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Solving the partial differential equations by using SEJI integral transform

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Abstract

The integral transformations have become very important in solving differential equations, especially those that have an applied aspect in the physical, chemical, economic and other scientific fields.

In this article, the suggested transform which named by SEJI integral transform is developed for solving the partial derivatives and applied to solve several partial differential equations.

This paper introduced the definition of the suggested transform with its properties that have the importance in this field. This new transform has an ability to applying in {PDEs) and

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finding the exact solution for these equations. Kinds of partial differential equations have solved by using this transform.

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1. Introduction

The integral transform has been widely utilized to solve differential equations; as a result, there are many words on the theory and uses of integral transforms, including Laplace, Fourier, Sumudu and SEE complex, Emad-Sara, Emad-Falih, and KAJ Transforms [1-17]. For the solution of ordinary and partial differential equations, SEJI integral transform was recently presented.

We develop the formula for the SEJI integral transform of partial derivatives in this paper and use it to address various starting value issues.

2. Basic Concepts

2.1 *Definition:* [18]

SEJI integral transform of the function $f(t)$ is formed as:

$$T_g^c\{f(t)\} = F_g^c(\mathfrak{s}) = p(\mathfrak{s}) \int_{t=0}^{\infty} e^{-iq(\mathfrak{s})t} f(t)dt. (1)$$

Where $t \geq 0, p(\mathfrak{s}) \neq 0$ and $q(\mathfrak{s}) > 0, p(\mathfrak{s}), q(\mathfrak{s}) \in \mathbb{R}, i = \sqrt{-1}$.

2.2 *SEJI Integral Transform for Functions:* [18]

A list of the new transform for several functions is given below in Table 1:

Table 1

Functions $f(t)$	$T_g^c\{f(t)\} = F_g^c(\mathfrak{s})$	**Functions $f(t)$**	$T_g^c\{f(t)\} = F_g^c(\mathfrak{s})$
1. 1	$\frac{-ip(\mathfrak{s})}{q(\mathfrak{s})}$	7. $\sin(dt)$	$\frac{-d\,p(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2 - d^2}$, $q(\mathfrak{s}) > \lvert d\rvert$
2. $t^j, j \in N$	$(-i)^{j+1} \frac{j!\ p(\mathfrak{s})}{[q(\mathfrak{s})]^{j+1}}$	8. $\cos(dt)$	$\frac{-i\,p(\mathfrak{s})\,q(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2 - d^2}$.

Contd...

3. $t^j, j > -1$	$(-i)^{j+1}\dfrac{\Gamma(j+1)\,p(\mathfrak{s})}{[q(\mathfrak{s})]^{j+1}}$	9. $\sinh(kt)$	$\dfrac{-k\,p(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2+k^2}$, $q(\mathfrak{s}) > 0.$
4. $e^{ct}, c \in \mathbb{R}$	$-p(\mathfrak{s})\left[\dfrac{c}{c^2+\left(q(\mathfrak{s})\right)^2}+i\dfrac{q(\mathfrak{s})}{c^2+\left(q(\mathfrak{s})\right)^2}\right], q(\mathfrak{s}) > c.$	10. $\cosh(kt)$	$\dfrac{-i\,p(\mathfrak{s})\,q(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2+k^2}.$

2.3. *The Inverse of SEJI Integral Transform* [18]

A list of the inverse of the transform is showed for several functions in Table2:

Table 2

$\boldsymbol{F_g^c(\mathfrak{s})}$	$\boldsymbol{f(t) = T_g^{c\,-1}\{F_g^c(\mathfrak{s})\}}$	$\boldsymbol{F_g^c(\mathfrak{s})}$	$\boldsymbol{f(t) = T_g^{c\,-1}\{F_g^c(\mathfrak{s})\}}$
1. $\frac{-ip(\mathfrak{s})}{q(\mathfrak{s})}$	1	5. $\frac{-l\,p(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2-l^2}$, $q(\mathfrak{s}) > \lvert l\rvert$	$\sin(lt)$
2. $(-i)^{j+1}\frac{p(\mathfrak{s})}{[q(\mathfrak{s})]^{j+1}}$, $n \in N$	$\dfrac{t^j}{j!}$	6. $\dfrac{-i\,p(\mathfrak{s})\,q(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2-l^2}$	$\cos(lt)$
3. $(-i)^{j+1}\dfrac{p(\mathfrak{s})}{[q(\mathfrak{s})]^{j+1}}$, $j > -1$	$\dfrac{t^j}{\Gamma(j+1)}$	7. $\dfrac{-\varepsilon\,p(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2+\varepsilon^2}$, $q(\mathfrak{s}) > 0.$	$\sinh(\varepsilon t)$
4. $-p(\mathfrak{s})\left[\frac{r}{r^2+\left(q(\mathfrak{s})\right)^2}+i\frac{q(\mathfrak{s})}{r^2+\left(q(\mathfrak{s})\right)^2}\right]$, $q(\mathfrak{s}) > r.$	$e^{rt}, r \in \mathbb{R}$	8. $\dfrac{-i\,p(\mathfrak{s})\,q(\mathfrak{s})}{\left(q(\mathfrak{s})\right)^2+\varepsilon^2}.$	$\cosh(\varepsilon t)$

