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Fixed points of multivalued nonexpansive random operator in banach spaces

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Abstract

In this paper, we show that a multivalued random operator has a fixed point in uniformly convex Banach space. In addition, the presence of a limit point in a multivalued random operator is shown, and a new iteration sequence for determining convergence of a multivalued random operator is introduced.

Subject Classification: *35R70, 47A06.*

Keywords: *Multivalued random operator, Random fixed point, Measurable space, Weak and strong convergence.*

1. Introduction

The random fixed point theorem (r.f.p) was started Prague School of Probability in 1950**.** The probability function is the main mathematical branch that is applied to solving problems; also, (zr. f.p) theory finds its roots in the work of [1]and [2]. Let Φ be a real Banach space. A subset Υ is called proximinal if $\forall u \in \Phi, \exists k \in \Upsilon$such that $d(u,k) = \inf\{\|u-v\| : v \in \Upsilon\} = d(u,\Upsilon)$. $\Pi(\Upsilon)$ the set of all bounded proximinal subsets of Υ, consistent with [3]. Let $CB(\Upsilon)$ be the class of all nonempty bounded and closed subsets of Υ. Let $B: \Psi \times \Upsilon \to \Pi(\Upsilon)$is called multivalued nonexpansive random operator if for all $\hat{e} \in \Psi$, $H\big(B(\hat{e},u), B(\hat{e},v)\big) \leq \|u-v\|$, we give some definitions

Definition 1.1[4] : A Banach space Φ is called opials condition if for any $\{S_n(\hat{e})\}$ in $\Phi, S_n(\hat{e}) \to u(\hat{e})$ implies that $\lim_{n\to\infty} sup\|S_n(\hat{e}) - u(\hat{e})\| < \lim_{n\to\infty} sup\|S_n(\hat{e}) - v(\hat{e})\|$ for all $v(\hat{e}) \in \Phi$, $v(\hat{e}) \neq u(\hat{e})$.

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Definition1.2: [5] Let $B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ be a random operator, then B is called demiclosed at $\zeta(\hat{e})$ if $\{S_n(\hat{e})\}$ converges weakly to $S(\hat{e})$ and $\{B(S_n(\hat{e})\}$ converges to $\zeta(\hat{e})$ imply that $S(\hat{e}) \in \Upsilon$ and $\zeta(\hat{e}) \epsilon B(S(\hat{e}))$ for each $\hat{e} \in \Psi$.

Lemma 1.3: [6] *Let Φ be a uniformly convex Banach space and $0 < p < t_n < q < 1$ for all $n \in N$. Let$\{S_n(\hat{e})\}\},\{\zeta_n(\hat{e})\}$ are two sequence of Φ such that $lim\ sup_{n\to\infty}\|S_n(\hat{e})\| \le a$, $lim\ sup_{n\to\infty}\|\zeta_n(\hat{e})\| \le a$ and $lim_{n\to\infty}\|t_nS_n(\hat{e}) + (1-t_n)\zeta_n(\hat{e})\| = a$ hold for some $a \ge 0$ .Then $lim_{n\to\infty}\|S_n(\hat{e})\} - \zeta_n(\hat{e})\| = 0$*

2. Main result

In this section, we have a convergence random fixed point**.**

Lemma 2.1: *Let $B : \Psi \times \Upsilon \to \Pi(\Upsilon)$be to multivalued random operator and $\Pi_B(\mathfrak{v}(\hat{e})) = \{\zeta(\hat{e}) \in B(\hat{e}, \mathfrak{v}(\hat{e})) = d(\mathfrak{v}(\hat{e}), B(\hat{e}, \mathfrak{v}(\hat{e}))\}$ Then the following are equivalent : (1)- $\mathfrak{v}(\hat{e}) \in F(B)$ (2)- $\Pi_B\big(\mathfrak{v}(\hat{e})\big) = \{\mathfrak{v}(\hat{e})\}$ (3)-$\mathfrak{v}(\hat{e}) \in F(\Pi_B)$ Moreover, $F(B) = F(\Pi_B)$*

