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Mathematical model studying the effect of vaccination rate on the spreading of respiratory system virus

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Abstract

This research try to study how vaccination rate and vaccine efficiency can affect the infection by respiratory system viruses using dynamical system. Depending on epidemiological state of the study community we divided it into four group : S (healthy),V (who get vaccine), I (who get infection) and Q (who attended to health quarantine). By solving all the equations of system we can found that the system has two equilibrium point. On the other hand the system was locally analyzed. And the numerical analysis of the system showed that rapid increase in infection rate when we have low vaccination rate and decrease in infection rate when increase vaccination rate and also explain how vaccine efficiency effect on development and spreading of disease.

Subject Classification: *00A71, 03C10.*

Keywords: *Dynamical system, Covid-19, Respiratory system viruses, Vaccine rate, Vaccine efficiency, SVIQ model.*

1. Introduction

The infectious respiratory disease caused by SARS-CoV-2 (severe acute respiratory syndrome Coronavirus 2), influenza virus and COVID-19 consider as an emergency outbreak to public health affecting most of the countries around

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the world , Iraq is one of them , it can be transmitted between individuals by direct contact with infected person or contaminated surfaces or by inhaling the respiratory droplets from infected persons[1,2]. The wide spread symptoms of infection are high fever, lose of smell and appetite, fatigue, pain of muscles, hard breathing and dry cough. People who were infected with one of these virus may developing a mild or severe respiratory symptoms and recover with specific treatment course but the older people complaining from several medical case such as cardiovascular disease, diabetes, chronic respiratory disease and cancer all of them very sensitive and developed sever infection[3,4]. In biological field there are many research study the effect of vaccine on infection rate and efficiency of vaccine in addition to that explain how vaccine reduce the transmission of these viruses among population and making a compression between contacting of infected persons with vaccinated and non-vaccinated individuals [5,6,12]. On the other hand in the field of mathematics there are many research have been demonstrated a mathematical model describe the transmission of viruses to understanding the dynamical behavior of spreading the virus while other studying the effect of quarantine on virus spreading [7,8,13]. In our study we demonstrated a mathematical model describe the effect of vaccine and vaccine efficiency on spreading of respiratory viruses throughout population.

2. Formulation of the Model

Depending on the epidemiological case of the studied population total number divided into four group: S (susceptible persons),V(vaccinated persons), I(infected persons) and Q (persons who attended to health care quarantine) , where the parameters are explained in Table 1.

Where the model given by

$$\begin{aligned} \frac{dS}{dt} &= \pi - \beta\, S\, I - (\mu + \Psi)S + (1-\gamma)\alpha\, I + \eta\, Q \\ \frac{dV}{dt} &= \Psi\, \mathrm{S} - \delta\beta \mathrm{V}\, I - \mu V \\ \frac{dI}{dt} &= \beta\, S\, I + \delta\beta \mathrm{V}\, I - (\alpha + \mu)I \\ \frac{dQ}{dt} &= \gamma\alpha\, I - (\eta + \mu)Q \end{aligned} \tag{1}$$

Table 1

The description of parameters that used in SVIQ model

Description	Symbol
Total birth rate	π
Transmission rate of disease	β
Vaccination efficiency $0 \le \delta \le 1$	δ
Recovered rate	α
Treatment rate of individuals in quarantine	η
Vaccination rate	Ψ
Death rate	μ
Progression from infected to quarantine $0 \le \gamma \le 1$	γ
Progression from infected to susceptible	$1-\gamma$

Theorem 1: *The solutions of system* (1) *that initial in* $\mathbb{R}_+^4$ *are uniformly bounded.*

Proof: $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dV}{dt} + \frac{dI}{dt} + \frac{dQ}{dt}$, this give, $\frac{dN}{dt} \leq \pi - \mu N$. using Gronwall – Bellman inequality[9] obtained $N(t) \leq \frac{\pi}{\mu}(1 - e^{-\mu t}) + N(0)e^{-\mu t}$. We have $N(t) \leq \frac{\pi}{\mu}$, as $t \to \infty$, where all of the system (1) solutions are initiate in $\mathbb{R}_+^4$.