2.4 *SEJI Integral Transform of Derivatives:* [18]

The application of the new transform to derivative of $f(t)$. Let $T_g^c\{f(t)\} = F_g^c(\mathfrak{s})$, we get:

$$T_g^c\{f^{(j)}(t)\} = \left(iq(\mathfrak{s})\right)^j F_g^c(\mathfrak{s}) - p(\mathfrak{s})\left[\sum_{k=1}^{j}\left(iq(\mathfrak{s})\right)^{k-1} f^{(n-k)}(0)\right].$$

3. SEJI Transform of partial derivatives $f(t)$:

We employ integration by parts as follows to generate the SEJI transform of the partial derivatives:

$$T_g^c\left\{\frac{\partial f}{\partial t}(x,t)\right\} = p(\mathfrak{s})\int_0^\infty \left(\frac{\partial f}{\partial t}\right) e^{-iq(\mathfrak{s})t}dt$$

$$= \lim_{r\to\infty} p(\mathfrak{s})\int_0^r \left(\frac{\partial f}{\partial t}\right) e^{-iq(\mathfrak{s})t}dt,$$

$$= \lim_{r\to\infty}\left(\left[p(\mathfrak{s})e^{-iq(\mathfrak{s})t}f(x,t)\right]_0^r + iq(\mathfrak{s})p(\mathfrak{s})\int_0^r e^{-iq(\mathfrak{s})t}f(x,t)dt\right),$$

$$T_g^c\left\{\frac{\partial f}{\partial t}(x,t)\right\} = iq(\mathfrak{s})F_g^c(x,\mathfrak{s}) - p(\mathfrak{s})f(x,0). \quad (2)$$

Suppose that f is a piecewise continuous with exponential order.

Now

$$T_g^c\left\{\frac{\partial f}{\partial x}(x,t)\right\} = p(\mathfrak{s})\int_0^\infty \left[\frac{\partial f}{\partial x}(x,t)\right] e^{-iq(\mathfrak{s})t}dt,$$

$= \frac{\partial}{\partial x}\left[p(\mathfrak{s})\int_0^\infty f(x,t)e^{-iq(\mathfrak{s})t}dt\right]$, (by using Leibnitz rule),

$$T_g^c\left\{\frac{\partial f}{\partial x}(x,t)\right\} = \frac{\partial}{\partial x}\left[F_g^c(x,\mathfrak{s})\right] = \frac{d}{dx}\left[F_g^c(x,\mathfrak{s})\right]. \quad (3)$$

Also we can find:

$$T_g^c\left\{\frac{\partial^2 f}{\partial x^2}(x,t)\right\} = \frac{d^2}{dx^2}\left[F_g^c(x,\mathfrak{s})\right]. \quad (4)$$

To find:

$$T_g^c\left\{\frac{\partial^2 f}{\partial t^2}(x,t)\right\}.$$

Let

$g = \frac{\partial f}{\partial t}$, then by using (2) we have

$$T_g^c\left\{\frac{\partial^2 f}{\partial t^2}(x,t)\right\} = T_g^c\left\{\frac{\partial g}{\partial t}(x,t)\right\} = iq(\mathfrak{s})T_g^c\{g(x,t)\} - p(\mathfrak{s})g(x,0),$$

$$= iq(\mathfrak{s})\left[iq(\mathfrak{s})F_g^c(x,\mathfrak{s}) - p(\mathfrak{s})f(x,0)\right] - p(\mathfrak{s})\frac{\partial f}{\partial t}(x,0),$$

$$T_g^c\left\{\frac{\partial^2 f}{\partial t^2}(x,t)\right\} = [iq(\mathfrak{s})]^2F_g^c(x,\mathfrak{s}) - iq(\mathfrak{s})p(\mathfrak{s})f(x,0) - p(\mathfrak{s})\frac{\partial f}{\partial t}(x,0). \quad (5)$$

4. Applications:

In this section, some (PDEs) are presented to solve them by applying the new transform for reaching to exact solution.

