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Hpre- Compactence and *Hpre*-Paracompactness

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Abstract

The topic *Hpre*-compact, which we covered in our article, is concerned with the research of *Hpre*-paracompactness. We employed *Hpre*-T_2 space, Hpre-locally compact, and a *Hpre*-regular in our work and relied on *Hpre*-open cover to provide our definition. These concepts were connected and utilized in presenting the fundamental research theories.

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1. Introduction

That's a topic compact and paracompactness, one of the main topics in topological spaces which researchers [1] always focus on in their work, and this is what prompted us to define a new type of it we named $Hpre-$ compact and $Hpre$-paracompactness using $Hpre$-open set provided by researchers Hadi, M.H. and Al-Yaseen, M [2] a subset $\mathcal{H}$ of a space X is called $Hpre$-closed set if

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$Cl_g(Int(\mathcal{H})) \subseteq \mathcal{H}$. $HpreC(X)$ is the means of all $Hpre$-closed sets, the complement of $Hpre$-closed is $Hpre$-open and signify the collective family $Hpre$open sets of a topological by $HpreO(X)$. In [3] Introduce $Hpre-T_2$ space such that let (X,τ)a topological space. So, the space (X,τ) is called $Hpre-T_2$ space iff for every distinct pair of points x_1 , x_2 in X, there exists $Hpre$-open sets U and V in X s.t. $x_1 \in U$ and $x_2 \in V$, and $U \cap V = \emptyset$. To learn more about the characteristics of $Hpre-$ closed, see [4,5]. In section two, we will review $Hpre$-compact, $Hpre$-locally compact, $Hpre$-regular We will associate it with $Hpre-T_2$ and $Hpre$-closed locally finite. We demonstrate the basic relationships between them.

2. Construction Foundations

Definition 2.1: Let $\mathcal{H}$ *be any set and* $W = \{W\alpha\}$ an $Hpre$-open covering of $\mathcal{H}$. A finite $Hpre$-subcover of W is a finite collection $W\alpha 1, W\alpha 2, \ldots, W\alpha k$ of the $W\alpha \ni \mathcal{H} \subseteq \cup\, k\, j = 1W\alpha j$.

Definition 2.2: Let X a topological and $K \subseteq X$. Say that K is $Hpre$-compact if any $Hpre$-open covering $W = \{W\alpha\}\alpha \in A$ of K has a finite $Hpre$-subcover.

Example 2.3: Let $X = \mathbb{R}$ with the usual topology. Let $K = \{1, 1/2, 1/3, 1/4, \ldots\} \cup \{0\}$. Then K is $Hpre$-compact. To see this, let $W = \{W\alpha\}$ be an $Hpre$-open cover K. So $\exists\, W\alpha 0$ contains 0. But $W\alpha 0$ contain all $1/j$ for $j > J0$, $\exists J0$ large. Now select $W\alpha 1$ that contains $1/1$, $W\alpha 2$ that contains $1/2$, on up to $W\alpha J0$ that contains $1/J0$. The $Hpre$-open sets $W\alpha 0, W\alpha 1, W\alpha 2, \ldots, WJ0$. Thus K is $Hpre$-compact.

Proposition 2.4: Let $f : X \to Y$ be a mapping of topological spaces. If $K \subseteq X$ is $Hpre$-compact, so $f(\mathcal{H}) \equiv \{f(h) : h \in \mathcal{H}\}$ is $Hpre$-compact.

Proof: Let $W = \{W\alpha\}\alpha \in A$ be an $Hpre$-open cover $f(\mathcal{H})$. So, since f is $Hpre$-continuous, $\{f^{-1}(W\alpha)\}\alpha \in A$ is an $Hpre$-open cover of $\mathcal{H}$. Therefore, there is a finite subcover, $f^{-1}(W\alpha 1), f^{-1}(W\alpha 2), \ldots, f^{-1}((W\alpha m)$. It follows then that $W\alpha 1, W\alpha 2, \ldots, W\alpha m$ is a subcover and finite of $\mathcal{H}$. Then $\mathcal{H}$ is $Hpre$-compact.

Proposition 2.5: A set $\mathcal{H} \subseteq \mathbb{R}$ is $Hpre$-compact iff it is $Hpre$-closed and $Hpre$-bounded.

Proposition 2.6: Let X be topological. A $Hpre$-closed of a $Hpre$-compact set in X is $Hpre$-compact.

Proposition 2.7: Let X be topological is $Hpre$-compact iff any family $Q = \{Q\alpha\}\alpha \in A$ of $Hpre$-closed in X with the finite intersection property satisfies $\cap\, \alpha \in AQ\alpha \neq \emptyset$.

Definition 2.8: Let (X, U) be topological. We say that X is $Hpre$-locally compact if every point X has a $Hpre$-nbh base consisting of $Hpre$-compact sets.

