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Micro topological space with grill-generalized open sets

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Abstract

In the present paper, we define the concept of micro-$\mathfrak{G}g$-open and micro-$\mathfrak{G}g$-closed sets by using micro topological space, and several properties of that set are studied, so we introduce micro-$\mathfrak{G}g\gamma$-open set and micro-$\mathfrak{G}g\varpi$- closed set and denoted of micro-Gg-open set by mic-$\mathfrak{G}gO(U)$, micro-$\mathfrak{G}g$-closed set by mic-$\mathfrak{G}gC(U)$ several theories that true and examples to the contrary are not true. We also studied a practical example of a micro-$\mathfrak{G}g$-open set and a micro-$\mathfrak{G}g$-closed set.

Subject Classification: *54A05, 54A99.*

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1. Introduction

The research was conducted by the grill on a topological space [1]. Thivager [2] knew Nano topological space by introduced the concept of lower approximation, upper approximation, and boundary. Chandrasekar [3] introduced the concept of micro topological space. Esmaeel and Mustafa [4] presented the idea of the $nano$-$\mathfrak{G}g$-open set and $nano$-$\mathfrak{G}g$-closed set. In the past few years, different kernel sets have been studied [5-9]. This paper defines the concept of micro-$\mathfrak{G}g$-open set and micro-$\mathfrak{G}g$-closed set by using micro topological space, and several properties of that set are studied to introduce micro-$\mathfrak{G}g\gamma$-open set and micro-$\mathfrak{G}g\varpi$-closed set and denoted of micro-$\mathfrak{G}g$-open set by mic-$\mathfrak{G}gO(U)$, micro-$\mathfrak{G}g$-closed set by mic-$\mathfrak{G}gC(U)$ several theories that

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true and examples to the contrary are not true. We also studied a practical example of a micro-$\mathfrak{G}g$-open set and a micro-$\mathfrak{G}g$-closed set.

2. Preliminaries

Definition 2.1. [1]: Grills are nonempty collections $\mathfrak{G}$ of nonempty subsets of a topological space $(U.\tau)$.

1. $h \in \mathfrak{G}$ and $h \subseteq \mathbb{B} \subseteq U$ then $\mathbb{B} \in \mathfrak{G}$.
2. $h.\mathbb{B} \subseteq$ U and $h \cup \mathbb{B} \in \mathfrak{G}$ then $h \in \mathfrak{G}$ or $\mathbb{B} \in \mathfrak{G}$.

Definition 2.2. [10]: If R is a nonempty finite set of objects referred to as the universe, and U is a nonempty finite collection of things called the universe, the indiscernibility relation is an equivalence relation on U. Then U is partitioned into disjoint equivalence classes. It is argued that elements ∈ to the same equivalence class are indistinguishable. The approximation space is defined as the pair *(U, R*).

1. The Set of all objects is the lower approximation of χ concerning R; it can be categorized as χ in relation to R and is indicated by $L_R(\chi)$ that is correct. "$L_R(\chi)$ =$\cup\{R(x)$:$R(x)\subseteq\chi\}$", where $R(\chi)$ denotes the determined equivalence class.
2. The Set of all objects that can be classed as χ with regard R and is indicated by $U_R(\chi)$ is the closest χ can be to R in its upper range, which is correct "$U_R(\chi)$ $=\cup\{R(x)$: $R(x)\cap\chi \neq\varphi\}$"
3. The collection of all possible objects categorized the two as *X* nor as not *X* concerning R is the border of the area *X* with relation to R, and it is denoted by $B_R(\chi)$. That is to say "$B_R(\chi)$ = $U_R(\chi)$– $L_R(\chi)$"

Definition 2.3. ([2],[10]): Let R be a relation of equivalence on U with U being the universe $\tau_R(\chi)$ =$\{U$,φ, $L_R(\chi)$, $U_R(\chi)$, $B_R(\chi)\}$ where $\chi \sqsubseteq U$

1. U, φ $\in\tau_R(\chi)$
2. The union of the elements of any subset of $\tau_R(\chi)$ is in $\tau_R(\chi)$
3. Any finite sub-collection of elements that intersect at $\tau_R(\chi)$ is in $\tau_R(\chi)$. The Nano topology on U is thus denoted by $\tau_R(\chi)$ concerning χ. The elements of $\tau_R(\chi)$ are called Nano open sets.