Example 1: To evaluate the solution of the first order (IVP):

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y,$$

$$y(x,0) = 6e^{-3x} \quad (6)$$

Where y is bounded for $x > 0$ and $t > 0$.

By taking SEJI transform of (6), we have

$$\frac{d}{dx}\left[F_g^c(x,\mathfrak{s})\right] = 2iq(\mathfrak{s})F_g^c(x,\mathfrak{s}) - 2p(\mathfrak{s})f(x,0) + F_g^c(x,\mathfrak{s}),$$

$$\frac{d}{dx}\left[F_g^c(x,\mathfrak{s})\right] - (2iq(\mathfrak{s})+1)\left[F_g^c(x,\mathfrak{s})\right] = -12p(\mathfrak{s})e^{-3x}.$$

We get linear ordinary differential equation.

The integrating factor is $I = e^{\int(-2iq(\mathfrak{s})-1)dx} = e^{-(2iq(\mathfrak{s})+1)x}$.

Therefore,

$$T_g^c\{y(x,t)\} = \frac{6p(\mathfrak{s})}{iq(\mathfrak{s})+2}e^{-3x} + cp(\mathfrak{s})e^{-(2iq(\mathfrak{s})+1)x}$$

Since $T_g^c\{y(x,t)\}$ is bounded, then $c = 0$. By the inverse of SEJI transform, we obtain:

$$y(x,t) = 6e^{-3x}T_g^{c^{-1}}\left\{\frac{p(\mathfrak{s})}{iq(\mathfrak{s})+2}\right\},$$

$$= 6e^{-3x}T_g^{c^{-1}}\left\{\frac{-p(\mathfrak{s})}{-2-iq(\mathfrak{s})}\cdot\frac{-2+iq(\mathfrak{s})}{-2+iq\mathfrak{s}}\right\},$$

$$= 6e^{-3x}e^{-2t} = 6e^{-2t-3x}.$$

Example 2: The Laplace equation is:

$$w_{xx} + w_{tt} = 0, u(x,0) = 0, w_t(x,0) = \cos x\,, x, t > 0 \quad (7)$$

Apply SEJI transform of (7), we have

$$\frac{d^2}{dx^2}\,T_g^c\{w(x,t)\} + [iq(\mathfrak{s})]^2T_g^c\{w(x,t)\} -$$

$$iq(\mathfrak{s})p(\mathfrak{s})w(x,0) - p(\mathfrak{s})\frac{\partial w}{\partial t}(x,0) = 0,$$

$$\frac{d^2}{dx^2}\,T_g^c\{w(x,t)\} + [iq(\mathfrak{s})]^2T_g^c\{w(x,t)\} = p(\mathfrak{s})\cos x.$$

The specific solution to this second order differential equation is of the form:

$$T_g^c\{w(x,t)\} = \frac{1}{D^2 + [iq(\mathfrak{s})]^2}[p(\mathfrak{s})\cos x],$$

$$T_g^c\{w(x,t)\} = \frac{-p(\mathfrak{s})}{1+[q(\mathfrak{s})]^2}\cos x \quad (8)$$

Taking the invers of the SEJI transform for (8), we get:

$$w(x,t) = \cos x\, T_g^{c^{-1}}\left\{\frac{-p(\mathfrak{s})}{1+[q(\mathfrak{s})]^2}\right\} = \cos x \sinh t.$$

Example 3: Solve the wave equation:

$$w_{xx} - 4w_{tt} = 0, u(x,0) = \sin \pi x\, , w_t(x,0) = 0, x, t > 0 \ (9)$$

Apply SEJI transform of (9):

$$\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\}$$
$$-4\left([iq(\mathfrak{s})]^2 T_g^c\{w(x,t)\} - iq(\mathfrak{s})p(\mathfrak{s})w(x,0) - p(\mathfrak{s})\frac{\partial w}{\partial t}(x,0)\right) = 0,$$
$$\frac{1}{4}\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} - [iq(\mathfrak{s})]^2 T_g^c\{w(x,t)\} = -iq(\mathfrak{s})p(\mathfrak{s}) \sin \pi x.$$

