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On some properties of Lambda ideal open sets

Hawra S. Abu Hemed Al-Ali *
Yiezi K. Al-Talkany †
Department of Mathematics
University of Kufa
Najaf-alashraf
Iraq

Abstract

This study's main goal is to provide a revised definition of λ-op.s.s by employment the definition ideal and Bi topspa in addition to studying its theories, characteristics and observations.

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1. Introduction

In this research, Kuratowski.K. in 1933[2] and Vaidyanathaswamy.R in 1945[3] researched the ideatopspa. (ideal topological spaces), a unique notion in the history of topology that we shall examine here.

The notion of local function of a subset A of X is one of the defining characteristics of the ideal topspa. Since then, researchers like Jankovic and Hamlett have dug into these ideas. T.R.[4], who presented research on a (topological) idea among the first to do so. Open sets are a notion first described by Njasted.O [1]. Kuratowski.K. in 1933[2] and Vaidyanathaswamy.R in 1945[3] researched the ideatopspa. (ideal topological spaces), a unique notion in the history of topology that we shall examine here.

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* *E-mail:* **hauraas.alali@uokufa.edu.iq (Corresponding Author)**
† *E-mail:* **yiezik.Altalkany@uokufa.edu.iq**

(topological) idea among the first to do so. Open sets are a notion first described by Njasted.O [1].

After that, a group of researchers presented a study on this concept in terms of properties and theories and invested it with many other topological concepts, the most prominent of which α_I-open set and α_I-closed set as well as the study of union relations and intersection on these concepts as well as giving some public and private observations to them and the most prominent scientists are Abdel-Monsef .M.E, Nasef,. and Radwan.A.E Esmaeel. A.B [7], Miguel Caldas , Al-Swidi-L.A[11] and Talkany, Y.K [6]

Kelly [5] has introduced the concepts of bitopological spaces by defining two topologies on a set. This concept was also invested by researchers and developed in different fields, as it was not limited to its application in mathematics, but also invested in physics and engineering sciences and from scientists who had a prominent role in this wonderful achievement Birsan ,T , Swart, J, Ivan L.Reilly , Al-Swidi-L.A [11]and Talkany, Y.K .[6]

2. Elementary Matearials

Definition 2.1: [4] A Bi topspa. is two topspa. defined on the space X. Our study deals with a special case of Bitopspa., which are introduced and study by some of aouthers, which were defined using (X, T) and $(X, T^{\propto})$ as a topspa.

Definition 2.2: [1] Let $A \subseteq X$, where (X,T) is topspa.is saied∝-op. if $A \subset \text{int}(\text{cl}(\text{int}(A))$.

Theorem 2.3: [1] *Every T-op. of a topspa. (X,T)is α-op. but not conversely.*

Definition 2.4: [5] Let $A \subseteq X$, where (X,T) topspa. Then A is saied λ-op. if to any$x \in A$ and to anyα-op.U, such that $A \subseteq U$, there exist $w \in T(x)$, satisfy $x \in w \subseteq U$.

Remark 2.5: [5] Let $A \subseteq X$, where (X,T) is topspa.then A is said to be λ- closed set if and only if its complement is λ-op.

Theorem 2.6: [5] *The collection of all λ-op. constructed a topspa.*

Definition 2.7: [3] The idea.I of subsets of X is a family. on X, achieves the following :

(1) $As \in I$, and $Bs \subseteq As \Rightarrow Bs \in I$ (heredity).

(2) $As \in I$, and $Bs \in I \Rightarrow As \cup Bs \in I$ (add with such a finite amount).

Definition 2.8: [3] suppose (X, T) be a topspa. "with a concept" An operator on sets, I on X$(.)^*: p(X) \Rightarrow p(X)$, defined as $A^*(1, T) = \{x \in X: A \cap U \notin I$, for every $U_x \in T\}$, And it is this that we refer to as the local function viaI and T..

Remark 2.9: [3] Let (X, T) be a topspa., with an idea. If we say that A is a subset of X and I define on X, we get:

1. $A \subset A^*$, $\longrightarrow A$ is named$*$ - dense in itself.
2. If $A^* \subset A$, $\longrightarrow$ A is namedA^* - closed.
3. If $A \in I$, $\longrightarrow A^* = \emptyset$, and(int $T(A)^* = \emptyset$.