Definition 2.4. [3]: (U,$\tau_R(\chi)$) is a nano topological space here $\mu_R(\chi)$ = {N∪($N'\cap\mathcal{M}$)}, N, N' ∈ $\tau_R(\chi)$ and called it micro topology of $\tau_R(\chi)$ by $\mathcal{M}$ where $\mathcal{M} \notin \tau_R(\chi)$.

Definition 2.5. [3]: The micro topology $\mu_R(\chi)$ satisfies the following axiom

1. U. Φ ∈ $\mu_R(\chi)$
2. Any sub-collection of $\mu_R(\chi)$ elements combine in $\mu_R(\chi)$
3. The juncture of the elements of any finite collection of $\mu_R(\chi)$ subsets are on $\mu_R(\chi)$

Then "$\mu_R(\chi)$" is named the micro topology in U relative to χ. The triple $(U.\tau_R(\chi).\mu_R(\chi))$" is said to be micro topological spaces key components of $\mu_R(\chi)$ as named micro open sets, the complement of micro open sets is said to be a micro closed set.

3. Mic γ – Set

Definition 3.1: $\mathbb{A}$ subset J of U is called Mic γ – set if $J = Mic\ ker(J)$.

Definition 3.2. $\mathbb{A}$ subset J of a micro topological space $(U.\tau_R(\chi).\mu_R(\chi))$ is called *Mic* - ϖ – closed if $J = I \cap Z$, which I a $Mic - \gamma$-set and Z is a called Mic_closed set.

Example 3.3. Let $U = \{f_1.f_2.f_3.f_4\}$, $U/R = \{\{f_2\}.\{f_4\}.\{f_1.f_3\}\}$, $X = \{f_1.f_3\}$, then $\tau_R(\chi) = \{\emptyset.U.\{f_2\}.\{f_2.f_1.f_3\}.\{f_1.f_3\}\}$, $\mathcal{M} = \{f_1\}$

$\mu_R(\chi) = \{\emptyset.U.\{f_2\}.\{f_2.f_1.f_3\}.\{f_1.f_3\}.\{f_1\}.\{f_1.\{f_1.f_2\}\}$

$\mu_R - closed = \{\emptyset.U.\{f_1.f_3.f_4\}.\{f_4\}.\{f_2.f_4\}.\{f_2.f_3.f_4\}.\{f_3.f_4\}\}$

$Mic - \gamma - sets = \{\emptyset.U.\{f_1\}.\{f_2\}.\{f_1.f_2\}.\{f_1.f_3\}.\{f_1.f_2.f_3\}$

$Mic - \varpi - closed = \{\emptyset.U.\{f_1\}.\{f_2\}.\{f_1.f_2\}.\{f_1.f_3\}.\{f_1.f_2.f_3\}.\{f_1.f_3.f_4\}.$
$\{f_4\}.\{f_2.f_4\}.\{f_2.f_3.f_4\}.\{f_3.f_4\}.\{f_2.f_3\}.\{f_3\}\}$

$Mic\ ker(f) = \{\emptyset.U.\{f_1\}.\{f_2\}.\{f_1.f_2\}.\{f_1.f_3\}.\{f_1.f_2.f_3\}$

4. Micro $\mathfrak{G}G$ -Closed set

Definition 4.1: Let a micro grill topological space $(U.\mu_R(\chi).\mathfrak{G})$, and let $\mathbb{E} \subseteq U$, $\mathbb{E}$ is called micro grill g-closed set indicated by "$Mic - \mathfrak{G}g$-closed" if $(\mathbb{E} - D) \notin \mathfrak{G}$ as well $(cl(\mathbb{E}) - D)) \notin \mathfrak{G}$ where every $D \subseteq U$ and D are micro open sets. $\mathbb{E}^c$ represent the micro grill-g-open set indicated by "$Mic\ \mathfrak{G}g$-open". The essential family of whole $Micro\ \mathfrak{G}g$-closed" sets is indicated from $Mic\ \mathfrak{G}g - C(U)$. The family of "$Micro\ \mathfrak{G}g$-open" sets is denoted by $Mic\ \mathfrak{G}g - O(U)$.