Above is the second (ODE) have the particular solution in the form

$$T_g^c\{w(x,t)\} = \frac{1}{\frac{1}{4}D^2 - [iq(\mathfrak{s})]^2}[-iq(\mathfrak{s})p(\mathfrak{s}) \sin \pi x],$$
$$= \frac{-iq(\mathfrak{s})p(\mathfrak{s})}{[q(\mathfrak{s})]^2 - \left(\frac{\pi}{2}\right)^2} \sin \pi x \ \ (10)$$

Use the invers of SEJI transform for (10) to get:

$$w(x,t) = \sin \pi x\, T_g^{c^{-1}}\left\{\frac{-iq(\mathfrak{s})p(\mathfrak{s})}{[q(\mathfrak{s})]^2 - \left(\frac{\pi}{2}\right)^2}\right\} = \sin \pi x \cos \frac{\pi}{2} t.$$

Example 4: The homogeneous heat equation in 1 dimension:

$$4w_t = w_{xx}, w(x,0) = \sin \frac{\pi}{2} x\, , x, t > 0 \ (11)$$

The SEJI transform of (9) is

$$4iq(\mathfrak{s})T_g^c\{w(x,t)\} - 4p(\mathfrak{s})w(x,0) = \frac{d^2}{dx^2}\, T_g^c\{w(x,t)\},$$
$$\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} - 4iq(\mathfrak{s})T_g^c\{w(x,t)\} = -4p(\mathfrak{s}) \sin \frac{\pi}{2} x.$$

To find the particular solution for solving $T_g^c\{w(x,t)\}$

$$T_g^c\{w(x,t)\} = \frac{1}{D^2 - 4iq(\mathfrak{s})}\left[-4p(\mathfrak{s}) \sin \frac{\pi}{2} x\right],$$
$$T_g^c\{w(x,t)\} = \frac{-4p(\mathfrak{s}) \sin \frac{\pi}{2} x}{-\left(\frac{\pi}{2}\right)^2 - 4iq(\mathfrak{s})},$$

$$T_g^c\{w(x,t)\} = \frac{p(\mathfrak{s})\sin\frac{\pi}{2}x}{\frac{\pi^2}{16} + iq(\mathfrak{s})} \ (12)$$

To get $w(x,t)$ apply the invers of SEJI transform for (12):

$$w(x,t) = \sin\frac{\pi}{2}x\, T_g^{c^{-1}}\left\{\frac{p(\mathfrak{s})}{\frac{\pi^2}{16} + iq(\mathfrak{s})}\right\}$$

$$w(x,t) = \sin\frac{\pi}{2}x\, T_g^{c^{-1}}\left\{\frac{-p(\mathfrak{s})}{-\frac{\pi^2}{16} - iq(\mathfrak{s})} * \frac{\frac{-\pi^2}{16} + iq(\mathfrak{s})}{\frac{-\pi^2}{16} + iq(\mathfrak{s})}\right\} = e^{\frac{-\pi^2}{16}t}\sin\frac{\pi}{2}x.$$

Example 5: Consider the telegraphers equation:

$$w_{tt} + 2\alpha w_t = \alpha^2 w_{xx}, 0 < x < 1, t > 0 \ (13)$$

where, $$w(x,0) = \cos x\,, w_t(x,0) = 0 \ (14)$$

Take SEJI transform of (13) and substituting (14) to get

$$[iq(\mathfrak{s})]^2 T_g^c\{w(x,t)\} - iq(\mathfrak{s})p(\mathfrak{s})\cos x + 2\alpha iq(\mathfrak{s})T_g^c\{w(x,t)\}$$
$$-2\alpha p(\mathfrak{s})\cos x = \alpha^2\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\},$$
$$\alpha^2\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} - \left([iq(\mathfrak{s})]^2 + 2\alpha iq(\mathfrak{s})\right) T_g^c\{w(x,t)\}$$
$$= -[iq(\mathfrak{s}) + 2\alpha]p(\mathfrak{s})\cos x.$$

