x) = 1 \ \forall\ x \in [\varsigma, \iota]$.

E^1It represents the collection of all fuzzy integers and a convex cone.

Definition 1.5 [4]: A couple $(\underline{\xi}, \underline{\xi})$of functions with $\underline{\xi}(\alpha), \underline{\xi}(\alpha)$ and $0 \leq \alpha \leq 1$ are considered as a fuzzy number $\tilde{\xi}$ in the parameterized form if they fulfil the next criteria:

1) $\underline{\xi}(\alpha)$ is a left-bounded, continuously non-decreasing function on the range [0, 1].

2) $\underline{\xi}(\alpha)$ is a left-bounded, continuously non-increasing function on the range [0, 1].
3) $\underline{\xi}(\alpha) \leq \underline{\xi}(\alpha)$ $,0 \leq \alpha \leq 1$.

Definition 1.6 [5]: The procedures are justified as following with every given fuzzy number $\tilde{\xi} = (\underline{\xi}(\alpha), \underline{\xi}(\alpha))$,$\tilde{\upsilon} = (\underline{\upsilon}(\alpha),\underline{\upsilon}(\alpha))$, $0 \leq \alpha \leq 1$and scalar k:

i. $\underline{(\xi + \upsilon)}(\alpha)$= $(\underline{\xi}(\alpha)$+$\underline{\upsilon}(\alpha))$,$\underline{(\xi + \upsilon)}(\alpha)$ = $(\underline{\xi}(\alpha)$+$\underline{\upsilon}(\alpha))$,
ii. $\underline{(\xi - \upsilon)}(\alpha)$= $(\underline{\xi}(\alpha)$-$\underline{\upsilon}(\alpha)$,$\underline{(\xi - \upsilon)}(\alpha)$ = $(\underline{\xi}(\alpha)$-$\underline{\upsilon}(\alpha))$,
iii. $k\xi$={$\left(k\underline{\xi}(\alpha), k\underline{\xi}(\alpha)\right), \quad k \geq 0 \ \left(k\underline{\xi}(\alpha), k\underline{\xi}(\alpha)\right), \quad k < 0$ (1.3)

$\tilde{\xi} \cdot \tilde{\upsilon} = \{\underline{\xi\upsilon}(\alpha) = max\{\underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha)\}\ \overline{\xi\upsilon}(\alpha) = min\{\underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha), \underline{\xi}(\alpha)\underline{\upsilon}(\alpha)\}$ (1.4)

Definition 1.7 [5]: For each random $\tilde{\xi}, \tilde{\upsilon} \in E^1$ fuzzy number

$$D(\tilde{\xi}, \tilde{\upsilon}) = max\{sup_{\alpha\in[0,1]} \left|\underline{\xi}(\alpha) - \underline{\upsilon}(\alpha)\right|, sup_{\alpha\in[0,1]} \left|\underline{\xi}(\alpha) - \underline{\upsilon}(\alpha)\right|\}$$

Theorem 1,1 [6]: $\widetilde{\imath\,\xi}$ and ῦ are separated by this distance, and (E^1, D) is a whole metric space.

Definition 1.8 [7, 8]: The nonempty compact convex fuzzy subset of R^n is represented by the class of E^n and $I = [\zeta, \varsigma]$ then:
$E^n = \{\psi: R^n \to I\}$ One that fulfils the criteria below
i) ψ is fuzzy convex,
ii) ψ is normal,
iii) ψ is upper semi-continuous,
iv) $[\psi]^0$= $cl\{x \in R^n |\ \psi\ (x) > 0\}$ is compact.

and the α−level sets $[\psi]^\alpha$ are given as $[\psi]^\alpha$= $\{x \in R^n |\ \psi(x) \geq \alpha\}$ for $0 < \alpha \leq 1$ and $[\psi]^0$ for $\alpha = 0$. Consequently, it implies that $[\psi]^\alpha \in E^n\ \forall\ 0 \leq \alpha \leq 1$ from (i)–(iv).

Definition 1.9 [9]: When a fixed $K > 0$ occurs, a function $F: I \to E^n$ is considered to be bound because $D\left(F(x), \tilde{0}\right) \leq M$ for every x in I.

Definition 1.10 [9]: A function F:$I \to E^n$ is considered continuous if for every constant $x_0 \in I$ and $\varepsilon > 0$ there occurs $\delta > 0$ such that $|x - x_0| < \delta$ then $D\left(F(x), F(x_0)\right) < \varepsilon$.

