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Medical applications of the new-transform

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Abstract

Solving differential equations by integral transformations is a highly general method. Using these powerful methods, mainly intractable problems may be conquered relatively easily. Because of the difficulty of solving ordinary differential equations, this study employs a novel integral transformation called the h-transform Related ideas, characteristics, and methods for evaluating significance are also elaborated upon. Due to the precision of the necessary transformations, the study of differential equations has inspired a new way of thinking called $\widetilde{HA}$ -fuzzy transformation, which is used to solve, identify, and classify impacted cells. Check whether the proposed method performs compared to other translations, and discover what the results reveal about it.

Subject Classification: *03Cxx, 03-XX, 03Exx.*

Keywords: $\widetilde{HA}$ *-transform, Properties* $\widetilde{HA}$ *-transform, Derivatives* $\widetilde{HA}$ *-transform, Solve equation using* $\widetilde{HA}$ *-transform, Cconvert* $\widetilde{HA}$ *fuzzy transform.*

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This article has been corrected with minor changes. These changes do not impact the academic content of the article.

1. Introduction

Mathematicians use a method known as an integral transform to convert the values of one set of functions to another. If you have a function that has been transformed, you can put it back to its original form by using the inverse transform[1,2,3]. We construct a novel integral transform, which we call the $\widetilde{HA}$-transform, and establish certain theorems concerning it in this study [4-6, 12-18]. Fuzzy approximation models frequently employ fuzzy transformations as a technique. A fuzzy derivative was first proposed by Chang and Zadeh [7,8,8] in 1972, while fuzzy differential equations [10,11] were first proposed by Kandel and Byatt [8,9]. Abbasbandy and Allahviranloo (2002) [5] were the first to suggest the idea of using a numerical solution to solve fuzzy differential equations. This study builds on prior work by introducing an unique fuzzy transform, the H-transform, that can be utilized to more precisely solve fuzzy differential equations. It is shown that the measured drug concentration may be interpreted using this fuzzy method. The plasma concentration is commonly used as an indicator of a drug's effectiveness. The $\widetilde{HA}$-transform kernel may be improved by including in this article.

Definition 1: Let $\Im(\delta)$ be an integrable function defined for $\delta \geq 0$, let $\mu(\varepsilon) = \cot\varepsilon, \varepsilon \neq 0$ be non-negative real functions, and $\psi(\varepsilon)$ be a positive function, then $G(\mu(\varepsilon), \psi(\varepsilon))$ exists for all $\psi(\varepsilon) > \mu$. Since

$$\| G(\mu(\varepsilon), \psi(\varepsilon)) \| = \left| \mu(\varepsilon) \int_0^\infty \Im(\delta) e^{-\psi(\varepsilon)\delta} d\delta \right|$$

$$\leq \mu(\varepsilon) \int_0^\infty |\Im(\delta)| e^{-\psi(\varepsilon)\delta} d\delta \leq \mu(\varepsilon) \int_0^\infty \mathrm{A} e^{\mu\delta} e^{-\psi(\varepsilon)\delta} d\delta \leq \frac{\mu(\varepsilon) A}{\mu - \psi(\varepsilon)'}$$

we have the following formula to describe its general integral transform $\Im(\delta)$:

$$\widetilde{HA}\left\{\Im(\delta), \varepsilon\right\} = \mu(\varepsilon) \int_0^\infty \Im(\delta)\ e^{-\psi(\varepsilon)\delta} d\delta = \cot\varepsilon \int_0^\infty \Im(\delta)\ e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta} d\delta\ ,$$

Definition 2: Inverse HA-transform of is represented by $\widetilde{HA}^{-1}[G(\varepsilon)]$ is a function that breaks up into pieces $\Im(\delta)$ on $[0, \infty)$ which satisfies $\widetilde{HA}\left|\Im(\delta)\right| = G(\varepsilon)$

2. H-Transform for Some Basic Functions:

Let us assume that $\Im(\delta)$ the integral in equation (1) holds true for any function. Therefore, the H-transform's case is (Table 1):