To find the particular solution for solving $T_g^c\{u(x,t)\}$

$$T_g^c\{w(x,t)\} = \frac{1}{\alpha^2 D^2 - \left([iq(\mathfrak{s})]^2 + 2\alpha iq(\mathfrak{s})\right)}(-[iq(\mathfrak{s}) + 2\alpha]p(\mathfrak{s})\cos x),$$
$$= \frac{[iq(\mathfrak{s}) + 2\alpha]p(\mathfrak{s})\cos x}{\left(\alpha + iq(\mathfrak{s})\right)^2} \ (15)$$

Then, we get $u(x,t)$ by using the invers of SEJI transform for (15):

$$w(x,t) = \cos x\, T_g^{c^{-1}}\left\{\frac{p(\mathfrak{s})(iq(\mathfrak{s}) + 2\alpha)}{\left(\alpha + iq(\mathfrak{s})\right)^2}\right\},$$
$$w(x,t) = (e^{-\alpha t} + \alpha t e^{-\alpha t})\cos x = (1 + \alpha t)\, e^{-\alpha t}\cos x.$$

Example 6: Consider the linear telegraph equation:

$$w_{xx} = w_{tt} + 2w_t + w, w(x,0) = e^x, u_t(x,0) = -2e^x \ (16)$$

Apply SEJI transform of (16) to get

$$\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} = [iq(\mathfrak{s})]^2 T_g^c\{w(x,t)\} - iq(\mathfrak{s})p(\mathfrak{s})e^x + 2p(\mathfrak{s})e^x + 2iq(\mathfrak{s})T_g^c\{w(x,t)\} - 2p(\mathfrak{s})e^x + T_g^c\{w(x,t)\},$$

$$\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} - ([iq(\mathfrak{s})]^2 + 2iq(\mathfrak{s}) + 1)\, T_g^c\{w(x,t)\} = -iq(\mathfrak{s})p(\mathfrak{s})e^x.$$

Then,

$$T_g^c\{w(x,t)\} = \frac{1}{D^2 - ([iq(\mathfrak{s})]^2 + 2iq(\mathfrak{s}) + 1)}(-iq(\mathfrak{s})p(\mathfrak{s})e^x),$$

$$= \frac{p(\mathfrak{s})}{iq(\mathfrak{s}) + 2} e^x \ (17)$$

Taking the invers of the SEJI transform for (17), we get:

$$w(x,t) = e^x T_g^{c^{-1}}\left\{\frac{p(\mathfrak{s})}{iq(\mathfrak{s}) + 2}\right\} = e^{x-2t}.$$

Example 7: Consider the second order linear homogenous Klein-Gordan equation:

$$w_{tt} = w_{xx} + w_x + 2w, -\infty < x < \infty, t > 0,$$

Subject to:

$$w(x,0) = e^x,\ u_t(x,0) = 0 \ (18)$$

By taking the SEJI transform of (18), we have

$$[iq(\mathfrak{s})]^2 T_g^c\{w(x,t)\} - iq(\mathfrak{s})p(\mathfrak{s})e^x = \frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} + \frac{d}{dx}\, T_g^c\{w(x,t)\} + 2T_g^c\{w(x,t)\},$$

$$\frac{d^2}{dx^2}\, T_g^c\{w(x,t)\} + \frac{d}{dx}\, T_g^c\{w(x,t)\} + (2 - [iq(\mathfrak{s})]^2)\, T_g^c\{w(x,t)\} = -iq(\mathfrak{s})p(\mathfrak{s})e^x.$$

Hence,

$$T_g^c\{w(x,t)\} = \frac{1}{D^2 + D + (2 - [iq(\mathfrak{s})]^2)}(-iq(\mathfrak{s})p(\mathfrak{s})e^x),$$

$$= \frac{-iq(\mathfrak{s})p(\mathfrak{s})}{4 + [q(\mathfrak{s})]^2} e^x \ (19)$$

By taking the invers of SEJI transform for (17), we get:

$$w(x,t) = e^x T_g^{c^{-1}}\left\{\frac{iq(\mathfrak{s})p(\mathfrak{s})}{4 + [q(\mathfrak{s})]^2}\right\} = e^x \cosh 2t.$$

5. Discussion and Conclusion

In this essay, we introduce and explain the SEJI transform of partial derivatives and show how to use it to solve various initial value issues. We were able to quickly arrive at the correct answer.

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