***Proof**:* (1) → (2) since $\mathfrak{v}(\hat{e}) \in F(B) \Longrightarrow d(\mathfrak{v}(\hat{e}), B(\mathfrak{v}(\hat{e})) = 0$ for any $y \in \Pi_B\big(\mathfrak{v}(\hat{e})\big) = \|\mathfrak{v}(\hat{e}) - \zeta(\hat{e})\| = d(\mathfrak{v}(\hat{e}), B(\mathfrak{v}(\hat{e})) = 0 \Longrightarrow \mathfrak{v}(\hat{e}) = \zeta(\hat{e})$. That is $\Pi_B\big(\mathfrak{v}(\hat{e})\big) = \{\mathfrak{v}(\hat{e})\}$. (2)→ (3) Since $\mathfrak{v}(\hat{e}) = \zeta(\hat{e})$For any $\zeta(\hat{e}) \in \Pi_B\big(\mathfrak{v}(\hat{e})\big) \Longrightarrow \mathfrak{v}(\hat{e}) \in \Pi_B\big(\mathfrak{v}(\hat{e})\big) \Longrightarrow \mathfrak{v}(\hat{e}) \in F(\Pi_B)$. (3)→(1) since $\mathfrak{v}(\hat{e}) \in \Pi_B\big(\mathfrak{v}(\hat{e})\big) \Longrightarrow d(\mathfrak{v}(\hat{e}), B\big(\mathfrak{v}(\hat{e})\big) = 0 \Longrightarrow \mathfrak{v}(\hat{e}) \in B\big(\mathfrak{v}(\hat{e})\big)$ then $\mathfrak{v}(\hat{e}) \in F(B)$

Theorem 2.2: *Let Φ be a normed space and Υ a nonempty convex closed subset of Φ , let $B : \Psi \times \Upsilon \to \Pi(\Upsilon)$ be a multivalued random operator where$F(B) \neq \emptyset$ and Π_B is a nonexpansive random operator, let $\{ S_n(\hat{e})\}$ be defined by $S_1(\hat{e}) \epsilon \Upsilon S_{n+1}(\hat{e}) = (1-\lambda)\zeta_n(\hat{e}) + \lambda\varsigma_n(\hat{e}), \eta_n(\hat{e}) = (1-\mu)S_n(\hat{e}) + \mu\zeta_n(\hat{e})$, $n \epsilon N$ where $\zeta_n \in \Pi_B\big(S_n(\hat{e})\big), \varsigma_n(\hat{e}) \in \Pi_B(\eta_n(\hat{e}))$ and $0 < \lambda, \mu < 1$.(2.1) .Then $lim_{n\to\infty}\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\|$ exists for all $\mathfrak{v}(\hat{e}) \in F(B)$.*

Proof: Let $\mathfrak{v}(\hat{e}) \in F(B)$ thus $\mathfrak{v}(\hat{e}) \in B(S(\hat{e}))$by lemma 2.1 follows from (2.1) that

$$\|S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})\| = \|(1-\lambda)\zeta_n(\hat{e}) + \lambda\varsigma_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \le (1-\lambda)\|\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| + \lambda\|\varsigma_n(\hat{e}) - \mathfrak{v}(\hat{e})\|(2.1) \le (1-\lambda)H\big(\Pi_B\big(S_n(\hat{e})\big), \Pi_B\big(\mathfrak{v}(\hat{e})\big) + \lambda H\big(\Pi_B\big(\eta_n(\hat{e})\big), \Pi_B\ (\mathfrak{v}(\hat{e})\big) \le (1-\lambda)\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| + \lambda\|\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})\|.$$

$$\text{Bu}\|\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| = \|(1-\mu)S_n(\hat{e}) + \mu\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \le (1-\mu)\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| + \mu\|\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \le$$

$$(1-\mu)\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\|\mu H\left(\Pi_B\big(S_n(\hat{e})\big), \Pi_B\big(\mathfrak{v}(\hat{e})\big)\right)\ (2.2)\ \le (1-\mu)\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| + \mu\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| = \|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\|$$

Thus$\|S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})\| \le (1-\lambda)\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| + \lambda\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| = \|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\|$and $\lim_{n\to\infty}\|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\|$ exists for each $\mathfrak{v}(\hat{e}) \in F(B)$.

Theorem 2.3: *Let Φ be a uniformly convex, separable ,Opial's condition and Banach space and Φ and Υ.B, F(B) $\neq \emptyset$, $\Pi_B$satisfytheorem2.2 and $\{S_n(\hat{e})\}$ as*

defined in (2.1). Then such that F(B) ≠ ∅. Let $I - \Pi_B$ satisfy definition1.2 with zero then $\{S_n(\hat{e})\}$ converges a point in F(B) also weakly.

