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On the φ-algebra

Naba Hasoon Jabir *
Areej Tawfeeq Hameed †
Depatment of Mathematics
Faculty of Eduation for Girls
University of Kufa
Najaf
MI 54001
Iraq

Abstract

We introduced a novel algebraic structure in this study with two operations, also known as ψ-algebras, defined its ideals and investigated a few of its properties. In particular, we showed that if is(χ; +, -, 0) is a ψ-algebra, then (χ; +) is a semigroup using identity 0. Additionally, we discussed how ideals and congruences relate to one another. We also discussed the subgroup,homomorphism and investigated some theories.

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Keywords: *ψ-algebras, ψ-subalgebra, ψ-deal, Homoorphis of ψ-algebra.*

1. Introduction

Y. Imai & $\mathcal{K}$. Is´ek i there are two types of abstract algebra were shown:

BC$\mathcal{K}$ −algebras and B $\boldsymbol{\mathcal{C}}$ I-algebras ([6,12, 16-24]). The type of BCK-algebra is recognized as a legitimate the class's subclass of BCI-algebras. In this paper, the notation is introduced of ψ-algebras, ψ-subalgebras, ψ ideals, homomorphism of ψ-algebras and investigate some related properties.

* *E-mail:* **nabaah.al-saedi@uokufa.edu.iq (Corresponding Author)**
† *E-mail:* **areej.tawfeeq@uokufa.edu.iq**

2. ψ -algebras

This paragraph introduces an algebraic structure called a ψ-algebra, as the following :

Definition 2.1: The algebraic system (X,+,-,0), the constant 0 and the operations + and - is known as ψ-algebras for every x,y, $z \in$ X

(ψ_1) $x - x = 0$,

(ψ_2) $(0 - x)+x=0$

(ψ_3) $(x - y)-z = x-(z + y)$,

(ψ_4) $(y + x)-(x - z)=y + z$.

We'll keep this short and simple (X;+,-,0) a ψ-algebra Binary relation in χ can be defined ($\leq$), $x \leq y \Leftrightarrow x - y = 0$

Example 2.2: Let the table below be a set for χ ={ 0, 1, 2, 3 }:

+	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

-	**0**	**1**	**2**	**3**
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Then (χ;+,-,0) is a ψ-algebra.

3. On ψ -subalgebras of ψ -algebras

We present the idea in this part of ψ-subalgebra in ψ algebra and give some examples and results of it.

Definition 3.1: Let (X;+,-,0) be a ψ-algebra and S be a nonempty set of X. S is known a ***ψ*-subalgebra of** X if $\chi + \gamma \in S$ and $\chi - \gamma \in S$, whenever χ, $\gamma \in S$.

Example 3.2: Let the table below be a set for (Z_4;$+_4$,$-_4$, 0):

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

-	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

(z_4;$+_4$, $-_4$, 0) is a ψ-algebra. It is simple to demonstrate ℓ_1={0, 2}=<2> and $\ell_2 = \mathcal{Z}_4$=<1> are ψ-subalgebras of ($\mathcal{Z}_4$;$+_4$,$-_4$, 0).

Proposition 3.3: Let $\{I_i$: $i\in\Lambda\}$ join family of ψ-subalgebras of ψalgebra (X;+,-,0), then $\bigcap_{i\in\Lambda} \ell_i$ is a ψ-sub algebras of X.

Proof: Since $\{I_i$: $i\in\Lambda\}$ be a family of ψ-subalgebras of X, su ose $x\in\bigcap_{i\in\Lambda} I_i$ and $\chi\in\bigcap_{i\in\Lambda} I_i$, then $\chi \in I_i$, $\chi \in I_i$, for every $i\in\Lambda$ but I_i is a ψ subalgebra of X, for every l $i\in\Lambda$. Then $x+y\in I_i$, and $\chi - \gamma \in I_i$ for all $i\in\Lambda$, therefore, $\chi + \gamma \in \bigcap_{i\in\Lambda} I_i$ and $\chi - \gamma \in \bigcap_{i\in\Lambda} I_i$. Hence $\bigcap_{i\in\Lambda} I_i$ is a ψ-subalgebr$a$ of X.

