

Computations for the groups $SL(2,81)$ and $SL(2,729)$

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Abstract

For any group G , we define G/H (read “ $G \bmod H$ ”) to be the set of left cosets of H in G and this set forms a group under the operation $(a)(bH) = abH$. The character table of rational representations study to gain the $K(SL(2,81))$ and $K(SL(2,729))$ in this work.

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1. Introduction

In group theory, a branch of abstract algebra, a character table is a two-dimensional table whose rows correspond to irreducible representations and whose columns correspond to conjugacy classes of group elements. The entries consist of characters and the traces of the matrices representing group elements of the column's class in the given row's group representation [1]. By stratifying

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the thought in [2-5], we gain $\mathcal{K}(\mathcal{SL}(2,81))$ and $\mathcal{K}(\mathcal{SL}(2,729))$. The group of all matrices of determinant 1 is $\mathcal{SL}(n, F)$, [6].

2. The Main Notion

Theorem 2.1: [5] $|\mathcal{SL}(2, q^n)| = q^n (q^{2n} - 1)$.

Theorem 2.3: [5] For any p -group G , $\mathcal{K}(G) = \mathbb{Z}_p$ and $\mathcal{K}(G) = \bigoplus_{i=1}^n \mathbb{Z}_{p^i}$.

3. The Outcomes

The searchers in [3] and [5] survey the character table of rational representations of $\mathcal{SL}(2, p^k)$. We stratify that for $\mathcal{SL}(2,81)$ and $\mathcal{SL}(2,729)$.

3.1 Outcomes for $\mathcal{SL}(2,729)$

$$|\mathcal{SL}(2,729)| = 387419760.$$

e and e' are divisors of 728 which are 1, 2, 4, 7, 8, 13, 14, 26, 28, 52, 56, 91, 104, 182, 364, and 728, such that $e, e' < 364$, so $e = e' = 1, 2, 4, 7, 8, 13, 14, 26, 28, 52, 56, 91, 104$, and 182. f and f' are divisors of 730 which are 1, 2, 5, 10, 73, 146, 365, and 730, such that $f, f' < 365$, so $f = f' = 1, 2, 5, 10, 73$, and 146, $\varepsilon = 1$.

The conjugacy classes are 1, z , c , zc , a , a^2 , a^4 , a^7 , a^8 , a^{13} , a^{14} , a^{26} , a^{28} , a^{52} , a^{56} , a^{91} , a^{104} , a^{182} , b , b^2 , b^5 , b^{10} , b^{73} and b^{146} .

C_g	1	z	$c = d$	$zc = zd$	a	a^2	a^4	a^7
$ C_g $	1	1	265720	265720	532170	532170	532170	532170
$ C_G(g) $	387419760	387419760	1458	1458	728	728	728	728
1_G	1	1	1	1	1	1	1	1
ψ	729	729	0	0	1	1	1	1
χ_1	210240	-210240	288	-288	0	0	-1152	0
χ_2	52560	52560	72	72	0	-144	144	0
χ_4	26280	26280	36	36	-36	36	0	216
χ_7	35040	-35040	48	-48	0	0	192	288
χ_8	26280	26280	36	36	36	0	216	-216
χ_{13}	17520	-17520	24	-24	0	0	96	0
χ_{14}	8760	8760	12	12	0	24	-24	-48
χ_{26}	4380	4380	6	6	0	12	-12	0
χ_{28}	4380	4380	6	6	6	-6	-12	0
χ_{52}	2190	2190	3	3	3	-3	0	-18
χ_{56}	4380	4380	6	6	-6	-12	-36	-72
χ_{91}	2920	-2920	4	-4	0	0	-16	16
χ_{104}	2190	2190	3	3	-3	0	-18	18
χ_{182}	730	730	1	1	0	-2	2	2
θ_1	209664	-209664	-288	288	0	0	0	0
θ_2	104832	104832	-144	-144	0	0	0	0
θ_5	52416	-52416	-72	72	0	0	0	0
θ_{10}	26208	26208	-36	-36	0	0	0	0

Contd...

