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Enforcement for the groups SL(2,125) and SL(2,3125)

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Abstract

The group for the multiplication of closets is the set G | N of all closets of N in G, if G is a group and N is a normal subgroup of G. The term "G by N factor group" describes this set. In the quotient group G | N, N is the identity element. In this paper, we procure $K(SL(2,125))$ and $K(SL(2,3125))$ from the character table of rational representations for each group.

Subject Classification: *05E15, 05E15.*

Keywords: *Diametrical matrix, Rational character table, Factor group, Character table, Representation.*

1. Introduction

Let N represent the group G's typical subgroup. N is normal in G, hence. Each right closet will therefore match its counterpart left closet. Therefore, there is no difference between the right and left closets; we will just refer to them as closets. Let G|N be the collection of all closets of N in G, i.e. $G|N=\{Na: a\in G\}$.

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Therefore, the collection of all closets that make up a normal subgroup constitutes a group about the composition of the multiplication of complexes [1].

Investigator in [2] define the representation of the group, The group of all matrices of determinant 1 is $\mathcal{SL}(n,\mathrm{F})$, [3] and [4], by enforce the thought in [5] and [6] we gain $\mathcal{K}(\mathcal{SL}(2,125))$ and $\mathcal{K}(\mathcal{SL}(2,3125))$.

2. The Major Concepts

Theorem 2.1: [3] *For any c $\wp$ –group G,* $\mathcal{K}(G) = \mathbb{Z}_{\wp}$ *and* $\mathcal{K}(G) = \bigoplus_{i=1}^{n} \mathbb{Z}_{p^i}$.

Theorem 2.3: [3] $|\mathcal{SL}(2,q^n)| = q^n(q^{2n} - 1)$.

3. The Consequence

The investigator in [6] and [8] research the character table of rational representations of $\mathcal{SL}(2,\wp^k)$. We enforce that for $\mathcal{SL}(2,125)$ and $\mathcal{SL}(2,3125)$.

3.1 *Consequence for $\mathcal{SL}(2,125)$*

$|\mathcal{SL}(2,125)| = 1953000$. e and e' are divisors of 124 which are 1,2,4,31,62, and 124, such that $e, e' < 62$, so $e = e' = 1,2,4$, and 31.

f and f' are divisors of 126 which are 1,2,3,6,7,9,14,18,21,42,63, and 126, such that $f, f' < 63$, so $f = f' = 1,2,3,6,7,9,14,18,21$, and 42, $\varepsilon = 1$.

The conjugacy classes are 1, z, $c = d$, $zc = zd$, a, a^2, a^4, a^{31}, b, b^2, b^3, b^6, b^7, b^9, b^{14}, b^{18}, b^{21} and b^{42}.

The characters are 1_G, ψ, χ_1, χ_2, χ_4, χ_{31}, θ_1, θ_2, θ_3, θ_6, θ_7, θ_9, θ_{14}, θ_{18}, θ_{21}, θ_{42}, ξ, and η.