Example 4.2: The following Table 1 explicate $Mic\ \mathfrak{G}g - C(U)$ and $Mic\ \mathfrak{G}g - O(U)$, where $\mathfrak{G} = \{U.\{f_2\}.\{f_2.f_1\}.\{f_2.f_3\}\}$

Table 1
Explicate of Mic in Example 4.2

A	$\tau_R(\chi)$	$\mu_R(\chi)$	$Mic\ \mathfrak{G}g - C(U)$	$Mic\ \mathfrak{G}g - O(U)$
φ	$\{\varphi.U\}$	$\{\varphi.U\}$	$\{\varphi.U.\{f_2\}.\{f_1.f_2\}.\{f_2.f_3\}$	$\{\varphi.U.\{f_1.f_3\}.\{f_3\}.\{f_1\}$
U	$\{\varphi.U\}$	$\{\varphi.U\}$	$\{\varphi.U.\{f_2\}.\{f_1.f_2\}.\{f_2.f_3\}$	$\{\varphi.U.\{f_1.f_3\}.\{f_3\}.\{f_1\}$
$\{f_1\}$	$\{\varphi.U.\{f_1.f_2\}\}$	$\{\varphi.U.\{f_2\}.\{f_1.f_2\}\}$	$\{\varphi.U.\{f_1\}.\{f_3\}.\{f_1.f_2\}.\{f_2.f_3\}.\{f_1.f_3\}\}$	$\{\varphi.U.\{f_2.f_3\}.\{f_1.f_2\}\{f_3\}.\{f_1\}.\{f_2\}\}$

Contd...

$\{r_2\}$	$\{\varphi.U.\{r_1.r_2\}\}$	$\{\varphi.U.\{r_2\}.\{r_1.r_2\}\}$	$\{\varphi.U.\{r_1\}.\{r_3\}.\{r_1.r_2\}.\{r_2.r_3\}.\{r_1.r_3\}\}$	$\{\varphi.U.\{r_2.r_3\}.\{r_1.r_2\}.\{r_3\}.\{r_1\}.\{r_2\}\}$
$\{r_3\}$	$\{\varphi.U.\{r_3\}\}$	$\{\varphi.U.\{r_3\}.\{r_2\}.\{r_2.r_3\}\}$	$\{\varphi.U.\{r_1\}.\{r_2\}.\{r_3\}.\{r_1.r_2\},\{r_2.r_3\}.\{r_1.r_3\}\}$	$\{\varphi.U.\{r_2.r_3\}.\{r_1.r_3\}.\{r_1.r_2\}.\{r_3\}.\{r_1\}.\{r_2\}\}$
$\{r_1.r_2\}$	$\{\varphi.U.\{r_1.r_2\}\}$	$\{\varphi.U.\{r_1.r_2\}.\{r_2\}\}$	$\{\varphi.U.\{r_1\}.\{r_3\}.\{r_1.r_2\}.\{r_2.r_3\}.\{r_1.r_3\}\}$	$\{\varphi.U.\{r_2.r_3\}.\{r.r_2\}.\{r_3\}.\{I_1\}.\{I_2\}\}$
$\{r_2.r_3\}$	$\{\varphi.U.\{r_3\}.\{r_1.r_2\}\}$	$\{\varphi.U.\{r_3\}.\{r_1.r_2\}.\{r_2\}.\{r_2.r_3\}\}$	$\{\varphi.U.\{r_2\}.\{r_3\}.\{r_1.r_2\}.\{r_1\}.\{r_2.r_3\}.\{r_1.r_3\}\}$	$\{\varphi.U.\{r_1.r_3\}.\{r_1.r_2\}.\{r_3\}.\{r_2.r\}.\{r\}.\{r_2\}\}$
$\{r_1.r_3\}$	$\{\varphi.U.\{r_3\}.\{r_1.r_2\}\}$	$\{\varphi.U.\{r_3\}.\{r_1.r_2\}.\{r_2\},\{r_2.r_3\}\}$	$\{\varphi.U.\{r_2\}.\{r_3\}.\{r_1.r_2\}.\{r_1\}.\{r_2.r_3\}.\{r_1.r_3\}\}$	$\{\varphi.U.\{r_1.r_3\}.\{r_1.r_2\}.\{r_3\}.\{r_2.r_3\}.\{r_1\}.\{r_2\}\}$

Remark 4.3: For any $(U.\mu_R(\chi).\mathfrak{G})$ space

(1) Each micro "closed set" "is a $Mic\ \mathfrak{G}g$-closed.
(2) Each micro "open set" "is a $Mic\ \mathfrak{G}g$-open.