4. ψ -ideals of ψ -algebras

The paragraph introduces the idea of ψ-ideal in ψ-algebra and include examples and outcomes as well.

Definition 4.1: A nonempty subse$t$ I of ψ-algebra (χ+,-,0) is known as $\boldsymbol{\psi}$-idea$l$ as X if it gratifying: χ, Υ, $\mathcal{Z}\in X$,

(I_1) $0 \in I$,

(I_2) $(\Upsilon + \mathcal{Z}) \in I$ and $(\chi - \mathcal{Z}) \in I$ imply $(\Upsilon + \chi) \in I$

Example 4.2: Let ($\mathcal{Z}_6$;$+_6$,$-_6$, 0) comprise the following tables:

+	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{4}$	$\bar{4}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{5}$	$\bar{5}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$

-	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\bar{0}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$
$\bar{1}$	$\bar{1}$	$\bar{0}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$
$\bar{2}$	$\bar{2}$	$\bar{1}$	$\bar{0}$	$\bar{5}$	$\bar{4}$	$\bar{3}$
$\bar{3}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	$\bar{0}$	$\bar{5}$	$\bar{4}$
$\bar{4}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	$\bar{0}$	$\bar{5}$
$\bar{5}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{1}$	$\bar{0}$

Then (z_6;$+_6$, $-_6$, 0) is a ψ-algebra. It is simple to demonstrate $\ell_1=\{\bar{0}, \bar{3}\} = \langle\bar{3}\rangle$ and $\ell_2 = \{\bar{0}, \bar{2}, \bar{4}\} = \langle\bar{2}\rangle$ are ψ-ideals of ($\mathcal{Z}_6$;$+_6$,$-_6$,0)

Proposition 4.3: Let $\{\ell_i$: $i\in\nu\}$be a family of ψ-ideals of ψ-algeb$r$a (χ;+,-,0), the$n$ $\bigcap_{i\in\Lambda} \ell_i$ is ψ-$i$de$a$l of χ.

Proof: Since $\{\ell_i$: $i \in \nu$ $\}$be a fa$m$ily as ψ-$i$deals of χ, $t$hen $0\in\ell_i$, every $i \in \nu$, then $0\in\bigcap_{i\in\nu} \ell_i$. For any χ, γ, $\mathcal{Z}\in X$, su ose $\gamma + \mathcal{Z} \in \bigcap_{i\in\nu} \ell_i$ and $\chi - \mathcal{Z}\in\bigcap_{i\in\nu} \ell_i$, then $\gamma + \mathcal{Z} \in \ell_i$, $\chi - \mathcal{Z} \in \ell_i$, f$o$r every $i \in \nu$, but ℓ_i is ψ-ideal of χ, for all $i \in \nu$. The$n$$\gamma + \mathcal{X} \in \ell_i$, f$o$r every $i \in \nu$, then, $\gamma + \chi\in\bigcap_{i\in\nu} \ell$i. Hence $\bigcap_{i\in\nu} \ell$i is a ψ-$i$deal of X.

5. Homomorphism of ψ -algebras

This section looks at homomorphism of ψ-algebra and describe its properties.

Definition 5.1: Let (χ;+,-,0), (Y;$+'$,$-'$,$0'$) be ψ-algebras, the ma ing f:(X;+,-,0) → (Y;$+'$,$-'$, $0'$) is known a homo$morphism$ if gratifying: for every χ, γ∈X

1. $f(\chi+\gamma)$=$f(\chi)$$+'$$f(\gamma)$,
2. $f(\chi-\gamma)$=$f(\chi)$$-'$$f(\gamma)$,

We define ($ker\ \beta$)(χ)={ χ∈X: β (χ)=$0'$ }.

Theorem 5.2: *Let f: (χ;+,-,0) → (Y;$+'$,$-'$, $0'$) be a homomorphism of ψ-algebras, then*

(**F₁**) If I is a ψ-ideal of χ, then f (I) is a ψ-ideal of γ.
(**F₂**) If S is a ψ-subalgebra of X, then $\varphi(S)$ is a ψ-subalgebra of γ.
(**F₃**) If J is a ψ-ideal of Y, then φ^{-1} (J) is a ψ-ideal of X.
(**F₄**) If β is a ψ-subalgebra of Y, then φ^{-1} (β) is a ψ-subalgebra of X.