$A(1) = \frac{1}{2}\Phi\left(\frac{124}{1}\right) = \frac{1}{2}(60) = 30$, $B(1) = 2$, $\tau_1(1,1) = \frac{\Phi(124)}{\Phi(124)}\,\mu(124) = 0$, $\tau_1(1,2) = 2$, $\tau_1(1,4) = -2$, $\tau_1(1,31) = 0$, $B(2) = 1$, $A(2) = \frac{1}{2}\Phi\left(\frac{124}{2}\right) = 15$, $\tau_1(2,1) = \frac{\Phi(62)}{\Phi(62)}\,\mu(62) = 1$, $\tau_1(2,2) = \frac{\Phi(62)}{\Phi(31)}\,\mu(31) = -1$, $\tau_1(2,4) = \frac{\Phi(62)}{\Phi(16)}\,\mu(16) = 0$, $\tau_1(2,31) = \frac{\Phi(62)}{\Phi(2)}\,\mu(2) = -1$, $A(4) = \frac{1}{2}\Phi\left(\frac{124}{4}\right) = 15$, $B(4) = 1$, $\tau_1(4,1) = \frac{\Phi(31)}{\Phi(31)}\,\mu(31) = -1$, $\tau_1(4,2) = \frac{\Phi(31)}{\Phi(15.5)}\,\mu(15.5) = 0$, $\tau_1(4,4) = \frac{\Phi(31)}{\Phi(8)}\,\mu(8) = 0$, $\tau_1(4,31) = \frac{\Phi(31)}{\Phi(1)}\,\mu(1) = 1$, $A(31) = \frac{1}{2}\Phi\left(\frac{124}{31}\right) = 2$, $B(31) = 2$, $\tau_1(31,1) = \frac{\Phi(4)}{\Phi(4)}\,\mu(4) = 0$, $C(1) = \frac{1}{2}\Phi\left(\frac{126}{1}\right) = \frac{1}{2}\Phi(126) = \frac{1}{2}(36) = 18$, $E(125) = 2$, $\tau_2(1,1)\,\frac{\Phi(126)}{\Phi(126)}\,\mu(126) = 0$, $\tau_2(1,2) = \frac{\Phi(126)}{\Phi(36)}\,\mu(63) = 0$, $\tau_2(1,3) = \frac{\Phi(126)}{\Phi(42)}\,\mu(42) = -3$, $\tau_2(1,6) = \frac{\Phi(126)}{\Phi(21)}\,\mu(21) = 3$, $\tau_2(1,7) = \frac{\Phi(126)}{\Phi(18)}\,\mu(18) = 0$, $\tau_2(1,9) = \frac{\Phi(126)}{\Phi(14)}\,\mu(14) = 6$, $\tau_2(1,14) = \frac{\Phi(126)}{\Phi(9)}\,\mu(9) = 0$, $\tau_2(1,18) = \frac{\Phi(126)}{\Phi(7)}\,\mu(7) = -3$, $\tau_2(1,21) = \frac{\Phi(126)}{\Phi(6)}\,\mu(1) = 18$, $\tau_2(1,42) = \frac{\Phi(126)}{\Phi(3)}\,\mu(-1) = -18$, $C(2) = \frac{1}{2}\Phi\left(\frac{126}{2}\right) = 18$, $\tau_2(2,1) = \frac{\Phi(63)}{\Phi(63)}\,\mu(63) = 0$, $\tau_2(2,2) =$

