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Some generalizations of fuzzy soft ($\hat{k}^*$– $\hat{A}$)-quasinormal operators in fuzzy soft Hilbert spaces

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Abstract

This article is a continuation of generalizations in field operator theory in mathematics, such as we introduce new classes of fuzzy soft operator, which namely fuzzy soft ($\hat{k}^*$– $\hat{A}$)-quasinormal operator in fuzzy soft Hilbert space, and shortly FS-($\hat{k}^*$– $\hat{A}$)-quasinormal operator. Give the text of the most important of these theorems with explains important properties for this concept and relationships with other types in the same filed finally more characterizations of FS-($\hat{k}^*$– $\hat{A}$)-quasinormal operator have been introduced in this article.

Subject Classification: *47S40 Fuzzy operator theory.*

Keywords: *FS-($\hat{k}^*$– $\hat{A}$)-quasinormal qperator, Fuzzy soft Hermation operator, Fuzzy soft quasinormal operator, FS-convergent sequaence.*

1. Introduction

Many researchers in modern mathematics focus on there jumbos in the field of fuzzy mathematics and special cases of fuzzy soft mathematics, including specialists in the field of functional analysis. In the beginning In 1965, Zadeh [19] expanded the idea of sets into a fuzzy set-up on a function known as the membership function that has a crisp set as its domain. with the range of

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closed interval [0,1], also he defined the essential operations on this sets, Molodtsov [4] in 1999 study the other types of sets which is namely soft set dependent on function has domain set of parameters into the rang of power subset of universal set, after that, many studies were conducted in this regain until they discover The fuzzy soft set theory, like P.K. Maji, and etc. In 2001 [13], studied the fuzzy soft set theory, and in 2003 the same authors. [14], introduce paper has title Soft set theory, they explained the most important theories about fuzzy soft, up on of this give the meaning of the term "fuzzy soft vectors", with continued the study in this region, where in 2014, Yazar M. I., and etc.[8], introduced a new note of soft normed space up on suitable vector space, and N. Faried, and etc., in 2020 [9] submitted the fuzzy soft inner product spaces, with some important properties of this concept. Also, N. Faried, and etc., in 2020 generalized the concept Hilbert space into fuzzy sot Hilbert space with some important theorems, moreover some studies given in 2020 the concept of linearity Considering soft fuzzy operators in aropriate Hilbert spaces, by N Faried, also in the same year, N Faried, and H H Sakr [10] submitted fuzzy soft Hermation operators. Finally in 2021 Salim D. M. and Ali Q. J. [15], introduced a new types of fuzzy soft operator in subject of special class of operator which namely fuzzy soft normal operator, also in 2021 Salim D. M. and Ali Q. J., [16] given another kinds of projection operator It is suggested that a projection operator defined in fuzzy soft Hilbert spaces is fuzzy.

Finally In [2,5,6,17,18] Many researchers worked in different fields of this trend. The outline on this article is to study one more generalizations of soft fuzzy quasinormal operators, namely ((k^*)ˆ-Aˆ)-quasinormal operator, in suitable Hilbert space which is fuzzy soft Hilbert space with summarize FS-((k^*)ˆ-Aˆ)-quasinormal operator, as well as, give the operations on this concept, with some important relation of this operator with other classes of operator have been given before that, this study explain the new discover operator is strong that fuzzy soft operator are studied before, and using some examples to illustrate the theorems aeared in this search.

2. Some Basic concepts and theorems for fuzzy soft Hilbert spaces

In this part of article, we recall some basic definition for fuzzy soft set as well as some properties of this set.

Definition 2.1: [11], The set of order pair $\hat{A} = \{(x, \mu_{\hat{A}}(x)) | x \text{ belong } \mathcal{X}, \mu_{\hat{A}}(x) \text{ belong to } \mathfrak{T}\}$ which called fuzzy set define on $\mathcal{X}$ with $\mu_{\hat{A}}: \mathcal{X} \longrightarrow \mathfrak{T}$ a membership function, where $\mathfrak{T} = [0,1]$.

Definition 2.2: [4], The set $\mathcal{G}_A = \{\mathcal{G}(e) \in \mathcal{P}(\mathcal{X}): e \in A\}$, which namely soft set define on $\mathcal{X}$, and the set that contains the parameters E, where $\mathcal{P}(\mathcal{X})$, the set of all its subsets $\mathcal{X}$ such as $A \subseteq E$, $\mathcal{G}: A \to \mathcal{P}(\mathcal{X})$, and shortly $(\mathcal{G}, A)$ or $\mathcal{G}_A$.