θ_{73}	2912	-2912	-4	4	0	0	0	0
θ_{146}	1456	1456	-2	-2	0	0	0	0
ξ	730	730	1	1	-2	2	2	-2
η	1456	-1456	-1	1	0	0	0	0

The character table of rational representations is The diametrical matrix of the prior table is:

Cg	a 182	b 1	b 2	b 5	b 10	b 73	b 146
Cg	532170	530712	530712	530712	530712	530712	530712
CG(g)	728	730	730	730	730	730	730
1G	1	1	1	1	1	1	1
ψ	1	-1	-1	-1	-1	-1	-1
χ_1	0	0	0	0	0	0	0
χ_2	-10368	0	0	0	0	0	0
χ_4	2592	0	0	0	0	0	0
χ_7	2304	0	0	0	0	0	0
χ_8	2592	0	0	0	0	0	0
χ_{13}	576	0	0	0	0	0	0
χ_{14}	288	0	0	0	0	0	0
χ_{26}	72	0	0	0	0	0	0
χ_{28}	72	0	0	0	0	0	0
χ_{52}	18	0	0	0	0	0	0
χ_{56}	72	0	0	0	0	0	0
χ_{91}	16	0	0	0	0	0	0
χ_{104}	18	0	0	0	0	0	0
χ_{182}	2	0	0	0	0	0	0
θ_1	0	-288	-288	-1152	-1152	-20736	20736
θ_2	0	-144	576	576	1152	10368	20736
θ_5	0	-72	72	648	-648	5184	-5184
θ_{10}	0	36	72	-324	432	-2592	-2592
θ_{73}	0	-4	4	16	-16	-16	-16
θ_{146}	0	2	4	-8	-8	-8	-8
ξ	2	0	0	0	0	0	0
η	0	4	-4	4	-4	4	-4

Thus $K(\mathcal{SL}(2,729)) = \oplus \mathbb{Z}_p$,

$p = 387419760, 64569960, 5, 7, 7, 7, 13, 13, 13, 73, 6, 6, 6, 6, 4, 4, 4, 3, 3, 3, 2, 2, 2, 1$.

3.2 Outcomes for $\mathcal{SL}(2,81)$

$$|\mathcal{SL}(2,81)| = 531360.$$

$$\begin{aligned}
\frac{\Phi(\frac{80}{4})}{\Phi(\frac{80}{16})}\mu(5) &= -2, \tau_1(4,5) = \frac{\Phi(20)}{\Phi(4)}\mu(4) = 0, \tau_1(4,8) = \frac{\Phi(\frac{80}{4})}{\Phi(\frac{80}{32})}\mu(2.5) = \\
-4, \tau_1(4,10) &= \frac{\Phi(20)}{\Phi(2)}\mu(2) = -8, \tau_1(4,16) = \frac{\Phi(\frac{80}{4})}{\Phi(\frac{80}{64})}\mu(1.25) = 0, \tau_1(4,20) = \\
\frac{\Phi(20)}{\Phi(1)}\mu(1) &= 8, A(5) = \frac{1}{2}\Phi\left(\frac{80}{5}\right) = \frac{1}{2}\Phi(16) = \frac{1}{2}(8) = 4, B(5) = 2, \tau_1(5,1) = \\
\frac{\Phi(\frac{80}{5})}{\Phi(\frac{80}{5})}\mu(16) &= 0, \tau_1(5,2) = \frac{\Phi(\frac{80}{5})}{\Phi(\frac{80}{10})}\mu(8) = 0, \tau_1(5,4) = \frac{\Phi(\frac{80}{5})}{\Phi(\frac{80}{20})}\mu(4) = 0, \\
\tau_1(5,5) &= \frac{\Phi(\frac{80}{5})}{\Phi(\frac{80}{25})}\mu(3.2) = -4, A(8) = \frac{1}{2}\Phi\left(\frac{80}{8}\right) = \frac{1}{2}\Phi(10) = \frac{1}{2}(4) = 2, B(8) = \\
1, \tau_1(8,1) &= \frac{\Phi(\frac{80}{8})}{\Phi(\frac{80}{8})}\mu(10) = 1, \tau_1(8,2) = \frac{\Phi(\frac{80}{8})}{\Phi(\frac{80}{16})}\mu(5) = -1, \tau_1(8,4) = \\
\frac{\Phi(\frac{80}{8})}{\Phi(\frac{80}{32})}\mu(2.5) &= -2, A(10) = \frac{1}{2}\Phi\left(\frac{80}{10}\right) = \frac{1}{2}\Phi(8) = \frac{1}{2}(4) = 2, B(10) = 1, \\
\tau_1(10,1) &= \frac{\Phi(\frac{80}{10})}{\Phi(\frac{80}{10})}\mu(8) = 0, \tau_1(10,2) = \frac{\Phi(\frac{80}{10})}{\Phi(\frac{80}{20})}\mu(4) = 0, \tau_1(10,4) = \\
\frac{\Phi(\frac{80}{10})}{\Phi(\frac{80}{40})}\mu(2) &= -4, \tau_1(10,5) = \frac{\Phi(\frac{80}{10})}{\Phi(\frac{80}{50})}\mu(1.6) = -4, \tau_1(10,16) = \frac{\Phi(\frac{80}{10})}{\Phi(\frac{80}{160})}\mu(0.5) = \\
4, A(16) &= \frac{1}{2}\Phi\left(\frac{80}{16}\right) = \frac{1}{2}\Phi(5) = \frac{1}{2}(4) = 2, B(16) = 1, \tau_1(16,1) = \frac{\Phi(\frac{80}{16})}{\Phi(\frac{80}{16})}\mu(5) = \\
-1, \tau_1(16,2) &= \frac{\Phi(\frac{80}{16})}{\Phi(\frac{80}{32})}\mu(2.5) = -2, \tau_1(16,16) = \frac{\Phi(\frac{80}{16})}{\Phi(\frac{80}{256})}\mu(0.3125) = 4, \\
A(20) &= \frac{1}{2}\Phi\left(\frac{80}{20}\right) = \frac{1}{2}\Phi(4) = \frac{1}{2}(2) = 1, B(20) = 1, \tau_1(20,1) = \frac{\Phi(\frac{80}{20})}{\Phi(\frac{80}{20})}\mu(4) = 0, \\
\tau_1(20,2) &= \frac{\Phi(\frac{80}{20})}{\Phi(\frac{80}{40})}\mu(2) = -2, C(1) = \frac{1}{2}\Phi\left(\frac{82}{1}\right) = \frac{1}{2}\Phi(82) = \frac{1}{2}(40) = 20, \\
B(1) &= 2, \tau_2(1,1) = \frac{\Phi(\frac{82}{1})}{\Phi(\frac{82}{1})}\mu(82) = 1, \tau_2(1,2) = \frac{\Phi(\frac{82}{1})}{\Phi(\frac{82}{2})}\mu(41) = -1, C(2) = \\
\frac{1}{2}\Phi\left(\frac{82}{2}\right) &= \frac{1}{2}\Phi(41) = \frac{1}{2}(40) = 20, B(2) = 1, \tau_2(2,1) = \frac{\Phi(\frac{82}{2})}{\Phi(\frac{82}{2})}\mu(41) = -1, \\
\tau_2(2,2) &= \frac{\Phi(\frac{82}{2})}{\Phi(\frac{82}{4})}\mu(20.5) = 0, E(1) = 2.
\end{aligned}$$