Proof:

(1) Let $\mathbb{E}$ be a micro closed set; now we prove that $\mathbb{E}$ is a $Mic\ \mathfrak{G}g$ -closed set, $(\mathbb{E}-D)\notin \mathfrak{G}$, for all $D\in\mu_R(\chi)$, but $(cl(\mathbb{E})-D)=(\mathbb{E}-D)$ $\because cl(\mathbb{E})=\mathbb{E}$, So that $(cl(\mathbb{E})-D)\notin\mathfrak{G}$.
(2) Let $F\in\mu_R(\chi)$, now we prove that F is $Mic\ \mathfrak{G}g$-open, F^c is Mic -closed Set $\Rightarrow$ by (1) F^c is $Mic\ \mathfrak{G}g$-closed, so that F is $Mic\ \mathfrak{G}g$-open set.
The converse is not true in Remark 4.3.

Example 4.4: $A=\{r_1\}$ then $\{r_1.r_2\}$ a $Mic\ \mathfrak{G}g$-closed set, but his set is not Mic-"closed set" and $\{r_3\}$ a $Mic\ \mathfrak{G}g$-O set, but his set is not Mic-O set.

Theorem 4.5: *Any two finite sets of Mic $\mathfrak{G}g$-closed sets together Mic $\mathfrak{G}g$-closed Set.*

Proof: Let $r_1,r_2\in Mic\ \mathfrak{G}g$-closed sets, prove that r_1 union r_2 be belong to $Mic\ \mathfrak{G}g$-closed. ($(r_1\cup r_2)\in Mic\ \mathfrak{G}g$-closed)

Let $\left((r_1\cup r_2)-D\right)\notin\mathfrak{G}$, also $\left((r_1-D)\cup(r_2-D)\right)\notin\mathfrak{G}$, then $(r_1-D)\notin\mathfrak{G}$ and $(r_2-D)\notin\mathfrak{G}$, assume that $cl(r_1\cup r_2)-D=((cl(r_1)-D)\cup(cl(r_2)-D)\in\mathfrak{G}$

D belong to $\mu_R(\chi)$ then either $(cl(r_1)-D)\in\mathfrak{G}$ or $(cl(r_2)-D)\in\mathfrak{G}$, where that basic since r_1, r_2 belong to $Mic\ \mathfrak{G}g$ -closed sets, then $(r_1-D)\notin\mathfrak{G}$ imply that $(cl(r_1)-D)\notin\mathfrak{G}$ and $(r_1-D)\notin\mathfrak{G}$

Imply that $(cl(r_2)-D)\notin\mathfrak{G}$, for all $D\in\mu_R(\chi)$ also $(cl(r_1\cup r_2)-D)\notin\mathfrak{G}$.

This property is not realized in the intersection, as shown in the following example, from example 4.2, $A = \{\mathfrak{r}_1\}$ then $Mic\ \mathfrak{G}g$-closed sets $\{\mathfrak{r}_1.\mathfrak{r}_2\} \cap \{\mathfrak{r}_2.\mathfrak{r}_3\} = \{\mathfrak{r}_2\}$ is $\notin$ $Mic\ \mathfrak{G}g$-closed set.

Corollary 4.6: *A $Mic\ \mathfrak{G}g - O$ set is the intersection of any finite sets of "$Mic\ \mathfrak{G}g - O$.*

Proof: Let J_1, J_2 belong to $Mic\ \mathfrak{G}g - O(U)$, we prove that $(J_1 \cap J_2)$ belong to $Mic\ \mathfrak{G}g - O(U)$. J_1^c, J_2^c belong to $Mic\ \mathfrak{G}g - C(U)$. Then $((J_1)^c \cup (J_2)^c)$ belong to $Mic\ \mathfrak{G}g - C(U)$ by theorem 4.5, then $((J_1)^c \cup (J_2)^c)^c$ belong to $Mic\ \mathfrak{G}g - O(U)$

$(J_1 \cap J_2)$ belong to $Mic\ \mathfrak{G}g - O(U)$.