Definition 2.3: [13], A fuzzy soft set $(\mathcal{G}, A)$ is represented as ($\mathcal{FS}$-set) define on $\mathcal{X}$, where $\mathcal{G}$ is describe as$\mathcal{G}: A \to \mathfrak{T}^{\mathcal{X}}$, has the range $\{\mathcal{G}(e) \in \mathfrak{T}^{\mathcal{X}}: e$ belong to $A\}$, and the family of all FS-sets, summarized by $\mathcal{FSS}(\widetilde{\mathcal{X}})$.

It is helpful to quickly review the definition of a fuzzy soft point using an FS-point, and it is clear that all fuzzy soft points are fuzzy soft singleton sets, even though a fuzzy soft set is not always a fuzzy soft point.

Definition 2.4:[14], let $\mathcal{G}_A$ be $\mathcal{FS}$ –set over $\mathcal{X}$, then $\mathcal{FS}$ –point written as $((x_{\mu_{\mathcal{G}(e)}}, A)$ or $\tilde{x}_{\mu_{\mathcal{G}(e)}}$), such that when $e \in A$ and $x \in \mathcal{X}$, one can has

$$\mu_{\mathcal{G}(e)}(x) = \begin{cases} \lambda, if\ x = x_o \in \mathcal{X}\ and\ e = e_0 \in A \\ 0, if\ x \in \mathcal{X} - \{x_0\}\ or\ e \in A - \{e_0\} \end{cases}, \text{ where } \lambda \in (0,1]$$

Notation 2.5: [14], through the work, denoted the group of all fuzzy soft things complex sets as $\mathcal{C}(A)$ and the class of all soft, fuzzy soft real set as $\mathcal{R}(A)$. And $\mathcal{C}(A), \mathcal{R}(A)$ are fuzzy soft complex vector and fuzzy soft real vector, in general summarize by $\mathcal{FSV}(\widetilde{\mathcal{X}})$.

Definition 2.6: [8], assume that $\widetilde{\mathcal{X}}$ is $\mathcal{FS} - vector\ space$, and $\widetilde{\|.\|}: \widetilde{\mathcal{X}} \to \mathcal{R}(A)$ which is namely fuzzy soft norm define on the set $\widetilde{\mathcal{X}}$ ($\mathcal{FSN}$) if the down requirements are all fulfilled.

i) for all $\tilde{x}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$, we have $\widetilde{\left\|\tilde{x}_{\mu_{\mathcal{G}(e)}}\right\|} \geqq \tilde{0}$,,also $\widetilde{\left\|\tilde{x}_{\mu_{\mathcal{G}(e)}}\right\|} \cong \tilde{0} \Leftrightarrow \tilde{x}_{\mu_{\mathcal{G}(e)}} \cong \tilde{\theta}$

ii) for all $\tilde{x}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$, and $\tilde{r} \in \mathcal{C}(A)$, we have $\widetilde{\left\|\tilde{r} \cdot \tilde{x}_{\mu_{\mathcal{G}(e)}}\right\|} \cong |\tilde{r}| \widetilde{\left\|\tilde{x}_{\mu_{\mathcal{G}(e)}}\right\|}$,

iii) for all $\tilde{x}_{\mu_{1\mathcal{G}(e_1)}}, \tilde{y}_{\mu_{2\mathcal{G}(e_2)}} \in \widetilde{\mathcal{X}}$ we have $\widetilde{\left\|\tilde{x}_{\mu_{1\mathcal{G}(e_1)}} + \tilde{y}_{\mu_{2\mathcal{G}(e_2)}}\right\|} \leqq \widetilde{\left\| \tilde{x}_{\mu_{1\mathcal{G}(e_1)}}\right\|} + \widetilde{\left\|\tilde{y}_{\mu_{2\mathcal{G}(e_2)}}\right\|}$.

The $\mathcal{FS} - vector\ space$ with the maing of fuzzy soft norm $\widetilde{\|.\|}$ namely soft and fuzzy norm vector space ($\mathcal{FSN} -\ space$), with shortly $\left(\widetilde{\mathcal{X}}, \widetilde{\|.\|}\right)$.

