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About minimal and maximal via ipre- continuous functions

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Abstract

Examining the contemporary course of sets based on the idea of ***ipre***-open set. The significant concepts presented in work are min *i***pre**-open and max ***ipre***-open. Other than, unused sorts of topological spaces presented, which are called *Ti****pre*** max and *Ti****pre*** min in spaces. Moreover, we display two modern capacities of progression, which are called min ***ipre***-continuous and max ***ipre***-continuous.

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1. Introduction

Within the analysis of least squares equations, min and max sets are significant topological, F., and Oda N. presented concepts in [1,2,3] and they utilized them to examine numerous topological properties. In work, presented the idea of min $\boldsymbol{ipre}$-open and max $\boldsymbol{ipre}$-open and their apologies. In [1] Adequate open (closed) $L \subseteq X$ is min open (closed) set in case any open contained in $\mathfrak{L}$ is φ or $\mathfrak{L}$. An appropriate $\mathfrak{L} \subseteq X$ is max open (closed) set if any clopen contains $\mathfrak{L}$ is $\mathfrak{L}$ or X [3,4]. In [5] Let $A \subseteq X$, can be defined int on union open sets and cl on intersecting closed sets. Let $A \subseteq X$ A is called a $\boldsymbol{pre}$-open if $A \subseteq int(cl(A))$, the complement is $\boldsymbol{pre}$-closed [6]. We show its relationship to spaces using distance, and because continuous functions are related to real life, we will present these new ideas that help us understand the functions of minimal and maximal we will mention definitions, examples and some theories related to the topic.

2. ipre-Open Set and ipre-Continuous

Definition 2.1: Let $A \subseteq X$ be i$\boldsymbol{pre}$-open set if $\exists$ $\boldsymbol{pre}$-open set s.t. $\mathfrak{L}$ ($\mathfrak{L} \neq X, \varphi$) and $A \subseteq cl^*(A \cap \mathfrak{L})$.

Definition 2.2: Let X, Y are spaces and $f: X \to Y$ then f is a i$\boldsymbol{pre}$-continuous if $f^{-1}(A)$ is a i$\boldsymbol{pre}$-open in X, $\forall$ A in Y min and max i$\boldsymbol{pre}$ -open sets.

Definition 2.3: let $B \subseteq X$ is i$\boldsymbol{pre}$-open is a minimal i$\boldsymbol{pre}$-open if $\forall$i$\boldsymbol{pre}$-open contained in B is φ or B.

Definition 2.4: Let $B \subseteq X$ i$\boldsymbol{pre}$-open is a max i$\boldsymbol{pre}$-open if $\forall$ $\boldsymbol{ipre}$-open that contains is B in X or B.

Definition 2.5: let $F \subseteq X$ is a max i$\boldsymbol{pre}$-closed if $\forall$ i$\boldsymbol{pre}$-closed contains F is X or F.

Theorem 2.6: *Let* n $\subseteq X$*, then* n *is a* m_{in} *ipre-closed iff* $X - n$ *is* m_{ax}*ipre-open set.*

Proof: $\Longrightarrow$ X/n is ipre-open since n be a m$_{in}$ipre-closed. To prove X/n is m$_{ax}$ipre-open now assume not, then there is a ipre-open s.t. $D \subseteq X$ and $X/n \subset D$ so $X - D \subset n$ is contradicted reason n is m$_{in}$ ipre-closed.

$\Longleftarrow$ let $n \subseteq X$ be an ipre-closed, $\exists K \neq \varphi$ is a ipre-closed s.t. $K \subset n$ and $X - n \subset K^C$ but K^C is ipre-open. Contradiction reason $X - n$ is m$_{ax}$ ipre-open.

Proposition 2.7: Let $\mathfrak{N}$ and $\mathfrak{M}$ be m$_{ax}$ ipre-open subsets X, then $\mathfrak{N} \cup \mathfrak{M}$ =X or $\mathfrak{N} = \mathfrak{M}$.