This property is not realized in the union, as demonstrated in the example below; from example 4.2, $\mathbb{A} = \{\mathfrak{r}_1\}$ then $Mic\ \mathfrak{G}g$-open sets $\{\mathfrak{r}_1\} \cup \{\mathfrak{r}_3\} = \{\mathfrak{r}_1.\mathfrak{r}_3\}$ is $\notin$ $Mic\ \mathfrak{G}g$-open set.

5. Micro 𝔊G – Kernal of Set

Definition 5.1: Let a micro grill topological space $(U.\mu_R(\chi).\mathfrak{G})$ and $\mathfrak{r} \subseteq U$, micro $\mathfrak{G}g$ -kernal of $\mathfrak{r} = \cap\{A : \mathfrak{r} \subseteq A, A \in Mic\ \mathfrak{G}g - O(U)\}$ and symbolizes it $Mic\ \mathfrak{G}g - Ker(\mathfrak{r})$. It is obvious that $\mathfrak{r} = Mic\ \mathfrak{G}g - ker(\mathfrak{r})$. Whenever $\mathfrak{r} \in Mic\ \mathfrak{G}g - O(U)$.

Example 5.2: From example 4.2, as explained in Table 2

If $A = \{\mathfrak{r}_1.\mathfrak{r}_2\}$ then $\mu_R(\chi) = \{\emptyset. U. \{\mathfrak{r}_1.\mathfrak{r}_2\}\{\mathfrak{r}_2\}\}$ and

$$Mic\ \mathfrak{G}g - O(U) = \{\emptyset. U. \{\mathfrak{r}_1\}. \{\mathfrak{r}_2\}. \{\mathfrak{r}_3\}. \{\mathfrak{r}_1.\mathfrak{r}_2\}. \{\mathfrak{r}_2.\mathfrak{r}_3\}\}$$

$\mathfrak{r} \subseteq U$, as shown $Mic\ \mathfrak{G}g - ker(\mathfrak{r})$ of following:

Table 2

$\mathfrak{r}$	$Mic\ ker(\mathfrak{r})$	$Mic\ \mathfrak{G}g - ker(\mathfrak{r})$
$\emptyset$	$\emptyset$	$\emptyset$
U	U	U
$\{\mathfrak{r}_1\}$	$\{\mathfrak{r}_1.\mathfrak{r}_2\}$	$\{\mathfrak{r}_1\}$
$\{\mathfrak{r}_2\}$	$\{\mathfrak{r}_2\}$	$\{\mathfrak{r}_2\}$
$\{\mathfrak{r}_3\}$	$\{\mathfrak{r}_3\}$	$\{\mathfrak{r}_3\}$
$\{\mathfrak{r}_1.\mathfrak{r}_2\}$	$\{\mathfrak{r}_1.\mathfrak{r}_2\}$	$\{\mathfrak{r}_1.\mathfrak{r}_2\}$
$\{\mathfrak{r}_1.\mathfrak{r}_3\}$	U	$\{\mathfrak{r}_1.\mathfrak{r}_3\}$
$\{\mathfrak{r}_2.\mathfrak{r}_3\}$	$\{\mathfrak{r}_2.\mathfrak{r}_3\}$	$\{\mathfrak{r}_2.\mathfrak{r}_3\}$

Proposition 5.3: Let a micro grill topological space $(U.\mu_R(\chi).\mathfrak{G})$, then any $Mic - \mathfrak{G}g - ker(\mathfrak{r}) \subseteq Mic\ ker(\mathfrak{r})$.

Proof: Let $I \notin Mic\ ker(\mathfrak{r})$ then $I \notin \cap\{A: \mathfrak{r} \subseteq A\}$ implies $\exists\ A \in Mic - open\ set$: $I \notin A$ and since $Mic - open$ set $is\ Mic\ \mathfrak{G}g - open$ (by remark 4.3) implies A is $Mic\ \mathfrak{G}g - open, \mathfrak{r} \subseteq A: I \notin A$ then $I \notin \cap\{\{A: \mathfrak{r} \subseteq A. A \in Mic\ \mathfrak{G}gU(O)\}$. Then $I \notin Mic\ \mathfrak{G}g - ker(\mathfrak{r})$.