Theorem 2.10: [3] *Let (X, T) I and J are two ideas that can go in this slot. on X, and assuming that A and B are two different subsets of X, then:*

1. $A \subseteq B \; then \; A^* \subseteq B^*$
2. $\tilde{A}s^* = \mathrm{cl}(\tilde{A}s^*) \subseteq \mathrm{csl}(\tilde{A}s)$;
3. $(\tilde{A}s^*)^* \subseteq \tilde{A}s^*$
4. $(\tilde{A}s \cup Bs)^* = \tilde{A}s^* \cup \tilde{A}s^*$;
5. $\tilde{A}s^* - \mathrm{Bs}^* = (\tilde{A}s - \mathrm{Bs})^* - \mathrm{Bs}^* \subseteq (\tilde{A}s - \mathrm{Bs})^*$.
6. For each $I \in I$, $(\tilde{A}s \cup I) *= \tilde{A}s *= (\tilde{A}s - I) *$.
7. $U \in T, \Longrightarrow \; U \cap \tilde{A}s^* = U \cap (U \cap \tilde{A}s)^* \subseteq (U \cap \tilde{A}s)^*$
8. $I \subseteq J, \Longrightarrow \; \tilde{A}s^*(I) \subseteq \tilde{A}s^*(I)$

Proposition 2.11:

1. Let $U \subseteq X$, such that U is T- op. then in $t_T(U)^* \subseteq (\mathrm{int}\, t_T(U))^*$.

Proof: Let $x \in$ int $\;\;_T(U^*) \subseteq U^*$, then $x \in U^*$, hence by definition (1.8) U $M \notin I$, for any $M \in T(X)$, and since $U =$ int $\;\;_T(U)$, then int $t_T(U) \sqcap M \notin I$to any$M \in T(X)$, from that we get $x \in (\mathrm{int}_T(U))^*$

2. $Cl(A^*) \subseteq (Cl(A))^*$, for any $A \subseteq X$

Proof: Let $x \in Cl(A^*)$, then $A^* \cap H \neq \emptyset$, to any$H \in T(X)$, and by theorem (2.10)(2), we get $\mathrm{Cl}(A) \cap H \neq \emptyset$, to any$H \in T(X)$,hence $x \in (Cl(A))^*$

Theorem 2.12: **[7]** *Let (X, T, I)be a spa. and let {A_α:α∈Δ} be any family of subset of X, then we have :*

(a) $(\sqcup A_\alpha^*: \propto\in \Delta) \subseteq (\sqcup A_\alpha; \propto\in \Delta)^*$.
(b) $(\Pi A_\alpha: \alpha \in \Delta)^* \subseteq (\Pi A_\alpha^*: \alpha \in \Delta)$.
(c) $\left(\cap A_{\alpha;} \propto\in \Delta\right)^* \subseteq (\cap A_\alpha^*: \propto\in \Delta)$.

Theorem 2.13: [6] The constraints "If space (X,T,I) and If" $\tilde{A}1 \subset$ X, we have :

1. If I is empty ,$\Longrightarrow \tilde{A}1^*(\mathrm{I}) = \mathrm{Cl}(\tilde{A}1)$.
2. If I = p(X), $\Longrightarrow \tilde{A}1^*(\mathrm{I}) = \emptyset$.

Definition 2.14: [6] Let $A \subseteq X$, where(X,T,I) is ideatopspa. Then A is said to be (α_{I}-op.) iff there exists an op. U such that$U - A \in I$ and A - int $(Cl(U)) \in I$.

Theorem 2.15: [6] *Every α-op. (op) of a topspa.(X,T) is α_I-op.*

Proposition 2.16: [6] Let (X,T)be a topspa. and I be an idea. defined on X, If $I = p(X)$, then $\propto_I-$op. =p(X)

Theorem 2.17: [6] *Let (X,T,I)be a space and*Ã1, $B \subseteq X$ *such that* Ã1 $\in I$, *then* Ã1Ã1 *is α_I-op.*

Proposition 2.18: [6] Suppose (X,T,I)be an ideatopspa., then

1. if A and B are both α_{I}-op., then so is their union.
2. if $I = \{\phi\}$,then α-op. and α_I-op. are equivalent.

Remark 2.19: [6] Let (X,T,I) be an ideal topos, If $A \in I$, then A is $\propto_{\mathrm{I}}$-op.s.