Proposition 2.8: Let U is a m$_{ax}$ ipre-open and $V \subseteq X$ be a ipre-open then $U \cup V = X$ or $V = U$.

Proposition 2.9: Let $U \subseteq X$ be a m_{ax}ipre-open with $x \in U^C$then $U^C \subset V$, $\forall$ ipre-open of X and $x \in V$.

Theorem 2.10: *Let $\mathfrak{V}, V \subseteq X$ are a $m_{in}ipre$-closed. Then $\mathfrak{V} \cap V = \emptyset$ or $F = V$.*

Proposition 2.11: Let $\mathfrak{L}, \mathfrak{N}$, $\mathfrak{G} \subseteq X$ are m_{ax}ipre-open s.t. $\mathfrak{L} \neq V$, if $\mathfrak{L} \cap \mathfrak{N} \subseteq \mathfrak{G}$, then either $\mathfrak{L} = \mathfrak{G}$ or $\mathfrak{N} = \mathfrak{G}$.

Proposition 2.12: Let $\mathfrak{L}, \mathfrak{N}$, $\mathfrak{G} \subseteq X$ are m_{ax} ipre-open which are different from each other, then $\mathfrak{L} \cap \mathfrak{N} \nsubseteq \mathfrak{L} \cap \mathfrak{G}$.

Theorem 2.13: *Let $\mathfrak{G} \subseteq X$ be a $m_{in}ipre$-closed, if $x \in \mathfrak{G}$then $\mathfrak{G} \subseteq K$, $\forall K \subseteq X$ $ipre$-closed containing x.*

Proof: Let$x \in K$ and $\mathfrak{G} \nsubseteq K$ *so* $\mathfrak{G} \cap K \subseteq \mathfrak{G}$ and $\mathfrak{G} \cap\ K \neq \varphi$, since $x \in \mathfrak{G} \cap K$, since $\mathfrak{G}$ *is* m_{in}ipre $-$closed so $\mathfrak{G} \cap K = \mathfrak{G}$ or $\mathfrak{G} \cap K = \varphi$. Then $\mathfrak{G} \cap K = \mathfrak{G}$ which $\mathfrak{G} \cap K \subseteq \mathfrak{G}$. So $\mathfrak{G} \subseteq K$.

Definition 2.14: Any space X is T_{ipremin} if any ipre-open of X is m_{in}ipre-open.

Definition 2.15: Any space X is T_{ipremax} if any ipre-open in X is m_{ax}ipre-open.

Example 2.16: Let $=X = \{\mathfrak{w}, \mathfrak{v}\}$ with $\mathfrak{J}$ = $\{\varphi, \{\ \mathfrak{w}\ \}$,X$\}$ and $\beta O(X) = i\beta O(X)\ = T$, it is$\{\mathfrak{w}\}$ is max and min ipre-open so a space X is both T_{ipremin} and T_{ipremax}.

Theorem 2.17: *A space X is $T_{ipremin}$ iff any adequate $\mathfrak{M} \subseteq X$ $ipre$-closed is m_{ax} $ipre$-closed in X.*

Proof: $\Longrightarrow$ Since $\mathfrak{M} \subseteq X$ be ipre-closed and $\mathfrak{M}$ is not max. Then there $K \subseteq X$ an ipre-closed with $K \neq X$ s.t. $\mathfrak{M} \subseteq K$. Thus $X - K \subseteq X - \mathfrak{M}$. So $X - \mathfrak{M}$ is an adequate ipre-open is not min and contradict of being X is T_{ipremin} space.

$\Longleftarrow$ Now $\mathfrak{N} \neq \emptyset$ and $X - \mathfrak{N} \subseteq X$ is ipre-open, then $X - \mathfrak{N}$ is ipre-closed. Then $X/\mathfrak{N}$ is m_{ax}ipre-closed in X. and by **(2.10)** $\mathfrak{N}$ is m_{in}ipre-open. So X is T_{ipremin}.