The converse is not true, as shown above when $\mathfrak{f} = \{\mathfrak{f}_1\}$ then $Mic\text{-}ker(\mathfrak{f}) = \{\mathfrak{f}_1.\mathfrak{f}_2\}$ and $Mic\ \mathfrak{G}g - ker(\mathfrak{f}) = \{\mathfrak{f}_1\}$ then $Mic\text{-}ker(\mathfrak{f}) \nsubseteq Mic - \mathfrak{G}g - ker(\mathfrak{f})$.

Definition 5.4: Let $(U.\mu_R(x).\mathfrak{G})$ be a micro grill topological-space and $\mathfrak{f} \subseteq U$ is called $Mic\ \mathfrak{G}g - \gamma$ −set if $\mathfrak{f} = Mic\ \mathfrak{G}g - ker(\mathfrak{f})$.

From example 4.2, $U.\emptyset.\{\mathfrak{f}_1\}.\{\mathfrak{f}_2\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}$ are $Mic\ \mathfrak{G}g - \gamma$-sets.

Remark 5.5: For $(U.\mu_R(\chi).\mathfrak{G})$, $\mathfrak{f} \subseteq U$, then $\mathfrak{f}$ is a $Mic\ \mathfrak{G}g - \gamma$ −set if $\mathfrak{f}$ is $\in\ Mic\ \mathfrak{G}g - O(U)$.

Definition 5.6: Let $(U.\mu_R(\chi).\mathfrak{G})$ be a micro grill topological space , any $\mathfrak{f} \subseteq U$ is a $Mic - \mathfrak{G}g - \varpi - closed$ set if $\mathfrak{f} = L \cap K$, where L is $Mic - \mathfrak{G}g - \gamma$-set and K is $Mic - \mathfrak{G}g - closed$ set.

Example 5.7: From example 4.2 and example 5.2, we got the result as shown in Table 3

Table 3

$\mathfrak{f}$	$Mic\ \mathfrak{G}g - ker(\mathfrak{f})$	$Mic\ \mathfrak{G}g - \gamma(\mathfrak{f})$	$Mic\ \mathfrak{G}g - C(U)$	$Mic\ \mathfrak{G}g - \varpi - closed$
$\emptyset$	$\emptyset$	$\emptyset$	$\{\emptyset.U.\{\mathfrak{f}_2\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}$	$\emptyset$
U	U	U	$\{\emptyset.U.\{\mathfrak{f}_2\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}$	U
$\{\mathfrak{f}_1\}$	$\{\mathfrak{f}_1\}$	$\{\mathfrak{f}_1\}$	$\{\emptyset.U.\{\mathfrak{f}_1\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}..\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_1\}$
$\{\mathfrak{f}_2\}$	$\{\mathfrak{f}_2\}$	$\{\mathfrak{f}_2\}$	$\{\emptyset.U.\{\mathfrak{f}_1\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_2\}$
$\{\mathfrak{f}_3\}$	$\{\mathfrak{f}_3\}$	$\{\mathfrak{f}_3\}$	$\{\emptyset.U.\{\mathfrak{f}_1\}.\{\mathfrak{f}_2\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_3\}$
$\{\mathfrak{f}_1.\mathfrak{f}_2\}$	$\{\mathfrak{f}_1.\mathfrak{f}_2\}$	$\{\mathfrak{f}_1.\mathfrak{f}_2\}$	$\{\emptyset.U.\{\mathfrak{f}_1\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_1.\mathfrak{f}_2\}$
$\{\mathfrak{f}_1.\mathfrak{f}_3\}$	$\{\mathfrak{f}_1.\mathfrak{f}_3\}$	$\{\mathfrak{f}_1.\mathfrak{f}_3\}$	$\{\emptyset.U.\{\mathfrak{f}_2\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_1\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_1.\mathfrak{f}_3\}$
$\{\mathfrak{f}_2.\mathfrak{f}_3\}$	$\{\mathfrak{f}_2.\mathfrak{f}_3\}$	$\{\mathfrak{f}_2.\mathfrak{f}_3\}$	$\{\emptyset.U.\{\mathfrak{f}_2\}.\{\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_2\}.\{\mathfrak{f}_1\}.\{\mathfrak{f}_2.\mathfrak{f}_3\}.\{\mathfrak{f}_1.\mathfrak{f}_3\}\}$	$\{\mathfrak{f}_2.\mathfrak{f}_3\}$

