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Developing the pentagonal membership function to estimate the fuzzy parameters of the exponential rayleigh model

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Abstract

In statistics, estimation procedures are applied to estimate the parameters of any model or distribution. In the present paper, an advanced procedure has been used to find the fuzzy parameters for one of the mixed models, the Exponential Rayleigh model (ERM). After deriving the classical estimation technique, the ordinary least square estimation method (OLSEM), the numerical values calculated using Newton-Raphson (NR) method, and the fuzzy interval estimation were applied to create the lower and the upper limits of the pentagonal membership function. Then, the ranking function was formed. Following that, the results were compared depending on the estimated values for survival function before and after the fuzzy with actual data via utilizing the mean squared error technique to manifest which result is the best.

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1. Introduction

The Exponential Rayleigh model (ERM) is a probability distribution often used to model data with a long right tail and a short left tail. Two scale parameters define the model. The scale parameters control the spread of the distribution. The fuzzy set theory has taken a significant part in the statistical fields discovered by Zadeh (in 1965 and 1968), who was the initial for the form probability measures in the fuzzy sets with respect to this becoming right likely incidents [2].

In this paper, estimating the parameters of the Exponential Rayleigh model using the ordinary least square estimation method (OLSEM) is provided by Newton-Raphson (NR) method. Then, confidence interval stimulation (CIE) determines the lower and upper intervals to find the fuzzy parameters. After that, employing proposed pentagonal membership function is employed, and the ranking function is created**.** Numerous researchers have introduced pentagonal fuzzy numbers with different kinds for clearing the nebulousness of the present issues, like [3] and [4]. Therefore, in this work, improving the structure membership function of pentagonal and determining the lower and upper bounds utilizing an approach are to transform uncertain parameters to ordinary parameters by employing the ranking function's concept, see [5]. The existence of fuzziness in most real-life problems is an objective fact. Many theories of reliability and models assume that the whole factors of the probability function of a lifetime are vigorous. Still, the truth is that the probability distribution factors can be denoted wrongly owing to personal judgment, human error, assessment, or unanticipated incidents, as the factors of lifetime distributions are fuzzy. So, the deal with the conventional survival (reliability) function may introduce problems for the reliability system.

Therefore, one can deal with a bigger survival definition than the conventional one. In real-world uses, it's essential to generalize the approaches of standard statistical assessment for fuzzy numbers. Fuzzy logic is a mathematical framework used to deal with uncertainty and imprecision. It allows for using linguistic variables to undertake a range of values rather than just discrete values. The fuzzy survival function is collected with fuzzy logic. In fuzzy survival analysis, the membership function's form and scale parameters are espoused as a certainty function to get the survival function's concrete data [6]. As the results in the present study, the estimated survival functions of this model before and after fuzzy from the lowest Mean Squared Error (MSE) were compared to know the supreme result. This paper has been prepared as follows: ERM with some properties about its functions is defined in Section 2. The discussion of the steps for (OLSEM) is given in Section 3. Section 4 illustrates certain basic definitions of the fuzzy set theory. Section 5 concentrates on the development of pentagonal fuzzy numbers. Section 6 concerns with the ranking function for converting the fuzzy parameters into crisp. Section 7 describes the real data. In Section 8, the numerical results are computed for comparing the MSE. Finally, Section 9 presented the conclusions.

2. Exponential Rayleigh Model (ERM) [7]

Some statistical properties of ERM are given in this section.

