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SEJI integral transform of Bessel's functions

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Abstract

Bessel's functions have great mathematical importance in solving differential equations, especially in the applied field. There is a lot of research that review some applied equations and their solutions using these functions. Since integral transforms have high efficiency in finding accurate solutions to distinct kinds of differential and integral equations, we use in this paper the proposed method "SEJI integral transform" for functions of Bessel. These certain applications have been taken for these functions and solved by SEJI integral transform that reached to the exact solutions.

In addition, some characteristics of this transformation were introduced that helped in arriving at accurate solutions.

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1. Introduction

Integral transformations have gained a wide position in the mathematical field due to their contribution to solving mathematical and scientific equations[1-6].

Functions of Bessel are now used in conjunction with a number of equations, including physical and mathematical equations to solve a wide range of scientific and engineering problems.

Functions of Bessel can be used to solve a variety of problems in mathematical and physical applications [7].

Functions of Bessel of order $n \in N$ is given by[8-17]

$$J_n(t) = \frac{t^n}{2^n n!}\left[1 - \frac{t^2}{4(n+1)} + \frac{(t^2)^2}{4.4.(n^2+3n+2)} - \frac{(t^2)^3}{4.4.6.(n^2+3n+2)(n+3)} + \cdots\right] \quad (1)$$

If $n = 0$, the infinite power series gives functions of Bessel of zero order, indicated by $J_0(t)$.

$$J_0(t) = 1 - \frac{t^2}{4} + \frac{(t^2)^2}{4^3} - \frac{(t^2)^3}{4^3.6^2} + \cdots. (2)$$

functions of Bessel of one order, represented by$J_1(t)$, is obtained when $n = 1$.

$$J_1(t) = \frac{t}{2} - \frac{t^3}{4^2} + \frac{t^5}{4^3.6} - \frac{t^7}{4.4^2.6^2.8} + \cdots. (3)$$

So, eq.(3) written as the form

$$J_1(t) = \frac{t}{2} - \frac{t^3}{2^3.2!} + \frac{t^5}{2^5.2!.3!} - \frac{t^7}{2^7.3!.4!} + \cdots. (4)$$

functions of Bessel of two orders, denoted by $J_2(t)$ and obtained for $n = 2$ is given by

$$J_2(t) = \frac{t^2}{2.4} - \frac{t^4}{4^2.6} + \frac{t^6}{4^3.6.8} - \frac{t^7}{4^3.6^2.8.10} + \cdots. (5)$$

To define SEJI integral transform of the function $f(t)$[13]

$$T_g^c\{f(t); q(s)\} = F_g^c(s) = p(s)\int_{t=0}^{\infty} e^{-iq(s)t} f(t)dt. (6)$$

Where $t \geq 0, p(s) \neq 0$ and $q(s) > 0, p(s), q(s) \in R, i = \sqrt{-1}$.

Properties of SEJI Transform:

Several crucial SEJI integral transform features are established in this section. We now display these characteristics:

2.1 Linearity Property:

Let $T_g^c\{f(t)\} = F_g^c(s)$ and $T_g^c\{h(t)\} = H_g^c(s)$, then for every u and v are arbitrary constants, then:

$$T_g^c\{uf(t) \pm vh(t)\} = uF_g^c(s) \pm vH_g^c(s)$$

Proof:

$$T_g^c\{uf(t) \pm vh(t)\} = p(s)\int_{t=0}^{\infty} e^{-iq(s)t}\big(uf(t) \pm vh(t)\big)dt,$$

$$= p(s)\int_{t=0}^{\infty} e^{-iq(s)t}\big(uf(t)\big)dt \pm p(s)\int_{t=0}^{\infty} e^{-iq(s)t}\big(vh(t)\big)dt,$$

$$= up(s)\int_{t=0}^{\infty} e^{-iq(s)t}f(t)dt \pm vp(s)\int_{t=0}^{\infty} e^{-iq(s)t}h(t)dt,$$

$$T_g^c\{uf(t) \pm vh(t)\} = uF_g^c(s) \pm vH_g^c(s).$$

2.2 *Change of Scale Property:*

Let $T_g^c\{f(t)\} = F_g^c(s)$ then SEJI integral transform of $f(\alpha t)$ is

$$T_g^c\{f(\alpha t)\} = \frac{1}{\alpha}F_g^c\left(\frac{q(s)}{\alpha}\right). (7)$$