Proposition 5.8: Let $(U.\mu_R(\chi).\mathfrak{G})$ be a micro grill topological space if U is a finite set and $\mathfrak{f} \subseteq U.\mathfrak{f}$ is a $Mic\ \mathfrak{G}g\varpi - closed$ set, then $\mathfrak{f} = Mic\ \mathfrak{G}g - ker(\mathfrak{f}) \cap K$, where K is a $Mic\ \mathfrak{G}g - closed$ set.

Proof: Since $\mathfrak{f}$ is a $Mic\ \mathfrak{G}g\varpi - closed$ set, then $\mathfrak{f} = L \cap K$ such that L is a $Mic\ \mathfrak{G}g\gamma$ −set and K is a $Mic\ \mathfrak{G}g - closed$ its imply $\mathfrak{f} \subseteq L = Mic\ \mathfrak{G}g - ker(L)$ and $\mathfrak{f} \subseteq Mic - \mathfrak{G}g - ker(\mathfrak{f})$ which is the smallest $Mic\ \mathfrak{G}g - open$ set consist of $\mathfrak{f}$, also $Mic\ \mathfrak{G}g - ker(\mathfrak{f}) \subseteq Mic\ \mathfrak{G}g - ker(L)$ and $\mathfrak{f} = L \cap K$ therefore $\mathfrak{f} = Mic\ \mathfrak{G}g - ker(\mathfrak{f}) \cap K$.

5.1 *Application in* $Mic\ \mathfrak{G}g - Set$

Message number 257 was sent to Ibn al Atheer Hospital "by the College of computer sciences and Mathematics at the University of Mosul to" gather information on the diagnosis of spleen enlargement in thalassemia patients. This section covers the usage of the message and how it was sent. A smattering of data serves as an example.

Example 5.9: We took the data from five thalassemia patients. Post-diagnosis results were the following:

In Table 4, let $U = \{f_1 . f_2 . f_3 . f_4 . f_5\}$, the set of patients are $\{B. L. H. E\}$ R should represent the equivalence relation on U. is given by $\frac{U}{R} = \{f_1 . f_3 . f_4\}. \{f_2\}. \{f_5\}$, let $X = \{f_1 . f_3 . f_5\} \subseteq U$,

$\tau_R(\chi) = \{\emptyset. U. \{f_5\}. \{f_1 . f_3 . f_4 . f_5\}. \{f_1 . f_3 . f_4\}\}$, $\mathcal{M} = \{f_2\} \notin \tau_R(\chi)$

$\mu_R(\chi) = \{\emptyset. U. \{f_5\}. \{f_1 . f_3 . f_4 . f_5\}. \{f_1 . f_3 . f_4\}. \{f_2\}. \{f_2 . f_5\}. \{f_1 . f_2 . f_3 . f_4\}\}$

$\mu_R(\chi) - closed = \{\emptyset. U. \{f_1 . f_2 . f_3 . f_4\}. \{f_2\}. \{f_2 . f_5\}. \{f_1 . f_3 . f_4 . f_5\}.$
$\{f_1 . f_3 . f_4\}. \{f_5\}$

$\mathfrak{G} = \{U. \{f_1\}. \{f_1 . f_2\}. \{f_1 . f_3\}. \{f_1 . f_4\}. \{f_1 . f_5\}. \{f_1 . f_2 . f_3\}. \{f_1 . f_2 . f_4\}. \{f_1 . f_2 . f_5\}.$
$\{f_1 . f_3 . f_4\}. \{f_1 . f_3 . f_4 . f_5\}. \{f_1 . f_3 . f\}. \{f_1 . f_4 . f_5\}. \{f_1 . f_2 . f_4 . f_5\}. \{f_1 . f_2 . f_3 . f_4\}.$
$\{f_1 . f_2 . f_3 . f_5\}. \{f_1 . f_4 . f_5\}\}$