$\frac{\Phi(63)}{\Phi(32)}\,\mu(32) = 0$, $C(3) = \frac{1}{2}\Phi\left(\frac{126}{3}\right) = 6$, $\tau_2(3,1) = \frac{\Phi(42)}{\Phi(42)}\,\mu(42) = -1$, $\tau_2(3,2) = \frac{\Phi(42)}{\Phi(21)}\,\mu(21) = 1$, $\tau_2(3,3) = \frac{\Phi(42)}{\Phi(14)}\,\mu(14) = 2$, $\tau_2(3,21) = \frac{\Phi(42)}{\Phi(2)}\,\mu(2) = -12$, $\tau_2(3,42) = \frac{\Phi(42)}{\Phi(6)}\,\mu(6) = 6$, $C(6) = \frac{1}{2}\Phi\left(\frac{126}{6}\right) = 6$, $\tau_2(6,1) = \frac{\Phi(21)}{\Phi(21)}\,\mu(21) = 1$, $\tau_2(6,2) = \frac{\Phi(21)}{\Phi(10)}\,\mu(10) = 3$, $\tau_2(6,3) = \frac{\Phi(21)}{\Phi(7)}\,\mu(7) = -2$, $\tau_2(6,6) = \frac{\Phi(21)}{\Phi(4)}\,\mu(4) = 0$, $\tau_2(6,7) = \frac{\Phi(21)}{\Phi(3)}\,\mu(3) = -6$, $\tau_2(6,9) = \frac{\Phi(21)}{\Phi(2)}\,\mu(2) = -12$, $\tau_2(6,14) = \frac{\Phi(21)}{\Phi(2)}\,\mu(2) = -12$, $\tau_2(6,18) = \frac{\Phi(21)}{\Phi(1)}\,\mu(1) = 12$, $\tau_2(6,21) = \frac{\Phi(21)}{\Phi(1)}\,\mu(1) = 12$, $\tau_2(6,42) = \frac{\Phi(21)}{\Phi(1)}\,\mu(1) = 12$, $C(7) = \frac{1}{2}\Phi(18) = \frac{1}{2}(6) = 3$, $\tau_2(7,1) = \frac{\Phi(18)}{\Phi(18)}\,\mu(18) = 0$, $\tau_2(7,2) = \frac{\Phi(18)}{\Phi(9)}\,\mu(9) = 0$, $\tau_2(7,3) = \frac{\Phi(18)}{\Phi(6)}\,\mu(6) = 3$, $\tau_2(7,6) = \frac{\Phi(18)}{\Phi(3)}\,\mu(3) = -3$, $\tau_2(7,7) = \frac{\Phi(18)}{\Phi(3)}\,\mu(3) = -3$, $\tau_2(7,9) == \frac{\Phi(18)}{\Phi(2)}\,\mu(2) = -6$, $\tau_2(7,14) = \frac{\Phi(18)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(7,18) = \frac{\Phi(18)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(7,21) = \frac{\Phi(18)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(7,42) = \frac{\Phi(18)}{\Phi(1)}\,\mu(1) = 6$, $C(9) = \frac{1}{2}\Phi\left(\frac{126}{9}\right) = 3$, $\tau_2(9,1) = \frac{\Phi(14)}{\Phi(14)}\,\mu(14) = 6$, $\tau_2(9,2) = \frac{\Phi(14)}{\Phi(7)}\,\mu(7) = -6$, $\tau_2(9,3) = \frac{\Phi(14)}{\Phi(3)}\,\mu(3) = -3$, $\tau_2(9,6) = \frac{\Phi(14)}{\Phi(2)}\,\mu(2) = -6$, $\tau_2(9,7) = \frac{\Phi(14)}{\Phi(2)}\,\mu(2) = -6$, $\tau_2(9,9) = \frac{\Phi(14)}{\Phi(2)}\,\mu(2) = -6$, $\tau_2(9,14) = \frac{\Phi(14)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(9,18) = \frac{\Phi(14)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(9,21) = \frac{\Phi(14)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(9,42) = \frac{\Phi(14)}{\Phi(1)}\,\mu(1) = 6$, $C(14) = \frac{1}{2}\Phi(9) = 3$, $\tau_2(14,1) = \frac{\Phi(9)}{\Phi(9)}\,\mu(9) = 0$, $\tau_2(14,2) = \frac{\Phi(9)}{\Phi(4)}\,\mu(9) = 0$, $\tau_2(14,3) = \frac{\Phi(9)}{\Phi(3)}\,\mu(3) = -3$, $\tau_2(14,6) = \frac{\Phi(9)}{\Phi(2)}\,\mu(2) = -6$, $\tau_2(14,7) = \frac{\Phi(9)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(14,9) = \frac{\Phi(9)}{\Phi(1)}\,\mu(1) = 6$, $\tau_2(18,3) = \frac{\Phi(7)}{\Phi(2)}\,\mu(-1) = -6$, $\tau_2(18,6) = \frac{\Phi(7)}{\Phi(1)}\,\mu(1) = -6$, $\tau_2(42,14) = \frac{\Phi(3)}{\Phi(1)}\,\mu(1) = 2$, $\tau_2(42,18) = \frac{\Phi(3)}{\Phi(1)}\,\mu(1) = \frac{2}{1}(1) = 2$, $\tau_2(42,21) = \frac{\Phi(3)}{\Phi(1)}\,\mu(1) = 2$, $\tau_2(42,42) = \frac{\Phi(3)}{\Phi(1)}\,\mu(1) = 2$, $E(125) = 2$.

The character table of rational representations is:

C_g	**1**	z	$c = d$	$zc = zd$	a	a^2	a^4	a^{31}
$\lvert C_g \rvert$	**1**	**1**	**7812**	**7812**	**15750**	**15750**	**15750**	**15750**
$\lvert C_G(g)\rvert$	**1953000**	**1953000**	**250**	**250**	**124**	**124**	**124**	**124**
1_G	1	1	1	1	1	1	1	1
ψ	125	125	0	0	1	1	1	1
χ_1	7560	-7560	60	-60	0	4	-4	0
χ_2	1890	1890	15	15	1	-1	0	-30
χ_4	1890	1890	15	15	-1	0	0	30

Contd...