Proof: From theorem2.2, $\lim_{n\to\infty} ||S_n(\hat{e}) - \mathfrak{v}(\hat{e})||$ exists for each $\mathfrak{v}(\hat{e}) \in F(B)$. Let $\lim_{n\to\infty} ||S_n(\hat{e}) - \mathfrak{v}(\hat{e})|| = c$for some $c \geq 0$. Since lim $\sup_{n\to\infty} ||\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq$ lim $\sup_{n\to\infty} H\left(\Pi_B\left(S_n(\hat{e})\right), \Pi_B\left(\mathfrak{v}(\hat{e})\right)\right) \leq$ lim $\sup_{n\to\infty} \|S_n(\hat{e}) - \mathfrak{v}(\hat{e})\| = c$,so $\lim_{n\to\infty} \sup \|\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \leq c$, (2.3) Similarly, $\lim_{n\to\infty} \sup \|\varsigma_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \leq c$,Applying Lemma 1.3, we get$\lim_{n\to\infty} ||\zeta_n(\hat{e}) - \varsigma_n(\hat{e})|| = 0$.by lim sup of (2.2), then $\lim_{n\to\infty} \sup \|\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})\| \leq c$ 2.4,Also$||S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})|| = ||(1 - \lambda)\,\zeta_n(\hat{e}) + \lambda\varsigma_n(\hat{e}) - \mathfrak{v}(\hat{e})|| = ||(\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})) + \lambda(\varsigma_n(\hat{e}) - \zeta_n(\hat{e}))|| \leq ||\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})|| + ||\zeta_n(\hat{e}) - \varsigma_n(\hat{e})||$ implies that $c \leq liminf||\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})||$.(2.5) Combining (2.3) and (2.5), we have$\lim_{n\to\infty} ||\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})|| = c$.Thus$||\zeta_n(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq ||\zeta_n(\hat{e}) - \varsigma_n(\hat{e})|| + ||\varsigma_n(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq ||\zeta_n(\hat{e}) - \varsigma_n(\hat{e})|| + H(\Pi_B(\eta_n(\hat{e})), \Pi_B(\mathfrak{v}(\hat{e}))) \leq ||\zeta_n(\hat{e}) - \varsigma_n(\hat{e})|| + ||\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})||$gives $c \leq liminf||\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})||$(2.6) and, in turn, by (2.4), we have $\lim_{n\to\infty} ||\eta_n(\hat{e}) - \mathfrak{v}(\hat{e})|| = c$.By lemma 1.3 once again $\lim_{n\to\infty} ||S_n(\hat{e}) - \zeta_n(\hat{e})|| = 0$. (2.7)Since $d\left(S_n(\hat{e}), B\left(S_n(\hat{e})\right)\right) \leq ||S_n(\hat{e}) - \zeta_n(\hat{e})||$, we have$\lim_{n\to\infty} d\left(S_n(\hat{e}), B\left(S_n(\hat{e})\right)\right) = 0$.

Let $\omega_1(\hat{e})$ and $\omega_2(\hat{e})$ be weak limits of $\{S_{ni}(\hat{e})\}$and $\left\{S_{nj}(\hat{e})\right\} \subseteq \{S_n(\hat{e})\}$, respectively. By (2.7), there exists $\zeta_n(\hat{e}) \in B\left(S_n(\hat{e})\right)$ such that $\lim_{n\to\infty}\left|S_n(\hat{e}) - \zeta_n(\hat{e})\right| = 0$. Since $I - \Pi_B$ is demiclosed at0, then$\omega_1(\hat{e}) \in F(\Pi_B) = F(B)$.we show that$\omega_2(\hat{e}) \in F(B)$.Next, suppose that $\omega_1(\hat{e}) \neq \omega_2(\hat{e})$, by Opial's condition, we have $\lim_{n\to\infty}\left|S_n(\hat{e}) - \omega_1(\hat{e})\right| = \lim_{n_i\to\infty}\left|S_{ni}(\hat{e}) - \omega_1(\hat{e})\right| < \lim_{n_i\to\infty}\left|S_{ni}(\hat{e}) - \omega_2(\hat{e})\right| = \lim_{n\to\infty}\left|S_n(\hat{e}) - \omega_2(\hat{e})\right| = \lim_{n_j\to\infty}\left|S_{nj}(\hat{e}) - \omega_2(\hat{e})\right| < \lim_{n_j\to\infty}\left|S_{nj}(\hat{e}) - \omega_1(\hat{e})\right| = \lim_{n\to\infty}\left|S_n(\hat{e}) - \omega_1(\hat{e})\right|$

Which is a contradiction. Then $\{S_n(\hat{e})\}$converges a point in F(B)also weakly.

Theorem 2.4: *Let Φ and Υ. B, F(B) ≠ ∅, Π_B satisfy theorem2.3 and $\{S_n(\hat{e})\}$ as defined in (2.1). Then$\{S_n(\hat{e})\}$converges to a point of $F(B)$also strongly if and only if $liminf_{n\to\infty} d(S_n(\hat{e}), F(B)) = 0$.*