Proof: $T_g^c\{f(t)\} = p(s)\int_{t=0}^{\infty} e^{-iq(s)t}f(t)dt. (8)$

Let $z = \alpha t$, and $t = \frac{z}{\alpha}$, then $dt = \frac{1}{\alpha}dz. (9)$

By substituting z in equation (8), we get:

$$T_g^c\{f(\alpha t)\} = p(s)\int_{t=0}^{\infty} e^{-iq(s)\left(\frac{z}{\alpha}\right)}f(z)\frac{1}{\alpha}dz,$$

$$= \frac{p(s)}{\alpha}\int_{t=0}^{\infty} e^{-i\left(\frac{q(s)}{\alpha}\right)z}f(z)dz,$$

$$= \frac{1}{\alpha}T_g^c\left\{f(t);\frac{q(s)}{\alpha}\right\}.$$

2.3 *Convolution Theorem:*

If SEJI integral transform of functions $g_1(t)$ is $T_g^c\{g_1(t)\}$ and $g_2(t)$ is $T_g^c\{g_2(t)\}$ then SEJI integral transform of their convolution $T_g^c\{g_1(t)\} \star T_g^c\{g_2(t)\}$ is defined by:

$$T_g^c\{g_1(t) \star g_2(t)\} = (p(s))^{-1}\, T_g^c\{g_1(t)\}T_g^c\{g_2(t)\}$$

$$= \frac{1}{p(s)}T_g^c\{g_1(t)\}T_g^c\{g_2(t)\}.$$

Where $g_1(t) \star g_2(t)$ is given by:

Proof:

$$T_g^c\{g_1(t)\star g_2(t)\} = p(s)\int_{t=0}^{\infty} e^{-iq(s)t}dt\int_0^t g_1(t-\zeta).g_2(\zeta)d\zeta.$$
$$= p(s)\int_0^{\infty} g_2(\zeta)d\zeta\int_0^{\infty} e^{-iq(s)t}g_1(t-\zeta)d\zeta.$$

Now, we assume that $t-\zeta = r$, we have:

$$T_g^c\{g_1(t)\star g_2(t)\} = p(s)\int_0^{\infty} g_2(\zeta)d\zeta\int_0^{\infty} e^{-iq(s)(r+\zeta)}g_1(r)dr.$$
$$= p(s)\int_0^{\infty} e^{-iq(s)\zeta}g_2(\zeta)d\zeta\int_0^{\infty} e^{-iq(s)r}g_1(r)dr.$$
$$= (p(s))^{-1}T_g^c\{g_1(t)\}T_g^c\{g_2(t)\}. \blacksquare$$

2.4 *SEJI Integral Transform of Derivatives:[13]*

Let $T_g^c\{f(t)\} = F_g^c(s)$, then:

$$T_g^c\{f^{(n)}(t)\} = \left(iq(s)\right)^n F_g^c(s) - p(s)\left[\sum_{k=1}^{n} \left(iq(s)\right)^{k-1} f^{(n-k)}(0)\right].$$

2.5 *SEJI Integral Transform for Functions:[13]*

A list the SEJE transform for several important functions is given below (Table 1):

Table (1)

Functions $f(t)$	$T_g^c\{f(t)\} = F_g^c(s)$	**Functions** $f(t)$	$T_g^c\{f(t)\} = F_g^c(s)$
1. 1	$\frac{-ip(s)}{(q(s))}$	6. $sin\ (mt)$	$\frac{-m\,p(s)}{\left(q(s)\right)^2 - m^2}, q(s) > \|m\|$
2. t	$\frac{-\,p(s)}{(q(s))^2}$	7. $cos\ (mt)$	$\frac{-i\,p(s)\,q(s)}{\left(q(s)\right)^2 - m^2}, q(s) > \|m\|.$
3. $t^{\delta}, \delta \in N$	$\frac{(-i)^{\delta+1}\delta!\ p(s)}{[q(s)]^{(\delta+1)}}$	8. $sinh\ (mt)$	$\frac{-m\,p(s)}{\left(q(s)\right)^2 + m^2}, q(s) > 0.$
4. $t^{\delta}, \delta > -1$	$(-i)^{\delta+1}\frac{\Gamma(\delta+1)\,p(s)}{[q(s)]^{\delta+1}}$	9. $cosh\ (mt)$	$\frac{-i\,p(s)\,q(s)}{\left(q(s)\right)^2 + m^2}, q(s) > 0.$
5. $e^{mt}, m \in R$	$-p(s)\left[\frac{m+iq(s)}{m^2+\left(q(s)\right)^2}\right],$ $q(s) > m.$		