Theorem 2.9: *The one point $Hpre$-compactification X^* of a topological X is $Hpre$-compact, and X is a subspace. The X^* is $Hpre$-T_2 if and only if X is $Hpre$-locally compact and $Hpre$-T_2.*

Proof: A set U is $Hpre$-open of X^* iff (a) $U \cap X$ is $Hpre$-open in X and (b) whenever $\infty\, \varepsilon\, U$, so $X \sim U$ is $Hpre$-compact. Consequently, finite intersections and unions of $Hpre$-open in X^* Intersect X in $Hpre$ open. If ∞ it is a member of the intersection of two $Hpre$-open subsets of X^*, then the complement of the intersection is the union of two $Hpre$-closed compacts of X and is therefore $Hpre$-closed and $Hpre$-compact. If $\infty \in \bigcup_{n=1}^{\infty} H_i$ $Hpre$-open of X^*, so $\infty \in U$ of family, and the complement of the union is a $Hpre$-closed of the $Hpre$-compact set $X \sim U$ and therefore $Hpre$-closed and $Hpre$-compact. Consequently X^* is topological and X a subspace. If u is an open cover of X^*, then ∞ is a member of some U in and $X \sim U$ is $Hpre$-compact, and hence there is a finite $Hpre$-subcover of u . Then X^* is compact. If X^* is a $Hpre$-T_2, then its $Hpre$-open subspace X is a $Hpre$-locally compact $Hpre$-T_2 space. Finally, it must be shown that $X^*$is a $Hpre$-T_2 space if X is a $Hpre$-locally compact $Hpre$-T_2 space. It is only necessary to show that, if $x\, \varepsilon\, X$, so there are disjoint $Hpre$-neighborhoods of x and ∞. But since X is $Hpre$-locally compact and $Hpre$-T_2 there is a $Hpre$-closed compact $Hpre$-neighborhood U of x in X and $X^* \sim U$ is the required neighbourhood of $Hpre - \infty$. If X is a $Hpre -$ compact, so ∞ is an isolated of the one-point $Hpre$ compactification (that is, $\{\infty\}$ is both $Hpre$ open and $Hpre$ closed). Conversely, if ∞ is an isolated point of X^*, so X is $Hpre$ closed of X^* and is, therefore $Hpre -$ compact.

Theorem 2.10: *The collection of all $Hpre$-compactifications of a topological is partially ordered b $\geq$If (f, Y) and (g, Z) are $Hpre$- T_2 spaces $Hpre$-compactifications of space and $(f, Y) \geq (g, Z) \geq (f, Y)$, then (f, Y) and (g, Z) are topologically equivalent.*

Proof: If $(f, Y) \geq (g, Z) \geq (h, U)$, where these are $Hpre$-compactifications of a space X, then there are $Hpre$-continuous functions j on Y to Z and k on Z to U such that $g = j \circ f$ and $h = k \circ g$ and hence $h = k \circ j \circ f$ and $(f, Y) \geq (h, U)$. Consequently $\geq$ partially orders the collection of all $Hpre$-compactifications of X. If (f, Y) and (g, Z) are $Hpre$- T_2 space $Hpre$-compactifications each of which follows the other relative to the ordering $\geq$, then both $f \circ g^{-1}$and $g \circ f^{-1}$ have $Hpre$-continuous extensions j and k to all Z and Y respectively. Since $k \circ j$ is the identity map on the $Hpre$-dense subset $g[X]$ of Z and Z is $Hpre$- T_2 space, $k \circ j$ is the identity of Z onto itself and similarly, $j \circ k$ is identity of Y onto Y. Consequently, (f, Y) and (g, Z) are equivalent.

Theorem 2.11: *If an $Hpre$-open cover has a $Hpre$-closed locally finite refinement, then it is an even cover. Consequently, each $Hpre$-open cover of a $Hpre$-compact regular is even.*

Proof: Let U be an $Hpre$-open cover of X and let be a $Hpre$-closed locally finite refinement. For any A in Q choose a member UA of $\ni A \subset UA$, and let $VA = (UA \times UA) \cup \{(X \sim A) \times (X \sim A)\}$. Evidently, VA is an $Hpre$- a neighbourhood of the diagonal in $X \times X$, and, if $x \, \varepsilon \, A$, then $VA[x] = UA$. Letting $V = \cap \{VA: A\varepsilon Q\}$, it follows that for each point x the set $V[x] \subset VA[x] = UA$ and consequently the family form $V[x]$ is a refinement of U It remains to be proved that V is a $Hpre$-neighborhood of the diagonal. For each point (x, x) of the diagonal, choose a $Hpre$-nbh W of x such that W intersects only finitely many members of Q. If $W \cap A$ is void, then $W \subset X \sim A$ and $W \times W \subset VA$. It follows that V contains the intersection of $W \times W \subseteq VA$ with a finite VA and is so a $Hpre$-nbh of (x, x). Finally, if X is $Hpre$-compact and $Hpre$-regular, so each $Hpre$-open cover has a $Hpre$-closed finite refinement (cover X by $Hpre$-open subsets whose $Hpre$-closures refine U) and hence each $Hpre$-open cover is even.