3. The Existence of System Equilibrium Points .

In order to testing the equilibrium point of the system (1) we solving all the equations in the system. So that system (1) has at least two equilibrium points denote as $P^i = (S_i, V_i, I_i, Q_i), i = 0,1$. If $I = 0$, then we have an equilibrium point in system (1) which is called a disease free equilibrium point and symbol by $P^0 = (S_0, V_0, 0,0)$ where $S_0 = \frac{\pi}{\mu+\Psi}$, $V_0 = \frac{\Psi}{\mu} S_0$. However, if $I \neq 0$ so that, system(1) posses an endemic equilibrium point denoted by $P^1 = (S_1, V_1, I_1, Q_1)$ where S_1, V_1, I_1, Q_1 positive solution for the following equations,$\frac{dS}{dt} = \frac{dV}{dt} = \frac{dI}{dt} = \frac{dQ}{dt} = 0$,we get $Q_1 = dI_1$, $S_1 = \frac{\pi + ((1-\gamma)\alpha + \eta d)I}{\beta I + \mu + \Psi}$, and $V_1 = \frac{\Psi(\pi + ((1-\gamma)\alpha + \eta d)I)}{(\beta I + \mu + \Psi)(\delta \beta I + \mu)}$ where $d = \frac{\gamma \alpha\, I}{\mu + \alpha_1}$, Substituting S_1, V_1, Q_1, in third equation of (1) we get,

$$\xi_1 I^2 + \xi_2 I + \xi_3 = 0 \tag{2}$$

Where $\xi_1 = \delta \beta^2 (\pi + \eta - \gamma\alpha - \mu)$.

$$\xi_2 = -\beta(\alpha\mu(\gamma + \delta) + \alpha\gamma\delta\Psi + \mu(\mu(1 + \delta) + \delta\Psi - (\pi + \eta))) < 0.$$

$$\xi_3 = \delta\beta(\pi + \eta)\Psi - \mu(\alpha + \mu)(\mu + \Psi) .$$

From the equation (2) using the Descartes rule of signs [10] possess a positive roots which is given by I and the equilibrium point P^1 exist uniquely in $\mathbb{R}_+^4$ if and only if the following conditions hold.

(i) $\xi_1 > 0$ and $\xi_3 < 0$ (ii) $\xi_1 < 0 and\ \xi_3 > 0$.

4. Local Stability Analysis of System

The local stability of each equilibrium point P^0, P^1 of system (1) were studied as in the following theorems

Theorem 2: *The equilibrium point* $P^0 = (S_0 V_0, 0,0)$ *in the system* (1) *is locally stable provided that:*

$$\pi\beta(\mu + \delta\Psi) < \mu(\mu + \Psi)(\mu + \alpha)$$

Proof: By using Jacobian matrix of the system (1) where (P^0) can be written $J(P^0) = [\rho_{ij}]_{4*4}$ .Clearly, $J(P^0)$ which have the following eigenvalue:

$$\lambda_S = -(\mu + \Psi), \lambda_V = -\mu\ , \lambda_I = \beta S_0 + \delta\beta V_0 - (\mu + \alpha), \lambda_Q = -(\mu + \eta).$$

Therefore P^0 is locally asymptotically stable if and only $\frac{\pi\beta(\mu+\delta\Psi)}{\mu(\mu+\Psi)(\mu+\alpha)} <$ 1,where the proof is complete.

Theorem 3: *If endemic equilibrium point* $P^1 = (S_1, V_1, I_1, Q_1)$ *of system* (1) *exist then* P^1 *is locally asymptotically stable providing the following :*

$$\beta S_1 + \delta\beta V_1 < (\mu + \alpha) \quad (1)$$

$$\frac{\beta S_1(\mu+\delta\Psi)+\delta\beta V_1(\eta+\delta\beta)}{2(\eta+\delta\Psi)(\mu+\alpha)} < 1 \quad (2)$$

Proof: The Jacobean matrix of system (1) at the endemic equilibrium point P^1 can be written $J(P^1) = [\rho_{ij}]_{4*4}$ .Then characteristics equation of Jacobean matrix is given by

$\lambda^4 + \omega_1\lambda^3 + \omega_2\lambda^2 + \omega_3\lambda + \omega_4 = 0$, where $\omega_1 = -\rho11 - \rho22 - \rho33 - \rho44.$

$$\omega_2 = \rho11\rho22 + \rho11\rho33 + \rho11\rho44 + \rho22\rho33 + \rho33\rho44 + \rho22\rho44 - \rho23\rho32\ .$$

$$\omega_3 = \rho13\rho21\rho32 + \rho11\rho23\rho32 - \rho11\rho22\rho33 - \rho11\rho22\rho44 + \rho23\rho32\rho44 - \rho11\rho33\rho44 - \rho22\rho33\rho44.$$

$$\omega_4 = \rho14\ \rho43\ \rho21\ \rho32 - \rho11\ \rho21\ \rho32\ \rho44 - \rho11\ \rho23\ \rho32\ \rho44 + \rho11\ \rho22\ \rho33\ \rho44\ .$$

$\Delta = \omega_1\omega_2\omega_3 - \omega_3^2 - \omega_1^2\omega_4$. Depending on (Routh-Hureitz) criterion [11] P^1 can be locally stable provided $\omega_1, \omega_2, \omega_3, \omega_4, \Delta > 0$, clearly $\omega_1 > 0$ and $\omega_2, \omega_3, \omega_4, \Delta > 0$ if provide the conditions (1& 2) hold.