Theorem 2.18: *If any set X two topologies $O1$ and $O2$ are defined, then identity mapping idx is an $m_{in}ipre$ continuous of $(X, O1)$ to $(X, O2)$ iff the $O1$ is finer than the topology O2.*

Proof: let X is space, R is real line with the natural topology, and I is the m_{in}ipre closed unit interval with the natural. From the equivalence of theorems **(2.13)** and **(2.17)**, it follows that a mapping f of X to R or I is a m_{in}ipre continuous iff $\forall x \in X$ and any $\epsilon >\ 0$ has a neighbourhood U of x s.t. $|f(x) -$

$f(x^-)| < \epsilon$,$\forall x^- \in U$. In particular, f is a an m_{in}ipre continuous mapping of R to R if for every $x \in R$ and any $\epsilon > 0$,$\exists \delta > 0$ s.t. $|f(x) - f(x^-)| < \epsilon$ for every x^-satisfying $|x - x'| < \delta$. An m_{in}ipre continuous to R or I will be called an m_{in}ipre continuous functions. This terminology is consistent with the terminology of real analysis. From what we observed above, it for any m_{in}ipre continuous $f: X \to R$, the $|f|$, where $|f|(x) = |f(x)|$, is an m_{in}ipre continuous. Indeed, $|f|$is the composition of f and the absolute value function is a an m_{in}ipre continuous function from R to R. Likewise, one can be checked that for the an m_{in}ipre continuous functions $f, g: X \to R$, the functions $f \pm g$, $f * g$, $min(f, g)$ and $max\,(f.g)$, If a m_{in}ipre continuous function $f: X \to R$ vanishes at no point in X, then the function $1\,/\,f$, where $(1\,/\,f)\,(x) = 1\,/\,f\,(x)$, is m_{in}ipre continuous. The same holds for functions to the interval I, provided that the corresponding operations are performable.

Theorem 2.19: *For every $m_{in}ipre$ continuous $f: X \to R$ there a countable set $xo \subset X$ as $f(x) = f\,(xo)$ for each $x \in X/X_0$. The set $Xi = X/f^{-1}\,((f\,(xo) - 1/i, f\,(xo) + 1/i))$ is closed and does not contain xo; thus, it is a finite set. One can readily verify that the set $Xo = \sqcup_{i=1}^{\infty} Xi$; has the required property.*

Proof: Let X is space and $\{fi\}$ is a sequence from X to R or I. We say that sequence $\{fi\}$ is uniformly converged uniformly to a real-valued function f if for every $\epsilon > 0$, there exists a k s.t. we have $|f(x) - fi(x)| < \epsilon$ for each $x \in X$ and $i \geq k$; we write this in symbols as $f = lim\, fi$.

Theorem 2.20: *A mapping f of X to a space Y whose topology is generated by a family of $\{f_s\}_{s\in S}$ ses, where f_s , is a map of Y to Y_s, is $m_{in}ipre$ continuous iff the composition fsf is $m_{in}ipre$ continuous $\forall\, s \in S$.*

Proof: Let us Suppose $fsf: X \to Y$, is continuous for all $s \in S$ and let us denote by P the subbase of the space Y consisting of all of form $f_s^{-1}(V_s)$, where V_s, is open in Y_s,. It is sufficient to show that the inverse images of the members of P under the mapping f are open in X. This follows, however from the equality $f^{-1} f_s^{-1}(V_s) = (fsf)^{-1}(V_s)$. If there is a m_{in}ipre continuous f of a space of X onto a space of Y, i.e., a mapping $f: X \to Y$ s.t. $f(X) = Y$, then we say that X can be mapped to Y or that Y is a m_{in}ipre continuous form of X.