As in section (3.1), $K(SL(2,81)) = \oplus \mathbb{Z}_{\mathcal{P}}$, $\mathcal{P} = 531360, 132840, 3, 3, 3, 3, 5, 4, 4, 2, 2, 2, 1$.

4. Conclusion

The factors of the order of the centralizer of the conjugacy classes except for the order of the centralizer of the conjugacy class 1 after removing the factors of the order of the group divided by 4 in section 3.2 and divided by 6 in section 3.1 and complete the remaining number by 1 are the reminder numbers

of the diagonal of the results matrix. Also, the first number is equal to the group's order, and the second number equal to the order of the group divided by 4 in section 3.2 and 6 in section 3.1.

References

- [1] Hall, Brian C. : Lie Groups, Lie Algebras, and Representations: An Elementary Introduction, Graduate Texts in Mathematics, vol. 222 (2nd ed.), Springer (2015).
- [2] Haytham R. H. and Niran S. J. : Exactness for a complex sequence in the skew shape $(8,8)/(1,0)$, *Journal of Interdisciplinary Mathematics* (2021), DOI:10.1080/09720502.2021.1892271.
- [3] Dunya M. H., Ahmed K. M., Intidhar Z. M. : Score for some Groups $SUT(2,p)$, *Int. J. Nonlinear Anal. Appl.*, Vol. 12, No. 2, pp.1-15 (2021).
- [4] Niran S.J., Sawsan J.K. and Ahmed I.A. : Enforcement for the partition $(7, 7, 4; 0, 0)$, *Journal of Interdisciplinary Mathematics* (2021), DOI:10.1080/09720502.2021.1892272.
- [5] Mohamed S. K. : On Rational -Valued Characters of Certain Types of Permutation Group, *Ibn Al-Haitham Journal for Pure and Applied Sciences*, Vol.19(4), pp.99-108 (2006).
- [6] Hassan H. R. : The Reduction of Resolution of Weyl Module From Characteristic-Free Resolution in Case $(4,4,3)$, *Ibn Al-Haitham Journal for Pure and Applied Science*, Vol. 25(3), pp.341-355 (2012).
- [7] Shubham Sharma & Ahmed J. Obaid. Mathematical modeling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.