Table 1

The H-transform's case for function $\Im(\delta)$

Function $\Im(\delta)=T^{-1}\{\widetilde{HA}(\varepsilon)\}$	$\widetilde{HA}(\varepsilon)=\widetilde{HA}\{\Im(\delta),\varepsilon\}$
$\Im(\delta)=1$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\cot\varepsilon}{\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}}$,
$\Im(\delta)=\delta$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\cot\varepsilon}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2}$
$\Im(\delta)=\delta^{\alpha}$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\Gamma(\alpha+1)\cot\varepsilon}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^{\alpha+1}}$, $\alpha\geq 0$.
$\Im(\delta)=\sin\delta$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\cot\varepsilon}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2+1}$
$\Im(\delta)=\cos\delta$	$\widetilde{HA}\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\cot\varepsilon\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}$
$\Im(\delta)=\sinh\delta$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\cot\varepsilon}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}$
$\Im(\delta)=\cosh\delta$	$\widetilde{HA}\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\cot\varepsilon\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2}{\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}$
$\Im(\delta)=e^{\delta}$	$\widetilde{HA}\{\Im(\delta),\varepsilon\}=\dfrac{\cot\varepsilon}{\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}-1}$

Theorem 1 : *Let $\Im(\delta)$ is a continuous function with respect to $\delta\geq 0$ and act as a genuinely useful function, $\psi(\varepsilon)=\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}$, $\varepsilon\neq 0$ be positive function then:*

(I) $\widetilde{HA}\{\Im(\delta)',\varepsilon\}=\left(\dfrac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\widetilde{HA}(\varepsilon)-\cot\varepsilon\ \Im(0)$,

(III) $\widetilde{HA}\{\Im(\delta)^{(n)},\varepsilon\}=\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^{n}\widetilde{HA}(\varepsilon)-\cot\varepsilon\sum_{k=0}^{n-1}\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^{n-1-k}\Im(0)^{k}$.

Proof:

(I) since $\widetilde{HA}\{\Im(\delta)',\varepsilon\}=\text{ot}\cot\varepsilon\int_0^\infty\Im(\delta)'e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}d\delta,$ integrating by parts:

$$\widetilde{HA}\{\Im(\delta)',\varepsilon\}=\cot\varepsilon\left[\Im(\delta)e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}\right]_0^\infty+\int_0^\infty\Im(\delta)e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}d\delta$$

$$=\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\widetilde{HA}(\varepsilon)-\cot\varepsilon\ \Im(0)\ .$$

(II) since $\widetilde{HA}\{\Im(\delta)'',\varepsilon\}=\cot\varepsilon\int_0^\infty\Im(\delta)''e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}d\delta,$ integrating by parts:

$$=\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\widetilde{HA}\{\Im(\delta)',\varepsilon\}-\cot\varepsilon\Im(0)'=\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^{2}\widetilde{HA}(\varepsilon)-\cot\varepsilon$$

$$\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\Im(0)-\cot\varepsilon\ \Im(0)^{\backslash}\ ,$$

(III) For this n - th derivative can be prove using mathematical induction.

Theorem 2: *Suppose that* $\Im_1(\delta)$ *and* $\Im_2(\delta)$ *functions that can be integrated with* $\widetilde{HA}$ *-transforms on [0, +∞),* $\widetilde{HA}$ *-transforms are established in the context of* $\varepsilon\neq 0$, $\widetilde{HA}\{\Im_1(\varepsilon)\}=\wp$, $\widetilde{HA}\{\Im_2(\varepsilon)\}=\mathcal{H}$, *Then the* $\widetilde{HA}$ *-transform of their convolution* $\Im_1(\varepsilon)*\Im_2(\varepsilon)$ *can defined by:* $\widetilde{HA}\{\Im_1*\Im_2\}(\varepsilon)=\frac{1}{\varepsilon}\wp(\varepsilon)\mathcal{H}(\varepsilon)$

Proof:

$$\widetilde{HA}(\Im_1(\varepsilon)*\Im_2(\varepsilon))=\cot\varepsilon\int_0^\infty(\Im_1(\varepsilon)*\Im_2(\varepsilon))e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}$$

$$=\cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}\int_0^\infty\Im_1(\delta)\Im_2(\delta-\tau)d\tau$$

$$=\cot\varepsilon\int_0^\infty\Im_1(\delta)\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}\Im_2(\delta-\tau)d\tau=\frac{1}{\varepsilon}H_1(\varepsilon)H_2(\varepsilon).$$