Theorem 2.21: *A Map $f: X \to \prod X$, is $m_{ax}ipre$ continuous iff $\prod_{\gamma} o\, f$ is $m_{ax}ipre$ continuous for all $a \in A$.*

Proof: Let m_{ax}ipre continuous maps is m_{ax}ipre continuous. On the contrary, suppose $\prod_{\gamma} o\, f$ it is a m_{ax}ipre continuous for each $\gamma \in A$. The sets of the form $\prod_{\gamma}^{-1}(U_{\gamma})\,, \gamma \in A$(U), and U_{γ}, m_{ax}ipre open in X_{γ}, form a subbase of the topology on $\prod X_{\gamma}$. But $f^{-1}\left(\prod_{\gamma}^{-1}(U_{\gamma})\right) = \left(\prod_{\gamma} of\right)^{-1}(U_{\gamma})$. So, f^{-1} of subsets m_{ax}ipre open are m_{ax}ipre open in X, by a m_{ax}ipre continuous of $\prod_\gamma o f$. F. This is sufficient to show that f is m_{ax}ipre continuous.

The two earlier hypotheses from the penultimate argument for selecting Tychonoff topology in $\prod X_\gamma$ across the topology of box. These findings provide the basis of the most insightful research on the characteristics of product spaces, whether directly or indirectly. As is typically the case, the conclusion becomes the basis of the definition.

Theorem 2.22: *For every $a \in A$, let $f_\gamma: X \to X_\gamma$. Then map $e: X \to \prod X_\gamma$, is an embedding iff X having weak topology given the functionsf_γ, the sets $\{f_\gamma | \gamma \in A\}$ separates the points in X.*

Proof: For all $a \in A$, $\prod_\gamma o\, e = f_\gamma\}$. Now let e is embedding of X in $\prod X_\gamma$. Then $e(X)$ if X has weak topology given the functions, embedding. So, since e is a homeomorphism, by the functions $\prod_\gamma o\, e = f_\gamma\}$.Furthermore, if $x \neq y$, in X, then $e\,(x) \neq e\,(y)$ and hence $[e\,(x)]_\gamma \neq [e\,(y)]_\gamma$, i.e. $f_\gamma\,(x) \neq f_\gamma\,(y)$, for some $\gamma \in A$. Thus, the collection $\{f_\gamma | \gamma \in A\}$ separates the points. Now let that the structure on X a weak structure the functions f_γ, the set $\{f_\gamma | \gamma \in A\}$ separates the points on X. For every $\gamma \in A$, $\prod_\gamma o\, e = f_\gamma$ is m_{ax}ipre continuous. Thus, e is m_{ax}ipre continuous. If $x \neq y$ is in X, then for some $\gamma \in A$, $f_\gamma\,(x) \neq f_\gamma\,(y)$, i.e., $[e\,(x)]_\gamma \neq [e\,(y)]_\gamma$, and thus $e\,(x) \neq e\,(y)$. Hence e is one-to-one. We that e is m_{ax}ipre open; if U is m_{ax}ipre open of X, so $e(U)$ is m_{ax}ipre open in $e(X)$. Since e is 1-1, it is sufficient in order e(U) is m_{ax}ipre open when U is a subbasic in m_{ax}ipre open. So, we let U is of the from $f_\gamma^{-1}(V)$ for some $\gamma \in A$ and some m_{ax}ipre open set V in X_γ. But then $U = [\prod_\gamma | e(X) o e]^{-1}\,(V) = e^{-1}[(\prod_\gamma | e(X)^{-1}(V)]$ and hence $e(U) = [\prod_\gamma | e(X)]^{-1}\,(V) = \prod_\gamma^{-1}(V) \cap e(X)]$ is m_{ax}ipre open set in $e\,(X)$, since $\prod_\gamma^{-1}(V)$ is m_{ax}ipre open in $\prod X_\gamma$. (The last argument appears much neater if you just keep in mind that Π_γ should be limited to $e(X)$ rather than writing it out).

3. Discussion and Conclusion

We have provided some theories, properties, and examples of m_{in}ipre-open and max m_{ax}ipre -open and continuity. These ideas can be generalized to references [7,8]. Can also be generalized m_{ax}ipre and m_{in}ipre to the defined set mentioned in neutrosophic sets.

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