Definition 2.7: [11], If $\widetilde{\mathcal{X}}$ is, then $\mathcal{FSV} - space$. Then, if$\widetilde{\langle .,. \rangle}$ the following conditions are met, we will specifically map $\widetilde{\langle .,. \rangle} : \widetilde{\mathcal{X}} \times \widetilde{\mathcal{X}} \to (\mathcal{C}(A)\ or\ \mathcal{R}(A))$ by fuzzy soft inner product on $\widetilde{\mathcal{X}}$ ($\mathcal{FS}\mathfrak{T}$).

i) For all $\tilde{x}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$, must have $\widetilde{\langle \tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{x}_{\mu_{\mathcal{G}(e)}} \rangle} \geqq \tilde{0}$,
and $\widetilde{\langle \tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{x}_{\mu_{\mathcal{G}(e)}} \rangle} \cong \tilde{0} \Leftrightarrow \tilde{x}_{\mu_{\mathcal{G}(e)}} \cong \tilde{\theta}$

ii) For all $\tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$ must have $\widetilde{\langle \tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}} \rangle} \cong \overline{\widetilde{\langle \tilde{y}_{\mu_{\mathcal{G}(e)}}, \tilde{x}_{\mu_{\mathcal{G}(e)}} \rangle}}$

iii) For all $\tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$ and $\tilde{\alpha} \in \mathcal{C}(A)$, must have
$\widetilde{\langle \tilde{\alpha}\tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}} \rangle} \cong \tilde{\alpha}\, \widetilde{\langle \tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}} \rangle}$

vi) For all $\tilde{x}_{\mu_{\mathcal{G}(e)}}, \tilde{y}_{\mu_{\mathcal{G}(e)}}, \tilde{z}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$ must have
$\langle \tilde{x}_{\mu_{\mathcal{G}(e)}} \widetilde{+ \tilde{y}_{\mu_{\mathcal{G}(e)}}}, \tilde{z}_{\mu_{\mathcal{G}(e)}} \rangle \cong \langle \tilde{x}_{\mu_{\mathcal{G}(e)}} \widetilde{, \tilde{z}_{\mu_{3\mathcal{G}(e_3)}}} \rangle + \langle \tilde{y}_{\mu_{\mathcal{G}(e)}} \widetilde{, \tilde{z}_{\mu_{\mathcal{G}(e)}}} \rangle$

also, written as $\left(\widetilde{\mathcal{X}}, \widetilde{\langle .,. \rangle}\right)$ of a fuzzy soft inner product space with shortly $(\mathcal{FS}\mathfrak{T} - space)$.

Next, we recall the following theorem give the relation between definitions (2.6) and (2.7)

Theorem 2.8: [16], *The* $\mathcal{FS}\mathfrak{T} - space$ $\left(\widetilde{\mathcal{X}}, \widetilde{\langle .,. \rangle}\right)$ *can be thought of as* $\mathcal{FSN} - space$ *and the* $\mathcal{FS} - norm$ $\left\| \widetilde{\tilde{x}_{\mu_{\mathcal{G}(e)}}} \right\| \cong \sqrt{\langle \tilde{x}_{\mu_{\mathcal{G}(e)}} \widetilde{, \tilde{x}_{\mu_{\mathcal{G}(e)}}} \rangle}$, *for each* $\tilde{x}_{\mu_{\mathcal{G}(e)}} \in \widetilde{\mathcal{X}}$

3. Convergent In Fuzzy Soft Hilbert Space

In this part, we will discuss various definitions, characteristics, and examples of fuzzy soft operator which is bounded linear remembering in fuzzy soft Hilbert space.

Definition 3.1: [7]**,** A sequence contain of $\mathcal{FS} -$ points $\left\{\tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}}\right\}$ lie in $\mathcal{FSN} - space$ $\left(\tilde{X}, \widetilde{\|\cdot\|}\right)$ namely fuzzy soft convergent to $\tilde{x}^o_{\mu_{0\mathcal{G}(e_0)}}$ if $\lim_{n\to\infty} \left\| \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \widetilde{-} \tilde{x}^o_{\mu_{0\mathcal{G}(e_0)}} \right\| \cong \tilde{0}$ that means, $\forall \tilde{\epsilon} > \tilde{0}, \exists n_o \in N,$ such that $\left\| \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \widetilde{-} \tilde{x}^o_{\mu_{0\mathcal{G}(e_0)}} \right\| \widetilde{<} \tilde{\epsilon}, \forall n \geq n_o$, and shortly $lim_{n\to\infty}\, \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \cong \tilde{x}^o_{\mu_{0\mathcal{G}(e_0)}}$ or $\tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \to \tilde{x}^o_{\mu_{0\mathcal{G}(e_0)}}$ as $n \to \infty$.