- The probability density function (p.d.f) of the ERM is:

$$f(x;\alpha,\beta) = \begin{cases} (\alpha+\beta x)e^{-\left(\alpha x+\frac{\beta}{2}x^2\right)}, x \geq 0; \alpha, \beta > 0 \\ 0, o.w \end{cases} \tag{1}$$

- The cumulative density function (c.d.f) of the ERM is:

$$F(x) = \int_0^x f(u)du = 1 - e^{-\left(\alpha x+\frac{\beta}{2}x^2\right)}, x \geq 0; \alpha, \beta > 0 \tag{2}$$

- Survival function $S(t)$ of the ERM is:

$$S(t;\alpha,\beta) = 1 - F(t) = e^{-(\alpha t+\frac{\beta}{2}t^2)}, t \geq 0; \alpha, \beta > 0 \tag{3}$$

- The moment about the origin is:

$$E(x^r) = e^{-\alpha}\frac{2^{\frac{r+n}{2}}}{\beta^{\frac{r+n}{2}}}\left[\frac{\alpha}{\sqrt{2\beta}}\Gamma\left(\frac{r+n+1}{2}\right) + \Gamma\left(\frac{r+n+2}{2}\right)\right], r = 1,2,3 \ldots \tag{4}$$

If one gets the first moment ($r = 1$), which is called the mean, then

$$E(x) = e^{-\alpha}\frac{2^{\frac{1+n}{2}}}{\beta^{\frac{1+n}{2}}}\left[\frac{\alpha}{\sqrt{2\beta}}\Gamma\left(\frac{n+2}{2}\right) + \Gamma\left(\frac{n+3}{2}\right)\right] \tag{5}$$

- The moment-generating function is:

$$M_x(t) = e^{-(\alpha-t)}\frac{2^{\frac{n}{2}}}{\beta^{\frac{n}{2}}}\left[\frac{\alpha}{\sqrt{2\beta}}\Gamma\left(\frac{n+1}{2}\right) + \Gamma\left(\frac{n+2}{2}\right)\right] \tag{6}$$

3. Ordinary Least Square Estimation Method (OLSEM)

The ordinary least square method is one of the most common procedures which are used to estimate the parameters α and β in the Exponential-Rayleigh model. The aim of (OLSEM) is to minimize the sum of the squared discrepancies between the noticed and the anticipated data.

By utilizing the c.d.f of ERM, one gets:

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n}[(F(x_i) - \hat{F}(x_i)]^2$$

Where, $F(x_i) = \frac{i-0.5}{n}$ is the empirical cumulative density function, and $\hat{F}(x_i) = 1 - e^{-\left(\alpha x_i+\frac{\beta}{2}x_i^2\right)}$.

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n}\left[\frac{i-0.5}{n} - (1 - e^{-\left(\alpha x_i+\frac{\beta}{2}x_i^2\right)})\right]^2$$

$$\sum_{i=1}^{n} \epsilon_i^2 = \sum_{i=1}^{n}\left[e^{-\left(\alpha x_i+\frac{\beta}{2}x_i^2\right)} - \frac{n-i+0.5}{n}\right]^2$$

By substituting $\sum_{i=1}^{n} \epsilon_i^2$ as $S(\alpha,\beta)$, then

$$S(\alpha,\beta) = \sum_{i=1}^{n}\left[e^{-\left(\alpha x_i+\frac{\beta}{2}x_i^2\right)} - \frac{n-i+0.5}{n}\right]^2 \tag{7}$$

One gets the partial derivatives from (7) with regard to (α,β) as the following:

$$\frac{\partial S(\alpha,\beta)}{\partial \alpha} = 2\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - 2\sum_{i=1}^{n} x_i e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{8}$$

$$\frac{\partial S(\alpha,\beta)}{\partial \beta} = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^2 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - \sum_{i=1}^{n} x_i^2 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{9}$$

Equations (8) and (9) are a system of non-linear equations that are complicated to solve. Therefore, one can assume:

$$f(\alpha) = 2\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - 2\sum_{i=1}^{n} x_i e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{10}$$

$$f(\beta) = \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^2 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - \sum_{i=1}^{n} x_i^2 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{11}$$

The non-linear equations system can be solved by employing the numerical analysis way. One of the common methods is Newton-Raphson employed for solving such equations as well as getting the estimated parameters depending on the Jacobian matrix (J_m).