Proof: Since φ: (χ ;+,-,0) → (Y;$+'$,$-'$, $0'$) ishomo$morphism$ of ψ-algebras, (**F₁**) ℓet I is ψ-$i$deal of X, The fact that 0∈u I, by Theorem (5.2(A_1)), $0'$= φ(0)∈φ(I), so $0'$∈f(I).

Consider that now, for all χ, γ, τ∈X, $\varphi(\gamma)$$+'$ $\varphi(\tau)$=$\varphi(\gamma+\tau)$∈φ(I) and $\varphi(\mathcal{X})$$-'$ $\varphi(\tau)$=$\varphi(\mathcal{X}-\tau)$∈$\varphi$(I), thus,it follows that $\gamma+\tau\in$ I, $\chi-\tau\in$ I, because I is a ψ-$i$deal,thus it follows$\gamma+\chi\in$ I, which imply that $f(\gamma+\chi)$=$\varphi(\gamma)$$+'$ $\varphi(\chi)$∈φ(I). Hence φ(I) is a ψ-$i$deal of Y.

(**F₂**) Let S be a ψ-subalgebra of χ and α, λ∈S, since S is a ψ subalgebra we have $\alpha+\lambda$∈S and $\alpha-\lambda$∈S. Then there exist χ, $\gamma\in\varphi$(S) such that $\chi=\phi(\alpha)\ and\ \gamma=\varphi\ (\lambda)$. Hence $\varphi(\alpha+\lambda)$=φ (α)$+'$ $\varphi(\lambda)$=$\chi$$+'$ $\gamma\in\varphi(S)$ and $\varphi(\alpha-\lambda)$=φ (α)$-'$ $\varphi(\lambda)$=$\chi-\gamma\in\varphi(S)$. Thus ϕ (S) ψ-subalgebra of X.

(**F₃**) Let ϵ be a ψ-ideal of Y. Then 0'∈ϵ, and hence0=ϕ^{-1}($0'$)∈$\varphi^{-1}(\epsilon)$.

Now, for any χ, γ, τ∈X, let $\gamma+\tau\in\varphi^{-1}(\epsilon)$ and $\chi-\tau\in\varphi^{-1}(\epsilon)$. Thus, it follows $\varphi(\ \gamma+\tau)$=$\varphi(\gamma)$$+'$ $\varphi(\chi)$∈$\epsilon$$\phi(\chi-\tau)$=$\phi(\chi)$$-'$ $\phi(\tau)$∈ϵ. Since ϵ is a ψ-$i$deal l of Y, we obtain that $\varphi(\gamma)$$+'$$\varphi(\chi)$=$\varphi(\gamma+\chi)$∈$\phi(\epsilon)$. Consequently $\varphi(\gamma+\chi)$∈$\varphi(\epsilon)$ and $\gamma+\chi\in\varphi^{-1}(J)$, proving that $\varphi^{-1}(\epsilon)$ is a ψ-ideal of X.

(**F₄**) Let β be a ψ-sub algebra of Y and χ, $\gamma\in\varphi^{-1}(\beta)$. Let φ^{-1}(α)=χ and $\varphi^{-1}(\lambda)$=γ, for some α, $\lambda\in\beta$, thus $\varphi(\chi+\gamma)$ = $\varphi(\chi)$$+'$ φ (γ)=α$+'$ $\lambda\in\beta$, and $\varphi(\chi-\gamma)$=$\varphi(\chi)$$-'$ $\varphi(\gamma)$=α$-'$ $\lambda\in\beta$, as β is a ψ-subalg$e$bra. Thus $\chi+\gamma\in\varphi^{-1}(\beta)$ and $\chi-\gamma\in\varphi^{-1}(\beta)$. Hence $\varphi^{-1}(\beta)$ is ψ-sub alg$ebra$ of X.

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