Definition 3.2: [7], A sequence of $\mathcal{FS} -$ points$\left\{\tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}}\right\}$ in $\mathcal{FSN} - space$ $\left(\widetilde{\mathcal{X}}, \widetilde{\|\cdot\|}\right)$ namely fuzzy soft Cauchy sequence, if $lim_{n\to\infty} \left\| \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \widetilde{-} \tilde{x}^o_{\mu_{m\mathcal{G}(e_m)}} \right\| \cong \tilde{0}$ that means $\forall \tilde{\epsilon} > \tilde{0}, \exists n_o \in N$ such that $\left\| \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \widetilde{-} \tilde{x}^m_{\mu_{m\mathcal{G}(e_m)}} \right\| \widetilde{<} \tilde{\epsilon}, \forall n, m \geq n_o, n > m,$ sometime say that $\left\| \tilde{x}^n_{\mu_{n\mathcal{G}(e_n)}} \widetilde{-} \tilde{x}^m_{\mu_{m\mathcal{G}(e_m)}} \right\| \to \tilde{0}$ as $n, m \to \infty$.

Definition 3.3: [15], The $\mathcal{FSN} - space$ $\left(\widetilde{\mathcal{X}}, \widetilde{\|\cdot\|}\right)$ namely fuzzy soft complete $(\mathcal{FS} - complete)$ if each $\mathcal{FS}$-convergent sequence contains an FS-Cauchy sequence.

Definition 3.4: [16], The fuzzy soft Hilbert space ($\mathcal{FSH}$-space), also known as the $\mathcal{FS}$-complete inner product space $\left(\widetilde{\mathcal{X}}, \widetilde{\langle .,. \rangle}\right)$, with represented by the symbol $\left(\widetilde{\mathcal{H}}, \widetilde{\langle , \rangle}\right)$

Definition 3.5: [10], let $\widetilde{\mathcal{H}}$ be $\mathcal{FSH}-space$, the fuzzy operator $\tilde{\mathcal{T}}:\widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ namely fuzzy soft linear operator, with summarized $(\mathcal{FSL}-operator)$ if for each $\tilde{x}_{\mu_{1\mathcal{G}(e_1)}}, \tilde{y}_{\mu_{2\mathcal{G}(e_2)}} \in \widetilde{\mathcal{H}}$ with $\tilde{\alpha}, \tilde{\beta} \in \mathcal{C}(A)$, one can have $\tilde{\alpha}\tilde{\mathcal{T}}\left(\tilde{x}_{\mu_{1\mathcal{G}(e_1)}}\right) \tilde{+} \tilde{\beta}\tilde{\mathcal{T}}\left(\tilde{y}_{\mu_{2\mathcal{G}(e_2)}}\right) \cong \tilde{\mathcal{T}}\left(\tilde{\alpha}\tilde{x}_{\mu_{1\mathcal{G}(e_1)}} \tilde{+} \tilde{\beta}\tilde{y}_{\mu_{2\mathcal{G}(e_2)}}\right)$, also namely fuzzy soft bounded operator $\mathcal{FSB}-$ operator if $\exists\ \tilde{0} \tilde{>} \tilde{m} \in \mathcal{R}(A)$ and get the condition $\left\|\widetilde{\tilde{\mathcal{T}}\left(\tilde{x}_{\mu_{\mathcal{G}_e}}\right)}\right\| \lesssim \tilde{m}\left\|\widetilde{\tilde{x}_{\tilde{x}_{\mu_{\mathcal{G}_e}}}}\right\|$, with each $\tilde{x}_{\tilde{x}_{\mu_{\mathcal{G}_e}}} \in \widetilde{\mathcal{H}}$.

Now, we recall the definition of fuzzy soft adjoint of bounded linear operator.