Definition 1.11 [7]: It is a fuzzy number symbolized by the three points:$\widetilde{M} = [\zeta_1, \zeta_2, \zeta_3]$

Definition 1.12 "Fuzzy Number in the Trapezoidal "[10]: According to the definition below, the trapezoidal fuzzy number M**,** $\widetilde{M} = [\zeta_1, \zeta_2, \zeta_3, \zeta_4]$

Hypotheses:

1. $\tilde{f}(x)$ is bounded $\forall\, x \in [a, b]$.

$$|k\left(x,t,\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right)\right) - k\left(x,t,\tilde{G}_1\left(x,t,\tilde{\xi}^*(t)\right)\right)| \le$$
$$L^*|\left(\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right) - \tilde{G}_1\left(x,t,\tilde{\xi}^*(t)\right)\right)|$$

2. $|k\left(.,.,\tilde{G}_1\left(.,.,\tilde{\xi}(.)\right)\right)| < M_1^*, |G(t, \int_a^t \tilde{G}_2\left(t,s,\tilde{\xi}^*(s)\right) ds))| < M_2^*.$

3. $|(G(t, \int_a^t \tilde{F}_2\left(t,s,\tilde{\xi}(s)\right) ds - G(t, \int_a^t \tilde{F}_2\left(t,s,\tilde{\xi}^*(s)\right) ds))| \le$
$L(\int_a^x D(\tilde{F}_2\left(t,s,\tilde{\xi}(s)\right) - \tilde{F}_2\left(t,s,\tilde{\xi}^*(s)\right))ds.$

   i. $\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right), \tilde{G}_2\left(t,s,\tilde{\xi}(s)\right) : I \times I \times E^1 \to E^1$

   ii. $|\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right)| = M_1, |\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right)| = M_2.$

   iii. $|(\tilde{G}_1\left(x,t,\tilde{\xi}(t)\right) - \tilde{G}_1\left(x,t,\tilde{\xi}^*(t)\right))| \le L_1\, |\tilde{\xi}(t) - \tilde{\xi}^*(t)|$

   $|(\tilde{G}_2\left(t,s,\tilde{\xi}(s)\right) - \tilde{G}_2\left(t,s,\tilde{\xi}^*(s)\right))| \le L_1^*\, |\tilde{\xi}(t) - \tilde{\xi}^*(t)|$

4. $0 < \lambda([(b-a)\sigma + (b-a)M_2^*\beta] < 1, \sigma = L^* L_1(b-a), \beta = L L_1^*(b-a)$

The main result of the proposal fuzzy integral equation with fuzzy kernel multiplied by nonlinear fuzzy integral is discussed as following theorems.

Theorem 1.2: *Set the formula of fuzzy nonlinear integral to satisfy the assumptions (1.1), has a special solution after that.*

Proof: Suppose there are two distinct solutions to formula (1.2), $\tilde{\xi}$ *and* $\tilde{\xi}^*$:

$$D(\tilde{\xi}, \tilde{\xi}^*)$$
$$= D(\tilde{f}(x)$$
$$+ \lambda \int_a^x k((x,t,\tilde{G}_1\left(t,\tilde{\xi}(t)\right)).A(t, \int_a^t \tilde{G}_2\left(s,\tilde{\xi}(s)\right) ds)dt\ ,$$
$$\tilde{f}(x) + \lambda \int_a^x k((x,t,\tilde{G}_1\left(t,\tilde{\xi}^*(t)\right)).A(t, \int_a^t \tilde{G}_2\left(s\,,\tilde{\xi}^*(s)\right) ds)dt\ )$$
$$D(\tilde{\xi}\,, \tilde{\xi}^*)$$
$$= max\{sup_{a \le x,t \le b}\, |\underline{f}(x)$$
$$+ \lambda \int_a^x k((x,t,\underline{G_1}\left(t\,,\underline{\xi}(t)\right)).A(t, \int_a^t \underline{G_2}\left(\,s\,,\underline{\xi}(s)\right) ds)dt$$
$$-\underline{f}(x) + \lambda \int_a^x k((x,t,\underline{G_1}_1\left(\,t\,,\underline{\xi}^*(t)\right)).A(t, \int_a^t \underline{G_2}\left(\,s\,,\underline{\xi}^*(s)\right) ds)dt\ |\,,$$

$sup_{a\leq x,t\leq b}|\, \underline{f}(x)+$
$\lambda\int_a^x \; k((x,t,\underline{G_1}\left(t\,,\underline{\xi}(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s\,,\underline{\xi}(s)\right)ds)dt \; -\underline{f}(x)+$

$$\lambda\int_a^x \; k((x,t,\underline{G_1}(t,\underline{\xi}(t)).A(t,\int_a^t \; \underline{F_2}\left(s,\underline{\xi}^*(s)\right)ds)dt \;|\,\}$$

$$\leq max\{\lambda\,[\int_a^x \; |\, k((x,t,\underline{G_1}\left(t\,,\underline{\xi}(t)\right)).A(t,\int_a^t \; \underline{F_2}\left(s,\underline{\xi}(s)\right)ds)dt$$

$$-\int_a^x \; k((x,t,\underline{G_1}\; \left(t\,,\underline{\xi}^*(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s\,,\underline{\xi}^*(s)\right)ds]|dt \;,$$

$\lambda\,[\int_a^x \; |k(\left(x,t,\underline{G_1}\left(t\,,\underline{\xi}(t)\right)\right).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)dt$