Proposition 2.20: [6] Suppose $I$and J are two idea. on topspa. (X,T). If $J \subseteq I$, then every$\alpha_{,\mathrm{J}}$ -op is $\propto_{\mathrm{I}}$-op.

Proposition 2.21: [6] Let (X,T,I)be an ideatopspa., then:-

1. If A is α_{I}-$op.$ and $A \subseteq B$, then B is α_{I}-op.
2. If A is $\propto_{\mathrm{I}-}$ op, then so is$A \sqcup B$, for any subset B of X...

3. Study on λ_I-op.s

Definition 3.1: A subset A of a topspa. X is saiedλ_I - op.s iff, to any$x \in A$, there exist α_{I} - op.sU, such that $U - A \in$ I, $A \subseteq (\mathrm{in}\, t_T(U))^*$

Example 3.2: suppose $X = \{a,b,c\}$, while in lead. T={X,∅,{a}}, and I = $\{\emptyset,\{b\}\}$,Then λI −op.={X,ϕ,{a},{a,b},{a,c}}

Remark 3.3: If $A \in I$,then by remark (1.18),A is $\propto_I$ – op. and by remark (2.9)(4) $A \neq \left(\mathrm{int}_T(U)\right)^*$, and then A is not λI - op.

Proportion 3.4: Let $(\mathrm{X},\mathrm{T},T^\alpha)$ be a Bitopspa., and I be an idea. defined on X, such that T $\subset I$ then $O_{\lambda I}(X) = \phi$.

Proof: Suppose that $O_{\lambda I}(X) \neq \emptyset$,then ∃ a subset A of X is λI-op., so $\exists \alpha_I$ op.U, s.t $U - A \in I, A \subseteq \left(\mathrm{int}\, t_T(U)\right)^*$ and then if $x \in A$, so $x \in \left(\mathrm{int}_T(U)\right)^*$ and by def (1.8) int $_T(U) \sqcap M \notin I$, to any$T$ −op.sM of x but $int_T(U) \sqcap M \in T$ and by assume $T \subset I$ hence $int_T(U) \cap M$ ∈I,and this contradiction,hence $O_{\lambda I}(X) = \phi$

Remark 3.5:

1. $\emptyset$ is λI- op.s.
2. X is not necessary λI – op. as in the belowEXAMPLE.

Example 3.6: Let $X = \{a, b, c, d\}$ with a top.T={X,$\emptyset$,{a},{b},{a,b}{b,c}} and an ideal $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, then λI – op.= $\{\phi\}$.

Remark 3.7: Clearly that by theorem $(1,12)(2)$ if I is an idea. defined as $I = p(X)$ then, λI – op.= $\{\phi\}$.

Remark 3.8: One can deduce that : A need not be present if A isα_I- op. λI - op. as seen by the EXAMPLE below .

Example 3.9: Let $X = \{a, b, c\}$ with a top.$T = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ then,

α_{I} – op.= $\{x, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b\}\}$ but $\lambda 1$-op.= $\{\phi\}$

Theorem 3.10: *Let X be any set which has more than one element and$T$ is any top. defined on X, and $A \subseteq X$, and I is any ideal. on X, If A is * dense in itself and op. then A is λI - op.*

Proof: Let $A \subseteq X$, such that A is T - $\mathrm{op.s}$, so by Th. (2.3) and def. (2.14).

So there exist $\alpha_I - \mathrm{op.s} W = A, W - A \in I$, now by assume and by Th. (2.10) (1) we have $A^* = (\mathrm{int}_T(U))^*$ and since A is * -dense in itself we get that A is λ I -$\mathrm{op.s}$.

Remark 3.11: In general, if A is λ-$\mathrm{op.s}$, then A is not necessary is $\lambda\mathrm{I} - \mathrm{op.s}$ and if A is λI-$\mathrm{op.s}$, then A is not necessary $\mathrm{op.s}$ as shown by theEXAMPLE below.

Example 3.12: Let $X = \{a, b, c\}$with a top. $T = \{X, \phi, \{a\}\}$ and $I = \{\phi, \{b\}\}$, then λ-op. ={X, ϕ,{a},{b,c}}, λI-op.={ϕ,X,{a},{a, b},{a,c}}, {a,b},{a,c} is λI-op. but$\{a, b\}, \{a, c\}$ is not λ-op., so $\{b, c\}$ is λ-op. but not λI-op.

