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A note on intuitionistic fuzzy-n-normal operator on IFH-space

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Abstract

In this article, we have defined a new type of Normal operator on IFH-space, proven some important properties in it, and proved some mathematical operations based on the definition of this type.

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1. Introduction

Defined fuzzy by Zadeh [9] in1965.D.Foster [5] introduced fuzzy Topological space in 1979. In 1986, Atanassov [2] defined the intuitionistic fuzzy set and studied its properties applied to Fuzzy metric space. In 2010[1], Alzaraiqi introduced the definition of n-normal operator. Rasoul Eskandari, Farzollah Mirzapour, and Ali Morassaei were introduced in 2012 [4] —more on (α, β)-Normal operators in Hilbert space.

2. The basics

Definition 2.1 [5-11] : Let $\{W\} \neq 0$ be and let $I = [0,1]$. Fuzzy set $\aleph : W \rightarrow I = [0\,,1]$

Suppose $\aleph$ is fuzzy in W then $\aleph$ is described as the characteristic function which connects for any $x \epsilon W$ to real number $\aleph\,(x)$ in the interval $I.\aleph\,(x)$ the grade membership functions to x in $\aleph$ where $\aleph\,(x)$ is said to be the membership function for $\aleph$.

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Definition 2.2 [2]: Let $\{W\} \neq 0$IFH Z given by :

$Z = \{ (x, \aleph_z(x), v_z(x): x \in W)\}$, the mappings $\aleph_A: W \to I, v_A: W \to I$ the degree member ship the degree non-membership to the set Z, $0 \leq \aleph_z(x) + v_z(x) \leq 1$ for each $x \in W$.

Definition 2.3: t-norm (triangular-norm) and t – conform (triangular-conform) see [8]and[9].

Definition 2.4 [10]: The three-tuple $(W, N,*)$ is called a (FNS)

$(i.e. N: W \times (0,\infty) \to [0,1])$ if all $x, y \in W\ t, s > 0$, then

(Z1) $N(x,t) > 0$.

(Z2) $N(x,t) = 1 \Leftrightarrow x = 0$.

(Z3) $N(\beta x, t) = N(x, \frac{t}{|\beta|}), \forall \beta \in F/\{0\}$.

(Z4) $N(x + y, t + s) \geq N(x,t) * N(y,s)$.

(Z5) $N(x,.): (0,\infty) \to [0,1]$ *continuous.*

(Z6) $lim_{t\to\infty} N(x,t) = 1 \wedge lim_{t\to 0} N(x,t) = 0$.

And also

(Z7) $N(x,t) > 0, \forall t > 0 \Rightarrow x = 0$.

Definition 2.5[7]: The five-tuple $(W . \aleph, v,*,\diamond)$ is called be (IFNS) if $W \times (0,\infty)$ $(i.e. \aleph, v : x \times (0,\infty) \to [0,1]$) if $x, y \in W \wedge t, s > 0$, then

(E 1) $\aleph(x,t) + v(x,t) \leq 1$.

(E 2) $\aleph(x,t) > 0$.

(E 3) $\aleph(x,t) = 1$ iff $x = 0$.

(E 4) $\aleph(\propto x, t) = \aleph(x. \frac{t}{|\propto|})$ if $F\backslash\{0\}$.

(E 5) $\aleph(x + y, t + s) \geq \aleph(x,t) * \aleph(y,s)$.

(E 6) $M(x,.) : (0,\infty) \to [0,1]$ is continuoues.

(E 7) $lim_{t\to\infty} \aleph(x,t) = 1 \wedge lim_{t\to 0} \aleph(x,t) = 0$.

(E 8) $v(x,t) < 1$.

(E 9) $v(x,t) = 0$ if and only if $x = 0$.