$\therefore\ Mic - \mathfrak{G}gC(U) = \{\emptyset. U. \{f_1\}. \{f_2\}. \{f_5\}. \{f_1 . f_2\}. \{f_1 . f_3\}. \{f_1 . f_4\}. \{f_1 . f_5\}.$
$\{f_2 . f_5\}. \{f_1 . f_2 . f_3\}. \{f_1 . f_2 . f_4\}. \{f_1 . f_2 . f_5\}. \{f_1 . f_3 . f_4\}.$
$\{f_1 . f_3 . f_5\}. \{f_1 . f_2 . f_3 . f_4\}. \{f_1 . f_2 . f_3 . f_5\}. \{f_1 . f_2 . f_4 . f_5\}.$
$\{f_1 . f_3 . f_4 . f_5\}. \{f_1 . f_4 . f_5\}\}$

Table 4

(A) Patients	(B) hb	(L) Lower hormones	(H) Phyperfertin	(E) Result
f_1	+	-	-	yes
f_2	+	-	+	yes
f_3	+	-	-	yes
f_4	+	-	-	yes
f_5	+	-	-	no

If the result (B) and (L) , the equivalence relations on U will remain the same as the equivalence relations in the previous step, and accordingly, it will be the same $Mic - \mathfrak{G}gC(U)$.

If the result (H), so $U/R = \{\{f_1 . f_2 . f_3 . f_4\}. \{f_5\}\}$,

$$\tau_R(\chi) = \{\emptyset. U. \{f_5\}. \{f_1 f_2 . f_3 . f_4\}\},$$

$\mathcal{M} = \{f_2\} \notin \tau_R(\chi)$ then $\mu_R(\chi) = \{\emptyset. U. \{f_5\}. \{f_1. f_2. f_3. f_4\}. \{f_2\}. \{f_2. f_5\}\}$

$\mu_R(\chi) - closed = \{\emptyset. U. \{f_1. f_2. f_3. f_4\}. \{f_5\}. \{f_1. f_3. f_4 f_5\}. \{f_1. f_3. f_4\}\}$

$\therefore \ Mic - \mathfrak{G}gC(U) = \{\emptyset. U. \{f_1\}. \{f_5\}. \{f_1. f_2\}. \{f_1. f_3\}. \{f_1. f_4\}. \{f_1. f_5\}.$
$\{f_1. f_2. f_3\}. \{f_1. f_2. f_4\}. \{f_1. f_2. f_5\}. \{f_1. f_3. f_4\}. \{f_1. f_3. f_5\}.$
$\{f_1. f_2. f_3. f_4\}. \{f_1. f_2. f_3. f_5\}. \{f_1. f_2. f_4. f_5\}. \{f_1. f_3. f_4. f_5\}.$
$\{f_1. f_4. f_5\}\}$

If the result (E), so $\frac{U}{R} = \{f_1. f_3. f_4. f_5\}. \{f_2\}\}$, $\tau_R(\chi) = \{\emptyset. U. \{f_1. f_3. f_4. f_5\}\}$,
$\mathcal{M} = \{f_2\}$. Then $\mu_R(\chi) = \{\emptyset. U. \{f_1. f_3. f_4. f_5\}. \{f_2\}\}$

$\mu_R(\chi) - closed = \{\emptyset. U. \{f_2\}. \{f_1. f_3. f_4. f_5\}\}$

$\therefore \ Mic - \mathfrak{G}gC(U) = \{\emptyset. U. \{f_1\}. \{f_2\}. \{f_1. f_2\}. \{f_1. f_3\}. \{f_1. f_4\}. \{f_1. f_5\}.$
$\{f_1. f_2. f_3\}. \{f_1. f_2. f_4\}. \{f_1. f_2. f_5\}. \{f_1. f_3. f_4\}. \{f_1. f_3. f_5\}. \{f_1. f_2. f_3. f_4\}.$
$\{f_1. f_2. f_3. f_5\}. \{f_1. f_2. f_4. f_5\}. \{f_1. f_3. f_4. f_5\}. \{f_1. f_4. f_5\}\}$

The conclusion is that in removing $row(B)$ and describing (L), the result will be similar and the patient does not have an enlarged spleen.

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