Proposition 3.13: Let(X, T, I)be a Bi topspa., and I be an idea. defined on X. If A, B are both λI-op., then so is their union.

Proof: Since A,B are λI-op., so there exist U_1, U_2 are α_{I}-op., so by proposition $(2.18)(1)$ we have that, $U_1 \cup U_2$ is $\propto_{\mathrm{I}}$-op.s, and then

$$(U_1 \cup U_2) - (A \cup B) = (U_1 \cup U_2) \cap (A \cup B)^C$$
$$= [(U_1 - A) \sqcap B^C] \cup [(U_2 - B) \sqcap A^C] \in I$$

Now since

A $\subseteq (\mathrm{int}_T(U_1))^*$, and B $\subseteq (int_T(U_2))^*$, then AUB $\subseteq (int_T(U_1))^* \cup (int_T(U_2))^*$, and by theorem (2.10) [4], we get $A \cup B \subseteq (\mathrm{int}_T(U)_1)^* \cup (\mathrm{int}_T(U)_2)^*$, and by using interior proposition the following exist, $A \cup B \subseteq (int_T(U_1 \cup U_2))^*$, hence AUB is λI-op.s.

Remark 3.14: If I and J are two ideals defined on topspa., if $(X, T_1), (X, T_2)$ such that $I \subset J$, then it is not necessary that every λI-op. is λJ-op. as shown by the below EXAMPLE.

Example 3.15: Let $X = \{a, b, c, d\}$ with two top.$T_1 = p(X)$, $T_2 = \{\phi, X, \{a, b\}, \{c, d\}\}$, and $I \subseteq J, I = \{\phi\}, J = \{\phi, \{a\}\}$, then $\{a\}, \{c\}, \{d\}$ is λI-op.but notλJ-op.

Remark 3.16:

1. If A is λI-op. and $A \subseteq B$, And therefore, it's not required that B is λI-op.
2. If A is λ-op. and $B \subseteq A$, Thus, it is not required. B is λI-op. as in the belowEXAMPLE.

Example 3.17: Let $X = \{a, b, c, d\}$ with a top.$T = \{X, \phi, \{a, b\}, \{c, d\}\}$ and $I = \{\phi, \{a\}\}$, then λI-op. = {X,ϕ,{b},{a, b},{c, d},{b,c,d}} clearly that if A={a,b} we have $\{a\} \subseteq A = \{a, b\}$ also $\{a, b\}$ is λI-op. but $\{a, b\} \subseteq \{a, b, c\}, \{a, b, c\}$ is not λI-op.

Remark 3.18: By remark $(2-9)$ (4) which we do if$A \in I$, then A is not λI-op. as the belowEXAMPLE.

Example 3.19: Let $X = \{a, b, c\}$ by means of topology $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, then λI-op.s= $\{\phi\}$ now $A = \{a\} \in I$ but $\{a\} \notin \lambda I$-op.s.

Theorem 3.20: *If $I = \{\phi\}$ then every α-op.(α_I-op.) is λI-op.*

Proof: Let A is $\alpha-$ op., then $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$, and by theorem $(2.13)(1)A \subseteq (\mathrm{int}(A))^*$, also by theorem $(2.15)A$ is $\propto_I$-op., then $A - A \in I$, hence A is $\lambda I -$ op.

Now if$A$ is α_I-op., then there exist op.U s.t $U - A \in I, A - \mathrm{int}(\mathrm{cl}(U)) \in I$, and since $U - A \in I$, we get that $U \sqcap A^c = \emptyset$, so $U \subseteq A$ hence by theorem(2. 13)(1)$A - \mathrm{int}(U^*)$,and Since U op., so by theorem(2.1), $A - \left(\mathrm{int}_T(U)\right)^* = \emptyset$ and this is can that to any$x \in A, x \in \left(\mathrm{int}_T(U)\right)^*$, and then $A \subseteq \left(\mathrm{int}_T(U)\right)^*$.

4. Conclusion

In this research, we concluded the following:

TheλI-open and op.s are independent set in general, also λI-open and $\propto$-open, λI-open and $\propto_I$-open are independent,also when a subset A is belonged to the ideal I,then it is not necessary that A is λI-open.

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