$$\frac{\partial f(\alpha)}{\partial \alpha} = 4\sum_{i=1}^{n} x_i^2 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - 2\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^2 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{12}$$

$$\frac{\partial f(\alpha)}{\partial \beta} = 2\sum_{i=1}^{n} x_i^3 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^3 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{13}$$

$$\frac{\partial f(\beta)}{\partial \beta} = \sum_{i=1}^{n} x_i^4 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - \frac{1}{2}\sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^4 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{14}$$

$$\frac{\partial f(\beta)}{\partial \alpha} = 2\sum_{i=1}^{n} x_i^3 e^{-2\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} - \sum_{i=1}^{n} \frac{n-i+0.5}{n} x_i^3 e^{-\left(\alpha x_i + \frac{\beta}{2} x_i^2\right)} \tag{15}$$

Where, $J_m = \begin{bmatrix} \frac{\partial f(\alpha)}{\partial \alpha} & \frac{\partial f(\alpha)}{\partial \beta} \\ \frac{\partial f(\beta)}{\partial \alpha} & \frac{\partial f(\beta)}{\partial \beta} \end{bmatrix}$ is a square matrix that contains the partial derivatives respecting to (α, β),

One obtains the estimators for scale parameters $(\hat{\alpha}, \hat{\beta})$ of ERM from the equation:

$$\begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} = \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} - J^{-1} \begin{bmatrix} f(\alpha) \\ f(\beta) \end{bmatrix}, k = 0,1, \ldots\ldots, n \tag{16}$$

Where, the initial values (α_0, β_0) are assumed with the stop error term (ϵ) of (α, β) in the following formula:

$$\begin{bmatrix} \alpha_{k+1} \\ \beta_{k+1} \end{bmatrix} - \begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} = \begin{bmatrix} \epsilon_{k+1}(\alpha) \\ \epsilon_{k+1}(\beta) \end{bmatrix}, k = 0,1, \ldots\ldots, n \tag{17}$$

4. Fuzzy Set Theory

In this section, certain concepts in the fuzzy set theory are introduced.

- Let X represent a universe set (a classical set of an object), a fuzzy set $\tilde{A}$ in X is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) : X \to [0,1]\}$, where $\mu_{\tilde{A}}(x)$ is called the membership degree for the element x in $\tilde{A}$ [8].
- The crisp set of elements belonging to the fuzzy set $\tilde{A}$ whose membership degree of members isn't less than α is named the α-cut set and defined by $A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$ [8].

- The fuzzy numbers are the fuzzy set $\tilde{A}$ upon the real line R, satisfying the following conditions [9]:
  1. $\exists$ is at least a point $x_0 \in R$, such that $\mu_{\tilde{A}}\,(x_0) = 1$.
  2. $\mu_{\tilde{A}}(x)$ represents a piecewise continuous function.
  3. The fuzzy number is normal as well as convex.

5. Developing the Pentagonal Membership Function (DPMF)

The pentagonal membership function represents a type of fuzzy membership function that is shaped like a pentagon. It has five parameters: The lower bound, upper bound, lower left corner, upper right corner, and lower right corner. The minimum and maximum values the variable can undertake are the lower and upper bounds. Also, the lower left and upper right corners are the points where the membership function transitions are from 0 to 1 and 1 to 0, respectively.

The pentagonal membership function can be used for estimating the fuzzy parameters of the ERM. To do this, first define the linguistic variables for the scale parameters.