Definition 3.6: [12], assume that$\widetilde{\mathcal{H}}$ be $\mathcal{FSH}-space$ also $\tilde{\mathcal{T}}:\widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ be $\mathcal{FSB}-operator$, then The fuzzy soft adjoint operator $\tilde{\mathcal{T}}^*$ characterized by$\langle \widetilde{\tilde{\mathcal{T}}\tilde{x}_{\mu_{\mathcal{G}_e}}, \tilde{y}_{\mu_{\mathcal{G}_e}}} \rangle \cong \langle \widetilde{\tilde{x}_{\mu_{\mathcal{G}_e}}, \tilde{\mathcal{T}}^*\, \tilde{y}_{\mu_{\mathcal{G}_e}}} \rangle$, for each $\tilde{x}_{\mu_{\mathcal{G}_e}}, \tilde{y}_{\mu_{\mathcal{G}_e}}$ belong to $\widetilde{\mathcal{H}}$.

The subsequent theorem now brings to mind some characteristics of the fuzzy soft adjoint of the bounded linear operator.

Theorem 3.7: [12], *Assume* $\tilde{\mathcal{T}}, \tilde{R} \in \tilde{B}(\widetilde{\mathcal{H}})$, *where* $\widetilde{\mathcal{H}}$ *is* $\mathcal{FSH}-space$ *and* $\tilde{\beta} \in \tilde{\mathcal{C}}(A)$, *then*

i) $\tilde{\mathcal{T}}^{**} \cong \tilde{\mathcal{T}}$,
ii) $\left(\tilde{\beta}\tilde{\mathcal{T}}\right)^* \cong \bar{\tilde{\beta}}\tilde{\mathcal{T}}^*$,
iii) $\left(\tilde{\mathcal{T}} + \tilde{R}\right)^* \cong \tilde{\mathcal{T}}^* + \tilde{R}^*$
iv) $\left(\tilde{\mathcal{T}}\tilde{R}\right)^* \cong \tilde{R}^*\tilde{\mathcal{T}}^*$.
v) $\widetilde{\|\tilde{\mathcal{T}}^*\|} \cong \widetilde{\|\tilde{\mathcal{T}}\|}$
vi) $\widetilde{\|\tilde{\mathcal{T}}^*\tilde{\mathcal{T}}\|} \cong \widetilde{\|\tilde{\mathcal{T}}\|}^2$.

Definitions 3.8: [12], [15],[16], Let $\widetilde{\mathcal{H}}$ be $\mathcal{FSH}-space$, an $\mathcal{FSBL}-$ operator $\tilde{\mathcal{T}} : \widetilde{\mathcal{H}} \to \widetilde{\mathcal{H}}$ is namely

i) Fuzzy soft self adjoint operator, shortly ($\mathcal{FS}-$ self adjoint operator) if $\tilde{\mathcal{T}} \cong \tilde{\mathcal{T}}^*$.
ii) Fuzzy soft normal operator, shortly ($\mathcal{FS}-$ normal operator) if $\tilde{\mathcal{T}}\tilde{\mathcal{T}}^* \cong \tilde{\mathcal{T}}^*\tilde{\mathcal{T}}$.
iii) Fuzzy soft quasinormal operator, summarize ($\mathcal{FS}-$ quasinormal operator) when $\tilde{\mathcal{T}}\left(\tilde{\mathcal{T}}\tilde{\mathcal{T}}^*\right) \cong \left(\tilde{\mathcal{T}}^*\tilde{\mathcal{T}}\right)\tilde{\mathcal{T}}$.
v) Fuzzy soft projection operator, summarize ($\mathcal{FS}-$ projection operator) when $\tilde{\mathcal{T}}^2 \cong \tilde{\mathcal{T}}$
vi) Fuzzy soft unitary operator, summarize ($\mathcal{FS}-$ unitary operator) when $\tilde{\mathcal{T}}\tilde{\mathcal{T}}^* \cong \tilde{\mathcal{T}}^*\tilde{\mathcal{T}} \cong \tilde{I}$.