$-\int_a^x \; k((x,t,\underline{G_1}\left(t\,,\underline{\xi}^*(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds]|dt$
$\leq max\{\lambda\,[\int_a^x \; |\, k((x,t,\underline{G_1}\left(t\,,\underline{\xi}(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)dt \; -$
$k((x,t,\underline{G_1}\; \left(t\,,\underline{\xi}^*(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds]|dt \; +$
$k((x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)dt$
$-\int_a^x \; k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)).G(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)dt \;|\,\}$
$,\lambda\,(\int_a^x \; |\int_a^x \; |\, k((x,t,\underline{G_1}\left(t,\underline{u}(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{u}(s)\right)ds)dt \quad -$
$k((x,t,\underline{G_1}\; \left(t,\underline{\xi}^*(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)dt \;| \quad +$
$k((x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)dt \quad -$
$\int_a^x \; k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)).A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)dt \;|\,\}$

$$\leq max\{\; \lambda\int_a^x \; (|k((x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right))|.\,|A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)dt$$

$$|k((x,t,\underline{G_1}\; \left(t,\underline{\xi}^*(t)\right))|.\,|A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)dt \;|\,.|$$

$k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)-k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)\right)|),$
$sup_{a\leq x,t\leq b}\,\lambda\int_a^x \; (|k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right))|.\,|A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)-$
$A(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds) \;|$

$|\, G(t,\int_a^t \; \underline{G_2}\left(s,\underline{\xi}(s)\right)ds) \;|.|\; k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)-\; k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)|)$

$$\leq max\{sup_{a\leq x,t\leq b}\lambda|k\left(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)\right)|.$$

$$|\int_a^x A(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)\ dt-\int_a^x A(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)\ dt|$$

$$+\int_a^x |A(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)\ dt|.(|k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)-$$

$$k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)\right))|,$$

$$\lambda sup_{a\leq x,t\leq b}|(k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)|.|\int_a^x A(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)\ dt|$$

$$-\int_a^x A(t,\int_a^t \underline{G_2}\left(t,s,\underline{\xi}(s)\right)ds)\ dt|$$

$$+\int_a^x |G(t,\int_a^t \underline{F_2}\left(s,\underline{\xi}(s)\right)ds)\ dt|.|k((x,t,\underline{G_1}\left(t,\underline{\xi}^*(t)\right)$$

$$-\ k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)|$$

$$\leq \lambda M_1^*|(b-a)|sup_{a\leq x,t\leq b}|G(t,\int_a^t \underline{F_2}\left(s,\underline{\xi}(s)\right)ds)\ dt$$

$$-A(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)\ dt|$$

$$+(b-a)M_2^* sup_{a\leq x,t\leq b}|\ k(x,t,\underline{G_1}\left(x,t,\underline{\xi}^*(t)\right)-\ k(\left(x,t,\underline{G_1}\left(t,\underline{\xi}(t)\right)\right)|,$$

$$\lambda\ M_1^*|(b-a)|sup_{a\leq x,t\leq b}|G(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}(s)\right)ds)\ dt$$

$$-G(t,\int_a^t \underline{G_2}\left(s,\underline{\xi}^*(s)\right)ds)\ dt|$$

$$+(b-a)M_2^* sup_{a\leq x,t\leq b}|\ k((x,t,\underline{F_1}\left(x,t,\underline{\xi}^*(t)\right)-\ k(\left(x,t,\underline{F_1}\left(x,t,\underline{\xi}(t)\right)\right)|$$

$$\begin{aligned}\lambda M_1^*(b-a)\sigma\left|\tilde{\xi}(t)-\tilde{\xi}^*(t)\right|+\lambda(b-a)M_2^*\beta|\tilde{\xi}^*(t)-\tilde{\xi}(t)|\\ \leq \lambda(M_1^*(b-a)\sigma\\ +(b-a)M_2^*\beta)sup_{a\leq t\leq b}D(\left|\tilde{\xi}(t)-\tilde{\xi}^*(t)\right|,\tilde{0})\\ \leq \lambda(M_1^*(b-a)\sigma+(b-a)M_2^*\beta)sup_{a\leq t\leq b}D(\tilde{\xi}(t),\tilde{\xi}^*(t))\end{aligned}$$

Where $\sigma=L^*L_1(b-a)$, $\beta=LL_1^*(b-a)$ We get: $D\left(\tilde{\xi},\tilde{\xi}^*\right)\leq\ \gamma D\left(\tilde{\xi},\tilde{\xi}^*\right)$, So, $(1-\gamma)\ D\left(\tilde{\xi},\tilde{\xi}^*\right)\leq 0$. Because $0<\gamma<1$, hence $D\left(\tilde{\xi},\tilde{\xi}^*\right)=0$. Thus, $\tilde{\xi}=\tilde{\xi}^*$.

Conclusions

In this research, we employ a modified decomposition approach to resolve the fuzzy Volterra integral formula that is 2^{nd} –kind since it is quick and straightforward to achieve convergence for any formula system and any value of α. The exact answer must be a portion of a nonhomogenous function or set of nonhomogeneous functions when using the modified admin approach.

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