Assume that $\tilde{A}_{DPMF} = (a, b, c, d, e; r, w)$ be fuzzy numbers. r , and w are two weight functions, then the pentagonal membership function is defined as:

$$\mathcal{M}_{\tilde{A}_{DPMF}}(x) = \begin{cases} r\,(\dfrac{x-a}{b-a})\ a \le x < b \\ r + (w-r)(\dfrac{x-b}{c-b})\ b \le x < c \\ w\ x = c \\ r + (w-r)(\dfrac{d-x}{d-c})\ c < x \le d \\ r\,(\dfrac{e-x}{e-d})\ d < x \le e \\ 0\ x < a, x > e \end{cases}$$

Where, $0 < r < w < 1$, and $0 \le \lambda \le 1$. By using λ- cut, one gets:

- Let $\lambda = r\,\frac{x-a}{b-a} \rightarrow\ x = a + (\frac{b-a}{r})\,\lambda = inf_1\tilde{A}(\lambda)$
- Let $\lambda =\ r + (w-r)\,\frac{(x-b)}{(c-b)} \rightarrow x = b + (\frac{c-b}{w-r})\,(\lambda - r) = inf_2\tilde{A}(\lambda)$
- Let $\lambda = r + (w-r)\,\frac{(d-x)}{(d-c)} \rightarrow x = d - (\frac{d-c}{w-r})\,(\lambda - r) = sup_1\tilde{A}(\lambda)$
- Let $\lambda = r\,\frac{e-x}{e-d} \rightarrow\ x = e - (\frac{e-d}{r})\,\lambda = sup_2\tilde{A}(\lambda)$

$$A_\lambda = \begin{cases} a + \left(\frac{b-a}{r}\right)\lambda\ =\ inf_1\tilde{A}(\lambda)\ on\ [0, r\,] \\ b + \left(\frac{c-b}{w-r}\right)(\lambda - r) =\ inf_2\tilde{A}(\lambda)\ on\ [r, w] \\ d - \left(\frac{d-c}{w-r}\right)(\lambda - r) =\ sup_1\tilde{A}(\lambda)\ on\ [r, w] \\ e - \left(\frac{e-d}{r}\right)\lambda\ =\ sup_2\tilde{A}(\lambda)\ on\ [0, r\,] \end{cases} \quad (18)$$

Where, $0 < r < w < 1$, and $0 \le \lambda \le 1$. Then, the λ-cut of DPMF is defined as:

$$A_\lambda = \{x \in X: \mathcal{M}_{\tilde{A}_{DPMF}}(x) \geq \lambda\}$$

6. Ranking Function

Jain first introduced the ranking function in (1976). In 1981, Yager proposed some indices for ordering fuzzy numbers in the closed unit interval [0, 1]. It is denoted by F(R), which the maps R: F(R) → R being the fuzzy' numbers set defined upon an actual line.

Now, applying the proposed ranking function depending on DPMF gives:

$$R(\tilde{A}) = \frac{1}{4}\int_0^r inf_1\tilde{A}(\lambda)d\lambda + \frac{1}{4}\int_r^w inf_2\tilde{A}(\lambda)d\lambda + \frac{1}{4}\int_r^w sup_1\tilde{A}(\lambda)d\lambda + \frac{1}{4}\int_0^r sup_2\tilde{A}(\lambda)d\lambda$$

$$R(\tilde{A}) = \frac{1}{4}\int_0^r [a + \left(\frac{b-a}{r}\right)\lambda]\, d\lambda + \frac{1}{4}\int_r^w [b + \left(\frac{c-b}{w-r}\right)(\lambda - r)]\, d\lambda + \frac{1}{4}\int_r^w [\, d - \left(\frac{d-c}{w-r}\right)(\lambda - r)]d\lambda + \frac{1}{4}\int_0^r [\, e - \left(\frac{e-d}{r}\right)\lambda]\, d\lambda$$

$$R(\tilde{A}) = \frac{1}{4}\left[a\lambda + \left(\frac{b-a}{2r}\right)\lambda^2\right]\Big|_0^r + \frac{1}{4}\left[b\lambda + \left(\frac{c-b}{2(w-r)}\right)\lambda^2 - \frac{r(c-b)}{w-r}\lambda\right]\Big|_r^w + \frac{1}{4}\left[d\lambda + \left(\frac{c-d}{2(w-r)}\right)\lambda^2 - \frac{r(c-d)}{w-r}\lambda\right]\Big|_r^w + \frac{1}{4}\left[e\lambda + \left(\frac{d-e}{2r}\right)\lambda^2\right]\Big|_0^r$$