Now, A topological is $Hpre$-paracompact iff it is $Hpre$-regular and each $Hpre$-open cover has an $Hpre$-open locally refinement. The purpose of this section is to prove the equivalence of $Hpre$-paracompactness and several other conditions. The methods used are $Hpre$-closely related to those of (REF). Recall a family Q of a space is discrete iff there is a $Hpre$-neighborhood of each point of the space intersects at most one member of the family. A family is ($Hpre -$ σ-locally finite) iff it is the union of countably many discrete (respectively $Hpre$-locally) subfamilies. The principal theorem of the section can now be stated; its proof is given in the sequence of lemmas which follows the statement.

Theorem 2.12: *If X is $Hpre$-regular and any $Hpre$-open cover is a $Hpre$-locally finite refinement, so any $Hpre$-open cover is a $Hpre$-closed locally finite refinement.*

Proof: If is an $Hpre$-open cover X, then there an $Hpre$-open cover s.t. the family of $Hpre$-closures of members v refines u since X is $Hpre$-regular. ($\forall x$, if $x \, \varepsilon \, U$ there an $Hpre$- nbh V of x s.t. $V \subset U$. Let Q be a $Hpre$-locally finite of v Then β of all $Hpre$-closures of β is $Hpre$-locally finite, each member of β is a subset of $V$of some V in v . Hence is the required $Hpre$-closed locally refinement of u.

For any topological space an $Hpre$-open cover which has a $Hpre$-closed locally finite is even, according to Theorem 2.12. Hence statement (c) of the theorem implies (d). Before proving the next implication, we prove two lemmas which are of some interest in themselves. For convenience we review some of the facts which will be needed. If U is a subset of $X \times X$ and $x \, \varepsilon \, X$, then $U[x]$ is any $y \ni (x, y) \, \varepsilon \, U$. If $A \subseteq X$, then $U[A] = \{y: (x, y) \, \varepsilon \, U$ for some x in $A\}$; clearly $U[A]$ is a union of sets $U[x]$ for x in A. The set $\{(x, y):$

$(y,x)\ \varepsilon\ U\}$ is U^{-1}, and U is $Hpre$-symmetric and $U = U^{-1}$. The set $U \cap U^{-1}$ is always $Hpre$-symmetric. If U and V are subsets of $X \times X$, then $U \circ V$ is the set of all pairs (x,z) such that for some y in X it is true that $(x,y)\ \varepsilon\ V$ and $(y,z)\ \varepsilon\ U$. Also put, $(x,z)\ \varepsilon\ U\ oV$ iff $(x,z)\ \varepsilon\ V^{-1}[y] \times U[y]$ for some y, likewise UoV is the union $V^{-1}[y] \times U[y]$ s.t. $y \in X$. If V is $Hpre$-symmetric, so $Vo\ V = \cup \{V[y] \times V[y]: y\ \varepsilon X\}$ Finally, any subset A of X it is true $\ni$ $(U \circ F)[A] = U[V[A]]$.

Theorem 2.13: *Let X a topological such that each $Hpre$-open cover is even. If U is a $Hpre$-neighborhood of the diagonal in, $X \times X$ then there is a $Hpre$-symmetric $Hpre$-neighborhood V of the diagonal such that $V \circ V \subset U$.*

Proof: For each point x of X there is a $Hpre$-nbh $W(x) \ni W(x) \times W(x) \subset U$, because U is a $Hpre$-nbh of the diagonal. All sets of the form $W(x)$ is an $Hpre$-open cover of X and there is a $Hpre$-nbh R of the diagonal s.t. the family of all sets $R[x]$ refines, and hence $R[x] \times R[x] \subset U\ \forall x$. Finally, let $V = R \cap R^{-1}$. Then V is a $Hpre$-symmetric $Hpre$-neighborhood of the diagonal and $V[x] \times V[x] \subset U\ \forall x$. Since $Vo\ V$ is the union of the sets $V[x] \times V[x]$ Thus, it $VoV \subseteq U$.

Now, the preceding theorem has the following intuitive content. Let us say two points x and y are at most $HpreU$ $-$distance apart if $(x,y)\ \varepsilon\ U$. Then there is V such that if x and y, and y and z, are at most $HpreV$ $-$distance apart, then x and z are at most $HpreU$ $-$distance apart. The following lemma shows that paracompact spaces satisfy a very strong normality condition.

Theorem 2.14: *Let X be a topological such that each $Hpreopen$ cover is even and let be a $Hpre$ $-$locally finite (or discrete) subsets of X. Then there is a $Hpre$ $-$neighborhood V of the diagonal in $X \times X$ such that the family $V[A]$ for A in Q is $Hpre$ $-$locally finite (respectively discrete).*

3. Discussion and Conclusion

We proved one of the most important topics of topology, which is $Hpre$-compact, and we compared it with the usual compact, and we noticed through theorems that it is possible to create a $Hpre$-cover for any space using $Hpre$-open, and these ideas can be generalized in the topic of $Hpre$- connected as in the references [6,7].

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