4. Essential results of fuzzy soft $(\widehat{k^*}-\hat{A})$-quasinormal operators.

Here we generalized the definition aeared [15] in and discover some important properties and characterization for this classes of fuzzy soft operator, as well as introduce new theorem about this concept in order to get some results for it. next starting with the definition provided below:

Definition 4.1: Assume $\widetilde{\mathcal{H}}$ is $FSH-space$ with $\tilde{\mathcal{T}}:\widetilde{\mathcal{H}}\to\widetilde{\mathcal{H}}$ be $\mathcal{FSB}-operator$ is namely $\mathcal{FS}-(\widehat{k^*}-\hat{A})$ −quasi operators if satisfy $\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\cong\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}$ where $\tilde{k}\geq\tilde{0}$ and $\tilde{A}$ is $\mathcal{FS}$ −bounded operator such that $\tilde{A}:\widetilde{\mathcal{H}}\to\widetilde{\mathcal{H}}$

To illustrate this definition consider the following example, If $\tilde{\mathcal{T}}\cong\begin{bmatrix}\tilde{\imath} & \tilde{1}\\ \tilde{0} & -\tilde{\imath}\end{bmatrix}$ $\mathcal{FSH}-space$ $\tilde{\mathbb{C}}^2$ then $\tilde{\mathcal{T}}$ is $F\mathcal{S}-(\widehat{k^*}-\hat{A})$ −quasinormal operator.

Remark 4.2: If $\tilde{\mathcal{T}}$ is $\mathcal{FS}-(\widehat{k^*}-\hat{A})$ −quasinormal operators and if $\tilde{k}=\tilde{1}$ with $\tilde{A}=\tilde{I}$ then $\tilde{\mathcal{T}}$ is $\mathcal{FS}$ −quasinormal operator.

Now, we give proposition to show some properties of $\mathcal{FS}-(\widehat{k^*}-\hat{A})$ −quasinormal operator.

Proposition 4.3: Let $\tilde{\mathcal{T}}\in\tilde{B}(\widetilde{\mathcal{H}})$ be $\mathcal{FS}-(\widehat{k^*}-\hat{A})-$ quasinormal operator then $\tilde{\mathcal{T}}^n$ is $\mathcal{FS}-(\widehat{k^*}-\hat{A})$ −quasinormal operators where $n\in N$.

Proof: Let $\tilde{\mathcal{T}}$ be $\mathcal{FS}-(\widehat{k^*}-\hat{A})-$ quasinormal operators

We assume that the result is true when $m=1$

$$\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}=\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}} \text{ ...... (a)}$$

We assume the result for $m=n$

$$\left(\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\right)^n=\left(\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\right)^n \text{...... (b)}$$

Let us prove the result for $m=n+1$

$$\left(\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\right)^{n+1}=\left(\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\right)^n\tilde{\mathcal{T}}^{\tilde{k}}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}$$

$=\left(\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\right)^n\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}$ from (a) and (b), one can have

$=\left(\tilde{A}(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}})^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\right)^{n+1}$

Thus the result is true for $m=n+1$.

Then $\tilde{\mathcal{T}}^n$ is $\mathcal{FS}-(\widehat{k^*}-\hat{A})$ −quasinormal operator.

Remark 4.4: Let $\tilde{\mathcal{T}} \in \tilde{B}(\tilde{\mathcal{H}})$ be $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasi operators then $\tilde{\mathcal{T}}^{-1}$ is not necessary to be $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator. To express consider, $\tilde{\mathcal{T}} \cong \begin{bmatrix} -\tilde{\imath} & \tilde{0} \\ \tilde{0} & -\frac{\tilde{\imath}}{\tilde{4}} \end{bmatrix}$ on $\mathcal{FSH}-space$ $\tilde{\mathbb{C}}^2$ then $\tilde{\mathcal{T}}$ is $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator where$\tilde{k}=1$ and $\tilde{A}=\tilde{I}$ but $\tilde{\mathcal{T}}^{-1} \cong \begin{bmatrix} \tilde{\imath} & \tilde{0} \\ \tilde{0} & \tilde{2}\tilde{\imath} \end{bmatrix}$ is not $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator.

Next, to make the remake (4.4) is true see the following proposition.