$$R(\tilde{A}) = \frac{1}{4}\left[\frac{(a+b)r^2}{2r}\right] + \frac{1}{4}\left[\frac{2b(w-r)-2r(c-b)+(c-b)(w+r)}{2}\right] + \frac{1}{4}\left[\frac{2d(w-r)-2r(c-d)+(c-d)(w+r)}{2}\right] + \frac{1}{4}\left[\frac{(d+e)r^2}{2r}\right]$$

$$R(\tilde{A}) = \frac{1}{4}\left[\frac{(a+b+d+e)r}{2}\right] + \frac{1}{4}\left[\frac{(b+c)(w-r)}{2}\right] + \frac{1}{4}\left[\frac{(c+d)(w-r)}{2}\right]$$

$$R(\tilde{A}) = \frac{1}{8}[(a+b+d+e)r] + \frac{1}{8}[(b+2c+d)(w-r)]$$

$$R(\tilde{A}) = \frac{1}{2}\left[\frac{r}{4}(a+b+d+e) + \frac{(w-r)}{4}(b+2c+d)\right]. \quad (19)$$

7. Description for Data

This paper depends on real data to apply for the present work as a practical part; the real data was collected from Al-Yarmouk Teaching Hospital in Baghdad-Iraq for the patients having a Myocardial Infarction (MI) for the period from 1/1/2022 to 10/31/2022. Throughout the collection of data, it is noted that the patient's number through such period is (468), where (414) patients stayed alive, and (54) patients are dead; that means this type of disease has a failure time. So, the data related to the mortality of this disease was selected based on the number of days that the patient stayed in the hospital until dead and taken as complete data which is set as follows:

t_i={1,2,1,2,2,5,1,1,2,4,9,1,6,3,3,3,11,2,2,2,4,4,2,1,2,2,9,1,3,1,5,4,2,9,2,11,2,2,3, 2,2,2,4,3,3,3,1,3,4,2,3,2,7,1}

Where, t_i is the set of days that patients stayed in the hospital until dead, and $i = 1,2, \dots, n$ (n= 54 is the number of mortality).

8. Numerical Results

In this section, the numerical estimated values were computed for the parameters $(\hat{\alpha}, \hat{\beta})$ of ERM via the subsequent steps:

1. Beyond deriving the OLSEM for estimating the parameters, the NR method was implemented by MATLAB programming with complete data and sample size (n = 54).
2. The initial values (α_0, β_0) were set to obtain the numerical values for $(\hat{\alpha}, \hat{\beta})$ from equation (16), and then the estimated values were found for $\hat{S}(t)$.
3. The CIE was applied for the points estimation $(\hat{\alpha}, \hat{\beta})$ to determine the lower and upper limits as the following:

   $\tilde{A} =$ [lower, lower left, lower right, upper right, upper]

   $$\hat{\alpha} \pm t_{(n-p,1-\alpha)}\sqrt{var(\hat{\alpha})} = [lower(\hat{\alpha}), upper(\hat{\alpha})]$$

   $$\hat{\beta} \pm t_{(n-p,1-\alpha)}\sqrt{var(\hat{\beta})} = [lower(\hat{\beta}), upper\ (\hat{\beta})]$$

   Where, p is the number of the ERM parameters, n represents the size of the sample, α represents the level of significance, $t_{(n-p,1-\alpha)}$ is the t-test value, and $var(\hat{\alpha})$,$var(\hat{\beta})$ are got from J_m.
4. The fuzzy parameters $(\tilde{\alpha}, \tilde{\beta})$ were generated depending on the DPMF as follows:

   $$\tilde{\alpha} = [lower(\hat{\alpha}), \bar{X} - S^2, \bar{X}, \bar{X} + S^2, upper(\hat{\alpha})]$$

   $$\tilde{\beta} = \left[lower(\hat{\beta}), \bar{X} - S^2, \bar{X}, \bar{X} + S^2, upper(\hat{\beta})\right]$$