2.6 *Inverse of SEJI Transform [13]*

The inverse of SEJI transform for several functions is given by the following Table (2)

Table (2)

$F_g^c(s)$	$f(t) = T_g^{c^{-1}}\{F_g^c(s)\}$	$F_g^c(s)$	$f(t) = T_g^{c^{-1}}\{F_g^c(s)\}$
$\frac{-ip(s)}{q(s)}$	1	$\frac{-r\,p(s)}{(q(s))^2 - r^2}, q(s) > \lvert r\rvert$	$sin\,(rt)$
$\frac{-\,p(s)}{[q(s)]^2}$	t	$\frac{-i\,p(s)\,q(s)}{(q(s))^2 - r^2}, q(s) > \lvert r\rvert.$	$cos\,(rt)$
$(-i)^{\delta+1}\frac{p(s)}{[q(s)]^{\delta+1}}, \delta \in N$	$\frac{t^\delta}{\delta!}$	$\frac{-r\,p(s)}{(q(s))^2 + r^2}, q(s) > 0.$	$sinh\,(rt)$
$(-i)^{\delta+1}\frac{p(s)}{[q(s)]^{\delta+1}}, j > -1$	$\frac{t^\delta}{\Gamma(\delta+1)}$	$\frac{-i\,p(s)\,q(s)}{(q(s))^2 + r^2}, q(s) > 0.$	$cosh\,(rt)$
$-p(s)\left[\frac{r + iq(s)}{r^2 + (q(s))^2}\right], q(s) > r.$	$e^{rt}, r \in R$		

2. Relations Between $J_0(t)$ and $J_1(t)$[10, 12]

$$\frac{d}{dt}J_0(t) = -J_1(t). (10)$$

3. Relations Between $J_0(t)$ and $J_2(t)$[12]

$$J_2(t) = J_0(t) + 2J_0^{''(t)}. (11)$$

The SEJI integral transform for Bessel's Functions :

i. When $f(t) = J_0(t)$:

Apply SEJI transform of equation (2), we get

$$T_g^c\{J_0(t)\} = T_g^c\{1\} - \frac{1}{2^2}T_g^c\{t^2\} + \frac{1}{2^2.4^2}T_g^c\{t^4\} - \frac{1}{2^2.4^2.6^2}T_g^c\{t^6\} + \cdots,$$

$$= \frac{-ip(s)}{q(s)} - \frac{1}{2^2}\left(\frac{2!\,(-i)^3p(s)}{[q(s)]^3}\right) + \frac{1}{2^2.4^2}\left(\frac{4!\,(-i)^5p(s)}{[q(s)]^5}\right) - \frac{1}{2^2.4^2.6^2}\left(\frac{6!\,(-i)^7p(s)}{[q(s)]^7}\right) + \cdots,$$

$$= \left(\frac{-i \cdot p(s)}{q(s)}\right)\left[1 - \frac{1}{2}\left(\frac{-1}{[q(s)]^2}\right) + \frac{1.3}{2.4}\left(\frac{-1}{[q(s)]^2}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{-1}{[q(s)]^2}\right)^3 + \cdots\right],$$

$$T_g^c\{J_0(t)\} = \left(\frac{-i \cdot p(s)}{q(s)}\right)\left[1 + \left(\frac{-i}{q(s)}\right)^2\right]^{\frac{-1}{2}},$$

$$T_g^c\{J_0(t)\} = \frac{-i \cdot p(s)}{\sqrt{(q(s))^2 - 1}}. (12)$$

ii. When $f(t) = J_1(t)$:

Apply SEJI transform of equation (10), we get

$$T_g^c\{J_1(t)\} = -T_g^c\left\{\frac{d}{dt}J_0(t)\right\}. (13)$$

Applying this feature to the function in equation (13), which is the SEJI integral transform of derivatives, we now get

$$T_g^c\{J_1(t)\} = -\left[iq(s)T_g^c\{J_0(t)\} - p(s)J_0(t)\right]. (14)$$

By using eq. (2) and eq. (12) in eq. (14), we get

$$T_g^c\{J_1(t)\} = -iq(s) \cdot \left(\frac{-ip(s)}{\sqrt{\left(q(s)\right)^2 - 1}}\right) + p(s),$$

$$T_g^c\{J_1(t)\} = p(s) - \frac{q(s)p(s)}{\sqrt{\left(q(s)\right)^2 - 1}}. (15)$$

iii. When $f(t) = J_2(t)$:

Apply SEJI transform of equation (11), we get

$$T_g^c\{J_2(t)\} = T_g^c\{J_0(t)\} + 2T_g^c\{J_0''(t)\}. (16)$$

With the help of eq.(12) and eq.(16), we can now apply the property SEJI transform of derivatives for $f(t)$

$$T_g^c\{J_2(t)\} = \frac{-i \cdot p(s)}{\sqrt{(q(s))^2 - 1}} + 2\left[\left(iq(s)\right)^2 T_g^c\{J_0(t)\} - p(s)J_0'(t) - iq(s)p(s)J_0(t)\right]. (17)$$

By using eq. (2), eq. (10) and eq. (12) in eq. (14), we have

$$T_g^c\{J_2(t)\} = \frac{-i \cdot p(s)}{\sqrt{(q(s))^2 - 1}} + 2\left[\left(iq(s)\right)^2 \frac{-i \cdot p(s)}{\sqrt{(q(s))^2 - 1}} + p(s) \cdot J_1(0) - i \cdot q(s) \cdot p(s)\right]. (18)$$

Using eq. (3) in eq. (18), we get

$$T_g^c\{J_2(t)\} = \frac{-ip(s) + 2\left(q(s)\right)^2 p(s) - 2iq(s)p(s)\sqrt{(q(s))^2 - 1}}{\sqrt{(q(s))^2 - 1}}. (19)$$

iv. If $f(t) = J_0(at)$:
From eq.(12), the SEJI integral transform of $J_0(t)$ is given by

$$T_g^c\{J_0(t)\} = \frac{-ip(s)}{\sqrt{(q(s))^2 - 1}}.$$

By applying change of scale property for SEJI transform, we obtain

$$T_g^c\{J_0(at)\} = \frac{1}{\alpha} T_g^c\left\{J_0(t); \frac{q(s)}{\alpha}\right\},$$

$$= \frac{1}{\alpha} . \frac{-ip(s)}{\sqrt{\left(\frac{q(s)}{\alpha}\right)^2 - 1}},$$

$$T_g^c\{J_0(at)\} = \frac{-ip(s)}{\sqrt{(q(s))^2 - a^2}}. (18)$$

v. If $f(t) = J_1(at)$:
From eq.(17), the SEJI integral transform of $J_1(t)$ is given by

$$T_g^c\{J_1(t)\} = p(s) - \frac{q(s)\, p(s)}{\sqrt{(q(s))^2 - 1}}$$

By applying change of scale property for SEJI transform, we obtain

$$T_g^c\{J_1(at)\} = \frac{1}{\alpha} T_g^c\left\{J_1(t); \frac{q(s)}{\alpha}\right\},$$

$$T_g^c\{J_1(at)\} = \frac{1}{\alpha}.\left[p(s) - \frac{\frac{q(s)}{\alpha}.p(s)}{\sqrt{\left(\frac{q(s)}{\alpha}\right)^2 - 1}}\right],$$

$$T_g^c\{J_1(at)\} = \frac{p(s)}{a}\left[1 - \frac{q(s)}{\sqrt{(q(s))^2 - a^2}}\right].(19)$$

vi. If $f(t) = J_2(at)$:

From eq.(17), the SEJI integral transform of $J_0(t)$ is given by

$$T_g^c\{J_2(t)\} = \frac{-ip(s) + 2\left(q(s)\right)^2 p(s) - 2iq(s)\, p(s)\sqrt{(q(s))^2 - 1}}{\sqrt{(q(s))^2 - 1}}.$$

By applying change of scale property for SEJI transform, we obtain

$$T_g^c\{J_2(at)\} = \frac{1}{\alpha} T_g^c\left\{J_2(t); \frac{q(s)}{\alpha}\right\},$$

$$T_g^c\{J_2(at)\} = \frac{1}{\alpha}.\left[\frac{-ip(s) + 2\left(\frac{q(s)}{\alpha}\right)^2 p(s) - 2i\left(\frac{q(s)}{\alpha}\right)p(s)\sqrt{\left(\frac{q(s)}{\alpha}\right)^2 - 1}}{\sqrt{\left(\frac{q(s)}{\alpha}\right)^2 - 1}}\right],$$

$$T_g^c\{J_2(at)\} = \frac{p(s)}{a^2}.\left[\frac{-ia^2 + 2(q(s))^2 - 2iq(s)\sqrt{(q(s))^2 - a^2}}{\sqrt{(q(s))^2 - a^2}}\right].(22)$$

2. Applications:

This section introduces a few examples of applications that demonstrate how the SEJI integral transform affects assessing functions of Bessel.