Proposition 4.5: Let $\tilde{\mathcal{T}} \in \tilde{B}(\tilde{\mathcal{H}})$ be invertible $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator then $\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}$ is $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator where $\tilde{A} \cong \left(\tilde{A}^{-1}\right)^{\tilde{*}}$

Proof: Since $\tilde{\mathcal{T}}$ be $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator so $\tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}} \cong \tilde{A}\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}$

By taking inverse and adjoint of both sides we can have

$$\left(\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}\right)^{\tilde{k}}\left(\left(\tilde{\mathcal{T}}^{-1}\right)\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}\right)^{\tilde{k}} \cong \left(\tilde{A}^{-1}\right)^{\tilde{*}}\left(\left(\tilde{\mathcal{T}}^{-1}\right)\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}\right)^{\tilde{k}}\left(\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}\right)^{\tilde{k}}$$

Therefore; $\left(\tilde{\mathcal{T}}^{-1}\right)^{\tilde{*}}$ is $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator.

Remark 4.6: Let $\tilde{\mathcal{T}}, \tilde{\mathcal{S}} \in \tilde{B}(\tilde{\mathcal{H}})$ be two $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operators then $\tilde{\mathcal{T}}+\tilde{\mathcal{S}}$ and $\tilde{\mathcal{T}}\tilde{\mathcal{S}}$ not necessary to $\mathcal{FS}-\left(\tilde{k}-\tilde{A}\right)^{\tilde{*}}$-quasinormal operator. To show that considers the following examples.

Examples 4.7:

(1) Let $\tilde{\mathcal{T}} \cong \begin{bmatrix} \tilde{0} & \tilde{2}\tilde{\imath} \\ -\tilde{2}\tilde{\imath} & \tilde{0} \end{bmatrix}$, $\tilde{\mathcal{S}} \cong \begin{bmatrix} -\tilde{1} & \tilde{0} \\ \tilde{0} & -\tilde{\imath} \end{bmatrix}$ be two $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operators on $FS\mathcal{H}-space$ where $\tilde{k}=2, \tilde{A}=\tilde{I}$ But $\tilde{\mathcal{T}}+\tilde{\mathcal{S}} \cong \begin{bmatrix} \tilde{1} & \tilde{2}\tilde{\imath} \\ -\tilde{2}\tilde{\imath} & \tilde{1} \end{bmatrix}$ is not $FS-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operator.

(2) Let$\tilde{\mathcal{T}} \cong \begin{bmatrix} \tilde{0} & -\tilde{2} \\ \tilde{2} & \tilde{0} \end{bmatrix}$, $\tilde{\mathcal{S}} \cong \begin{bmatrix} \tilde{0} & \tilde{\imath} \\ -\tilde{\imath} & \tilde{1} \end{bmatrix}$ be two $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$quasinormal operators on $\tilde{\mathbb{C}}^2$ where $\tilde{k}=2, \tilde{A}=\tilde{I}$ But $\tilde{\mathcal{T}}\tilde{\mathcal{S}} \cong \begin{bmatrix} \tilde{2}\tilde{\imath} & -\tilde{2} \\ \tilde{0} & \tilde{2}\tilde{\imath} \end{bmatrix}$ is not $\mathcal{FS}-\left(\widehat{k^*}-\hat{A}\right)-$ quasinormal operators.

Now, the following theorem given the condition to make the remark (4.6) is true.

Theorem 4.8: *If $\tilde{\mathcal{T}}, \tilde{\mathcal{S}} \in \tilde{B}(\tilde{\mathcal{H}})$ be two $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operators then:*

i) $\tilde{\mathcal{T}} + \tilde{\mathcal{S}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operator if $\tilde{\mathcal{T}}\,\tilde{\mathcal{S}} \cong \tilde{\mathcal{S}}\tilde{\mathcal{T}} \cong \tilde{\mathcal{T}}^{\tilde{*}}\,\tilde{\mathcal{S}} \cong \tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{T}} \cong \tilde{\mathcal{T}}\,\tilde{\mathcal{S}}^{\tilde{*}} \cong \tilde{\mathcal{S}}\,\tilde{\mathcal{T}}^{\tilde{*}} \cong \tilde{0}$

ii) $\tilde{\mathcal{T}}\,\tilde{\mathcal{S}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right) -$ quasinormal operator if $\tilde{\mathcal{T}}\,\tilde{\mathcal{S}} \cong \tilde{\mathcal{S}}\tilde{\mathcal{T}}, \tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{T}} \cong \tilde{\mathcal{T}}\,\tilde{\mathcal{S}}^{\tilde{*}}$, $\tilde{\mathcal{S}}\,\tilde{\mathcal{T}}^{\tilde{*}} \cong \tilde{\mathcal{T}}^{\tilde{*}}\,\tilde{\mathcal{S}}$ and $\tilde{\mathcal{S}}$ is $\mathcal{FS} -$ quasi operators.