5. Numerical analysis

Numerical analysis has an important in the confirmation of the result we have obtained .This model indicating the effect of vaccination rate on spread of disease and depending on contact by infected persons with vaccinated and non-vaccinated persons in addition to that the effect of vaccine efficiency.

6. Discussion

The numerical solution of system (1) have been determined using a specific initial conditions. The effective way to controlling the respiratory viruses by compliance social distance, health rules and sanitation all these can help in controlling of transmission but this is consider as a difficult way in large population and cannot put the community in lock down for long time during the pandemic in addition to that get vaccine against these viruses has an important role in prevent and development of infection, so that we focus on the role of vaccine in helping reduce of infection and prevent transmission. Figure (1-a) showed the spreading of disease when there is no vaccinated persons we notice rapid increase in the number of infected persons. While in Figure (1-b) when the number of vaccinated persons increase to 20% we can notice a little decrease in the numbers of the infected persons. Also in Figure (1-c) showed when increase the rate of vaccination to 50% there was a decrease in number of infection when compare with Figure (1a and b) this because increase in vaccine rate, this explain the mechanism of vaccine in prevent spreading, development and reducing the symptoms of infection since vaccinated person showed the ability to not transmit the infection to other person as we showed in Figure (1-d) a highly increase in vaccination rate 80% and decrease in infection rate in all these Figure we consider the vaccine efficiency $\delta = 0.5$.

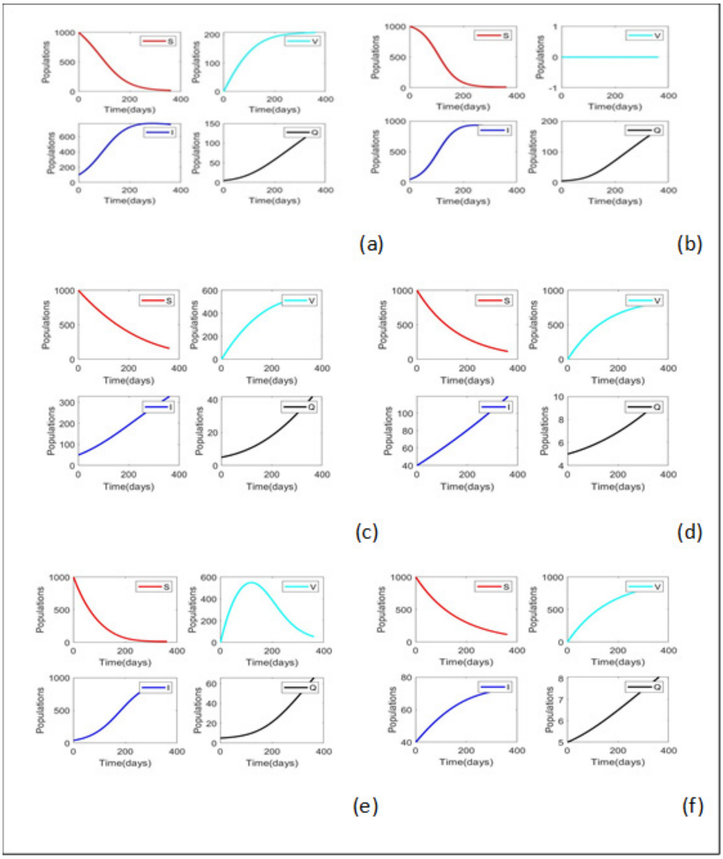

Figure 1

a) when vaccination rate 0 %, b) when vaccination rate increase to 20%, c) vaccination rate 50%, d)vaccination rate 80 %, e) vaccination rate 80% and $\delta = 0$, f) vaccination rate 80 % and $\delta = 1$.

On the other hand in Figure (1-e) we studied the effect of vaccine efficiency when vaccination rate was 80% we noticed that when vaccine active ($\delta = 0$), they were decrease in infection rate in contrast to Figure (1-f) showed when vaccine was inactive ($\delta = 1$) showed an increase in infection rate despite of the vaccination rate was 80%.

7. Conclusion

We can conclude that vaccine play an important role in controlling of transmission and development of diseases but in addition to that vaccine efficiency also has a major role in the processing of disease when vaccine was active that is mean the vaccine success in reduce the infection and symptoms but when vaccine was inactive failed in its role since there are many different factors may affect on vaccine efficiency may include manufacturing, transport and storage.

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