Application 1: Consider the integral

$$I(t) = \int_0^t J_0(u)J_0(t-u)dt.(23)$$

When both sides of eq.(23) are transformed using the SEJI integral, we have

$$T_g^c\{I(t)\} = T_g^c\left\{\int_0^t J_0(u)J_0(t-u)dt\right\}.(24)$$

Equation (24), the convolution theorem of SEJI transform, yields

$$T_g^c\{I(t)\} = \frac{1}{p(s)} T_g^c\{J_0(t)\} T_g^c\{J_0(t)\},$$

$$= \frac{1}{p(s)}\left(\frac{-ip(s)}{\sqrt{(q(s))^2 - 1}}\right)\left(\frac{-ip(s)}{\sqrt{(q(s))^2 - 1}}\right),$$

$$T_g^c\{I(t)\} = \frac{-p(s)}{(q(s))^2 - 1}. (25)$$

Then, $I(t) = T_g^{c^{-1}}\left\{\frac{-p(s)}{(q(s))^2-1}\right\} = sin\ t. (26)$

Application 2: Evaluate the integral

$$I(t) = \int_0^t \quad J_0(u)J_1(t-u)dt. (27)$$

When both sides of equation (27), we have used the SEJI integral transform, we have

$$T_g^c\{I(t)\} = T_g^c\left\{\int_0^t \quad J_0(u)J_1(t-u)dt\right\}, (28)$$

Equation (28), the convolution theorem of SEJI transform, yields

$$T_g^c\{I(t)\} = \frac{1}{p(s)} T_g^c\{J_0(t)\} T_g^c\{J_1(t)\},$$

$$= \frac{1}{p(s)}\left(\frac{-ip(s)}{\sqrt{(q(s))^2 - 1}}\right)\left(p(s) - \frac{q(s)p(s)}{\sqrt{(q(s))^2 - 1}}\right),$$

$$T_g^c\{I(t)\} = \frac{-ip(s)}{\sqrt{(q(s))^2 - 1}} + \frac{ip(s)q(s)}{(q(s))^2 - 1}. (29)$$

Then we get $I(t) = T_g^{c^{-1}}\left\{\frac{-ip(s)}{\sqrt{(q(s))^2-1}}\right\} + T_g^{c^{-1}}\left\{\frac{ip(s)q(s)}{(q(s))^2-1}\right\},$

$$I(t) = J_0(t) - coscos\ cos\ t. (30)$$

Application 3: Evaluate the following

$$I(t) = \int_0^t \quad J_1(t-u)dt. (31)$$

Take SEJI transform for eq. (31), we have

$$T_g^c\{I(t)\} = T_g^c\left\{\int_0^t \quad J_1(t-u)dt\right\}. (32)$$

Using the property (2.3) on eq.(32), we get

$$T_g^c\{I(t)\} = \frac{1}{p(s)} T_g^c\{1\} T_g^c\{J_1(t)\},$$

$$= \frac{1}{p(s)}\left(\frac{-ip(s)}{q(s)}\right)\left(p(s) - \frac{q(s)p(s)}{\sqrt{(q(s))^2 - 1}}\right),$$

$$T_g^c\{I(t)\} = \frac{-ip(s)}{q(s)} + \frac{ip(s)}{\sqrt{(q(s))^2 - 1}}. (33)$$

Then we obtain

$$I(t) = T_g^{c^{-1}}\left\{\frac{-ip(s)}{q(s)}\right\} + T_g^{c^{-1}}\left\{\frac{ip(s)}{\sqrt{(q(s))^2 - 1}}\right\},$$

$$I(t) = 1 - J_0(t). (34)$$

5. Discussion and Conclusion

We can use the SEJI integral transform of functions of Bessel in this essay. The examples described here illustrate the use of this transform for these functions to finding integrals containing functions of Bessel.

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