Proof:

i)
$$\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)^{\tilde{k}}\left(\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)^{\tilde{*}}\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)\right)^{\tilde{k}}$$
$$\cong \left(\tilde{\mathcal{T}}^{\tilde{k}} + \tilde{\mathcal{S}}^{\tilde{k}}\right)\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}} + \left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}} + \left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}} + \left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}} + \tilde{\mathcal{S}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{S}}\right)^{\tilde{k}}$$
$$\cong \tilde{A}\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}} + \tilde{A}\left(\tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{S}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)^{\tilde{*}}\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)\right)^{\tilde{k}}\left(\tilde{\mathcal{T}} + \tilde{\mathcal{S}}\right)^{\tilde{k}},$$

Hence $\tilde{\mathcal{T}} + \tilde{\mathcal{S}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right) -$ quasinormal operator.

ii)
$$\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{k}}\left(\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{*}}\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)\right)^{\tilde{k}} \cong \tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}$$
$$\cong \tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{S}}\right)^{\tilde{k}}$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{T}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\tilde{\mathcal{S}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\right)^{\tilde{k}}\left(\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\tilde{\mathcal{T}}^{\tilde{k}}\tilde{\mathcal{S}}^{\tilde{k}}\right)$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}^{\tilde{*}}\tilde{\mathcal{S}}^{\tilde{*}}\right)^{\tilde{k}}\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{k}}\right)\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{k}}$$
$$\cong \tilde{A}\left(\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{*}}\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)\right)^{\tilde{k}}\left(\tilde{\mathcal{T}}\tilde{\mathcal{S}}\right)^{\tilde{k}},$$

Hence $\tilde{\mathcal{T}}\tilde{\mathcal{S}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right) -$ quasinormal operator.

Now, the following lemma gives to illustrate the relation between $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right) -$ quasinormal operators with ($\mathcal{FS}$ −self-adjoint, $\mathcal{FS}$ −skew-adjoint, $\mathcal{FS}$ −normal and $\mathcal{FS}$ −unitary) operator.

Lemma 4.9: *Let* $\tilde{\mathcal{T}} \in \tilde{B}(\widetilde{\mathcal{H}})$ *then:*

i) If $\tilde{\mathcal{T}}$ is $\mathcal{FS}$ −self-adjoin operator so, $\tilde{\mathcal{T}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operator.

ii) If $\tilde{\mathcal{T}}$ is $\mathcal{FS}$ −skew-adjoin operator so, $\tilde{\mathcal{T}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operator.

iii) If $\tilde{\mathcal{T}}$ is $\mathcal{FS}$ −normal operator so, $\tilde{\mathcal{T}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operator.

vi) If $\tilde{\mathcal{T}}$ is $\mathcal{FS}$ −unitary operator so, $\tilde{\mathcal{T}}$ is $\mathcal{FS} - \left(\widehat{k^*} - \hat{A}\right)$ −quasinormal operator.

5. Discussion and Conclusion

Our initial findings are more extensive, generalized, and exact when fuzzy and soft sets are combined. Substantial research has been done on fuzzy soft normative spaces, fuzzy soft Hilbert spaces, and operators with fuzzy soft linearity. The fuzzy soft normal operator and fuzzy soft unitary operator are two members of a unique class of fuzzy soft linear operators that have recently been developed.

We obtain novel conclusions that are more extensive, broad, and exact by combining fuzzy and soft sets. The ideas underlying some of these general extension, or instance, we present new classes of fuzzy soft operators, such as the fuzzy soft $\mathcal{FS} - \left(\widehat{\mathrm{k}^*} - \widehat{\mathrm{A}}\right)$ quasinormal operator in fuzzy soft Hilbert space, or simply $\mathcal{FS} - \left(\widehat{\mathrm{k}^*} - \widehat{\mathrm{A}}\right)$)-quasinormal operator, have not received much attention from scholars. The text of the most significant of these theorems should be provided, together with an explanation of the concept's key features and its connections to other kinds in the same field. Moreover, new characterizations of the $\mathcal{FS} - \left(\widehat{\mathrm{k}^*} - \widehat{\mathrm{A}}\right)$quasinormal operator have been introduced in this article.

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