Proof: Suppose that $liminf_{n\to\infty} d\left(S_n(\hat{e}), F(B)\right) = 0$.Bytheorem3, now $||S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq ||S_n(\hat{e}) - \mathfrak{v}(\hat{e})||$, then $d(S_{n+1}(\hat{e}), F(B)) \leq d(S_n(\hat{e}), F(B))$. Then $\lim_{n\to\infty} d\left(S_n(\hat{e}), F(B)\right)$ exists, $liminf_{n\to\infty} d(S_n(\hat{e}), F(B)) = 0$. Then $lim_{n\to\infty} d(S_n(\hat{e}), F(B)) = 0$. We show that $\{S_n(\hat{e})\}$is a Cauchy sequence in Υ. Let $\varepsilon > 0$.Since $lim_{n\to\infty} d(S_n(\hat{e}), F(B)) = 0.$, $n \geq n_0$, we have $d\left(S_n(\hat{e}), F(B)\right) < \frac{\varepsilon}{4}$.In particular, $inf\left\{\left|\left|S_{n_0}(\hat{e}) - \mathfrak{v}(\hat{e})\right|\right| : \mathfrak{v}(\hat{e}) \in F(B)\right\} < \frac{\varepsilon}{4}$. There must exist a $\mathfrak{v}^*(\hat{e}) \in F(B)$ such that $\left|\left|S_{n_0}(\hat{e}) - \mathfrak{v}^*(\hat{e})\right|\right| < \frac{\varepsilon}{2}$., n ≥ n_0, then

$||S_{n+m}(\hat{e}) - S_n(\hat{e})|| \leq ||S_{n+m}(\hat{e}) - \mathfrak{v}^*(\hat{e})|| + ||S_n(\hat{e}) - \mathfrak{v}^*(\hat{e})|| \leq 2||S_{n_0}(\hat{e}) - \mathfrak{v}^*(\hat{e})|| < 2\left(\frac{\varepsilon}{2}\right) = \varepsilon$. Then $\{S_n(\hat{e})\}$ is a Cauchy sequence in Υ of Φ, so converge in Υ. Let $\lim_{n\to\infty} S_n(\hat{e}) = \mathrm{q}(\hat{e})$. Now $d\big(\mathrm{q}(\hat{e}), \Pi_B \mathrm{q}(\hat{e})\big) \leq ||S_n(\hat{e}) - \mathrm{q}(\hat{e})|| + d(S_n(\hat{e}), \Pi_B S_n(\hat{e})) + H\big(\Pi_B(S_n(\hat{e})), \Pi_B(\mathrm{q}(\hat{e})) \leq ||S_n(\hat{e}) - \mathrm{q}(\hat{e})|| + ||S_n(\hat{e}) - \zeta_n(\hat{e})|| + ||S_n(\hat{e}) - \mathrm{q}(\hat{e})|| \to 0\ as n \to \infty$ then $d(\mathrm{q}(\hat{e}), \Pi_B(\mathrm{q}(\hat{e})) = 0$. By Π_B is a nonexpansive mapping so. Then, $\mathrm{q}(\hat{e}) \in F(\Pi_B) = F(B)$.

Theorem 2.5: *In theorem2.4 if satisfy if a* $\mho : [0,\infty) \to [0,\infty)$*non decreasing,* $\mho(0) = 0, \mho(c) > 0$ *for* $\forall\, c \in (0,\infty)$*and* $d(\mathfrak{v}(\hat{e}), B(\mathfrak{v}(\hat{e})) \geq \mho(d(\mathfrak{v}(\hat{e}), F(B))$ *for each* $\mathfrak{v}(\hat{e}) \in \Upsilon$*then* $\{S_n(\varrho)\}$*converges to the point of* $F(B)$ *also strongly*.

Proof*:* By theorem2.4 and, $\lim_{n\to\infty} ||S_n(\hat{e}) - \mathfrak{v}(\hat{e})||$ exists for all $\mathfrak{v}(\hat{e}) \in F(B)$. Suppose $c > 0$. Now $||S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq ||S_n(a) - \mathfrak{v}(\hat{e}))||$ then $\inf_{\mathfrak{v}(\hat{e})\in F(B)} ||S_{n+1}(\hat{e}) - \mathfrak{v}(\hat{e})|| \leq \inf_{\mathfrak{v}(\hat{e})\in F(B)} ||S_n(\hat{e}) - \mathfrak{v}(\hat{e})||$ And $d(S_{n+1}(\hat{e}), F(B)) \leq d(S_n(\hat{e}), F(B))$ and so $\lim_{n\to\infty} d(S_n(\hat{e}), F(B))$ exists By and, $\lim_{n\to\infty} \mho(\, d(S_n(\hat{e}), F(B))) = 0$. Since$\mho$ is a non-decreasing function and $\mho(0) = 0$, it follows that $\lim_{n\to\infty} d(S_n(\hat{e}), F(B)) = 0$

3. Unresolved Issue

Examine the fixed point of the results presented in paper [7]–[16] when a multivalued nonexpansive random operator is used.

4. Conclusions

Fixed points under multivalued random operators have recently emerged. We first look for iterations that are sensitive to non-expansive random operators.

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