   Where, $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$ and $S^2 = \frac{\sum_{i=1}^{n}((x_i - \bar{X})^2}{n}$
5. The five fuzzy parameters $(\tilde{\alpha}, \tilde{\beta})$ were substituted into the ranking function for getting the crisp parameters. Then, $S(t)$ was found after the fuzzy.
6. Now, the MSE for the survival functions was obtained before fuzzy $\hat{S}(t)$ and after fuzzy $S(t)$ by the following equation:

   $$MSE[S] = \sum_{i=1}^{n}\left[S(t_i) - \hat{S}(t_i)\right]^2/(n-p) \quad (20)$$

   Where, $S(t_i)$ got from step (5), and $\hat{S}(t_i)$ got from step (2).

   The following Tables (1, 2, and 3) list the results:

Table 1
The estimated values for the parameters of ERM

OLSEM	$\alpha_0 = 0.5, \beta_0 = 0.1$	$\alpha_0 = 0.02, \beta_0 = 0.01$	$\alpha_0 = 0.08, \beta_0 = 0.04$
$\hat{\alpha}$	0.1975960	0.1895494	0.1739184
$\hat{\beta}$	0.1228102	0.1941517	0.1566837

Table 2
MSE values for the $\widehat{S}(t)$ before fuzzy

$\alpha_0 = 0.5, \beta_0 = 0.1$	$\alpha_0 = 0.02, \beta_0 = 0.01$	$\alpha_0 = 0.08, \beta_0 = 0.04$
MSE	MSE	MSE
0.348519200	0.410816179	0.364796097

Table 3
MSE values for the $S(t)$ after fuzzy

$\alpha_0 = 0.5, \beta_0 = 0.1$			$\alpha_0 = 0.02, \beta_0 = 0.01$			$\alpha_0 = 0.08, \beta_0 = 0.04$		
r	w	MSE	r	w	MSE	r	w	MSE
0.1	0.2	0.2400945	0.1	0.2	0.2608702	0.1	0.2	0.2421284
0.2	0.3	0.1888888	0.2	0.3	0.2282683	0.2	0.3	0.1898873
0.3	0.4	0.1506025	0.3	0.4	0.1615080	0.3	0.4	0.1511391
0.4	0.5	0.1207377	0.4	0.5	0.1286149	0.4	0.5	0.1209686
0.5	0.6	0.0968118	0.5	0.6	0.1023586	0.5	0.6	0.0968125
0.6	0.7	0.0773374	0.6	0.7	0.0811225	0.6	0.7	0.0771724
0.7	0.8	0.0613497	0.7	0.8	0.0638366	0.7	0.8	0.0610796
0.8	0.9	0.0481711	0.8	0.9	0.0497346	0.8	0.9	0.0478547

- In Table (2), when ($\alpha_0 = 0.5, \beta_0 = 0.1$), the MSE is equal to (0.348519200), but in Table (3), the MSE is equal to (0.0481711), which is the minimum value.
- In Table (2), when ($\alpha_0 = 0.02, \beta_0 = 0.01$), the MSE is equal to (0.410816179), but in Table (3) the MSE is equal to (0.0497346), which is the least one.
- Also, in Table (2), the MSE is equal to (0.364796097) when ($\alpha_0 = 0.08, \beta_0 = 0.04$), but in Table (3), the MSE is equal to (0.0478547), which is the smallest value.

9. Conclusion

1. When comparing the calculated MSE values, we can see that the results after fuzzy is more efficient and accurate than before fuzzy.
2. From the comparison for MSE after fuzzy, one can know the well-estimated parameters for this model based on the proposed pentagonal membership function and ranking function, which are employed at in acceptable level to increase the survival function values.

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