(E 10) $v(ax,t) = 0$ iff $= 0$,

(E 11) $v(x + y, t + s) \leq v(x,t) \diamond v(y,s)$,

(E 12) $v(x,.) : (0,\infty) \to [0,1]$ is continuous,

(E 13) $lim_{t\to\infty} v(x,t) = 0 \wedge lim_{t\to 0} v(x,t) = 1$,

assume that (W, μ, V,*, $\diamond$) satisfies the conditions:

(E14) $a * a = a \wedge a \diamond a = a, \forall a \in [0,1]$,

(E15) $\aleph(x, t) > 0 \wedge v(x, t) < 1, \forall t > 0 \Rightarrow x = 0$.

3. Main Results

In this section, we defined the IFn-Normal operator in IFHspace and explained several basic properties of the IF n-normal operator in IFHspace.

Definition 3.1: (Intuitionistic Fuzzy n-Normal operator) Let$\left(V, \mathcal{F}_{\mu,v}, \mathcal{T}\right)$ is IFH space with $\langle u, v\rangle$ = *suprmum* $\left\{\ t \in \mathrm{R}: \mathcal{F}_{\mu\,,v}\ (u, v, t) < 1\right\}$, $\forall\, u, v \in V$ and let $T \in \mathrm{IFB}(V), T$ is an IHF space if it commutes with it is IF-A joint. i.e $T^n\, T^* = T^*\, T^n$.

Theorem 3.2: *Let* $T \in \mathrm{IFB(H)}$ *be IF-n-normal operator IF* T^{-1} *exists then* (T^{-1}) *is IF-n-normal operator.*

Theorem 3.3: *IF* $S \in$ *IF-n-Normal operator the restriction* S/M *of* S *to any closed subspace M of H that reduced S is IF-n - Normal operator.*

Proof:

$$p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S}{M}\right)^{n}\left(\frac{S}{M}\right)^{*}\right)u,t\right)$$

$$=p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S^n}{M}\right)\left(\frac{S^*}{M}\right)\right)u,t\right)$$

$$=p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S^nS^*}{M}\right)\right)u,t\right)$$

$$=p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S^*S^n}{M}\right)\right)u,t\right)$$

$$=p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S^*}{M}\right)\left(\frac{S^n}{M}\right)\right)u\,,t\right)$$

$$=p_{\mu,v}\left(\left(\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}-\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{n}\right)u\,,t\right)=0$$

So

$$\left(\frac{S}{M}\right)^{*}\left(\frac{S}{M}\right)^{\mathrm{n}}=\left(\frac{S}{M}\right)^{\mathrm{n}}\left(\frac{S}{M}\right)^{*}$$

Proposition 3.4: Let $T \in \mathrm{IFB(H)}$ be IF-n-normal operator,then if $S \in \mathrm{IFB(H)}$ is unitary equivalents to *T*, then *S* IF-n-normal operator

Proof:

Let $S = UTU^*$, $S^n = UT^nU^*$

Then

$$p_{\mu,v}((S^*S^n - S^nS^*)u, t)$$
$$= p_{\mu,v}((UT^*U^*\ UT^nU^*) - (UT^nU^*\ \ UT^*U^*)u, t)$$
$$= p_{\mu,v}((UT^*U^*\ UT^nU^*) - (UT^n\ \ T^*U^*)u, t)$$
$$= p_{\mu,v}((UT^*U^*\ UT^nU^*) - (UT^*\ \ T^nU^*)u, t)$$
$$= p_{\mu,v}((UT^*U^*\ UT^nU^*) - (UT^*U^*\ \ U\,T^nU^*)u, t) = 0$$

Then

$$UT^*U^*\ UT^nU^* = UT^nU^*\ \ U\,T^*U^*$$

$$\text{So}\ \ S^*S^n = S^nS^*$$

Thus S is the IF-n-normal operator

Proposition 3.5: Let*, E, F* be an eternal IF-n-Normal, i.e $(E+F)^*$ eternal with $\sum_{k=1}^{n-1}\binom{n}{k} E^{n-k}F^k$, Then *(E+F)* is IF- n-normal- operator.

4. Conclusion

In this article, we presented a new study on the effects of a new type, so we defied the IFn-normal operator. We have presented some necessary properties and provided some important proof.

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