χ_{31}	252	-252	2	-2	0	-4	4	4
θ_1	4464	-4464	-36	36	0	0	0	0
θ_2	2232	2232	-81	-81	0	0	0	0
θ_3	1488	-1488	-12	12	0	0	0	0
θ_6	744	744	-6	-6	0	0	0	0
θ_7	744	-744	-6	6	0	0	0	0
θ_9	744	-744	-6	6	0	0	0	0
θ_{14}	372	372	-3	-3	0	0	0	0
θ_{18}	372	372	-3	-3	0	0	0	0
θ_{21}	248	-248	-2	2	0	0	0	0
θ_{42}	124	124	-1	-1	0	0	0	0
ξ	126	126	1	1	-2	2	2	-2
η	248	-248	-1	1	0	0	0	0

C_g	b	b^2	b^3	b^6	b^7	b^9	b^{14}	b^{18}	b^{21}	b^{42}
$\|C_g\|$	**15500**	**15500**	**15500**	**15500**	**15500**	**15500**	**15500**	**15500**	**15500**	**15500**
$\|C_G(g)\|$	**126**	**126**	**126**	**126**	**126**	**126**	**126**	**126**	**126**	**126**
1_G	1	1	1	1	1	1	1	1	1	1
ψ	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_1	0	0	0	0	0	0	0	0	0	0
χ_2	0	0	0	0	0	0	0	0	0	0
χ_4	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0
θ_1	0	0	6	-6	0	-12	0	12	-36	36
θ_2	0	0	-3	-9	0	6	9	0	18	36
θ_3	2	-2	-4	4	-12	6	12	24	24	-24
θ_6	-1	-3	2	0	6	12	12	-12	-12	-12
θ_7	0	0	-6	6	6	12	-12	-12	-12	-12
θ_9	-2	2	6	12	12	12	-12	-12	-12	-12
θ_{14}	0	0	3	6	-6	-6	-6	-6	-6	-6
θ_{18}	1	0	6	-6	-6	-6	-6	-6	-6	-6
θ_{21}	-2	2	4	-4	-4	-4	-4	-4	-4	-4
θ_{42}	1	2	-2	-2	-2	-2	-2	-2	-2	-2
ξ	0	0	0	0	0	0	0	0	0	0
η	4	-4	4	-4	4	4	-4	-4	4	-4

The diametrical matrix of the beforehand table is

$$\left(\begin{array}{cccccccccccccccccc}
1953000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 651000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)$$

Thus, $K(\mathcal{SL}(2,125)) = \oplus \mathcal{Z}_{p}$,

$p = 1953000, 651000, 7, 7, 5, 5, 5, 5, 3, 3, 3, 3, 2, 2, 2, 2, 1, 1.$

3.2 *Consequence for $\mathcal{SL}(2,3125)$*

$|\mathcal{SL}(2,3125)| = 30517575000$. e and e' are divisors of 3124 which are 1,2,4,11,22,44,71,142,284,781, 1562, and 3124, such that $e, e' < 1562$, so $e = e'$=1,2,4,11,22,44,71,142,284, and 781.

f and f' are divisors of 3126 which are 1,2,3,6,521,1042,1563, and 3126, such that $f, f' < 1563$, so $f = f'$ = 1,2,3,6,521, and 1042, $\varepsilon = 1$.

The conjugacy classes are 1, z, $c = d$, $zc = zd$, a, a^2, a^4, a^{11}, a^{22}, a^{44}, a^{71}, a^{142}, a^{284}, a^{781}, b, b^2, b^3, b^6, b^{521} and b^{1042}.

The characters are 1_G, ψ, χ_1, χ_2, χ_4, χ_{11}, χ_{22}, χ_{44}, χ_{71}, χ_{142}, χ_{284}, χ_{781}, θ_1, θ_2, θ_3, θ_6, θ_{521}, θ_{1042}, ξ, and η.

As the same idea in section (3.1), $K(\mathcal{SL}(2,3125)) = \oplus \mathcal{Z}_{p}$,

p =30517575000, 6103515000, 5, 5, 5, 5, 5, 5, 11, 11, 71, 71, 521, 3, 2, 2, 2, 4, 4, 1.

4. Conclusion

From our work, we can get that the first number in the diagonal of the results matrix is equal to the group's order. In contrast, the second number is equal to the group's order divided by 5. Finally, the reminder numbers are equal to the factors of the order of the centralizer of the conjugacy classes except for the order of the conjugacy class 1 after removing the factors of the group's order divided by five and completing the remaining number by 1.

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