Theorem 3: *Let* $\Im(\delta)$ *If the Laplace transform of a function is differentiable, then the function is said to* $\Im(\delta)$ *and* $\widetilde{HA}$ *is* $\widetilde{HA}$ *-Transform of* $\Im(\delta)$, *then:*

$$\widetilde{HA}(\varepsilon) = \cot\varepsilon F\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)$$

Proof: From Definition 1: $\widetilde{HA}(\varepsilon) = \widetilde{HA}[\Im(\delta);\varepsilon] = \cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta}\Im(\delta)d\delta$,

since Laplace transform is denoted by $F(p) = \ell[f(t);p] = \int_0^\infty e^{-pt} f(t)dt$, Then $\widetilde{HA}(\varepsilon) = \cot\varepsilon F\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)$.

Theorem 4: *Let* $\Im_1(\delta), \Im_2(\delta), \ldots, \Im_n(\delta)$ *functions that are continuous possess* $\widetilde{HA}$ *-transforms and* $\mu_1, \mu_2, \ldots\mu_n$ *linearity trait is defined if and only if are chosen at random*

$$\widetilde{HA}[\mu_1\Im_1(\delta)] + [\mu_2\Im_2(\delta)] + \ldots + [\mu_n\Im_n(\delta)] = \mu_1\widetilde{HA}[\Im_1(\delta)] + \mu_2\widetilde{HA}[\Im_2(\delta)] + \ldots + \mu_n\widetilde{HA}[\Im_n(\delta)]$$

Proof: Let $\Im_1(\delta)$ and $\Im_2(\delta)$ consisting of continuous functions for $\delta \geq 0$:

$$\begin{aligned}
&\widetilde{HA}[\mu_1\Im_1(\delta) + \mu_2\Im_2(\delta) + \ldots + \mu_n\Im_n(\delta)] \\
&= \cot\varepsilon\int_0^\infty [\mu_1\Im_1(\delta) + \mu_2\Im_2(\delta) + \ldots + \mu_n\Im_n(\delta)]\, \mathrm{e}^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta} d\delta \\
&= \cot\varepsilon\int_0^\infty [\mu_1\Im_1(\delta)]\, \mathrm{e}^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta} d\delta + \cot\varepsilon\int_0^\infty [\mu_2\Im_2(\delta)]\, \mathrm{e}^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta} d\delta \\
&\quad + \ldots + \cot\varepsilon\int_0^\infty [\mu_n\Im_n(\delta)]\, \mathrm{e}^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\delta} d\delta \\
&= \mu_1\widetilde{HA}[\Im_1(\delta)] + \mu_2\widetilde{HA}[\Im_2(\delta)] + \ldots + \mu_n\widetilde{HA}[\Im_n(\delta)]
\end{aligned}$$

4. Fuzzy H-Transform

Fuzzy differential equations have achieved extensive use over the past decades owing to their various and varied applications[7,9]. This work presents a novel approach to solving fuzzy differential equations[5,6] in an effort to stay up with the field's tremendous growth and advancement in recent years. In this part, we apply the H-Transform to a genuine problem—edge detection pictures from a blood film—after converting it into a fuzzy H-transform and introducing various attributes and fuzzy theories of the first and second order.

Definition 3: Let $\varphi(\sigma)$ be a continuous fuzzy-valued function Suppose that $\cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\varphi(\sigma)d\sigma$ is improper fuzzy Riemann-integrable on $[0,\infty)$, then $\cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\varphi(\sigma)d\sigma$ is called $\widehat{H}$-transform and it denoted by:

$$\widehat{H}[\varphi(\sigma)] = \widehat{H}(\varepsilon) = \cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\varphi(\sigma)d\sigma$$

Sine from theorem 2 to get :

$$\cot\varepsilon\int_0^\infty e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}\varphi(\sigma)d\sigma = \cot\varepsilon\int_0^\infty e^{-\left(\frac{i}{\sqrt[2]{\varepsilon}}\right)\sigma}\underline{\varphi}(\sigma;\phi)d\sigma, \cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$$

Using the definition of classic H-transform:

$$\widehat{H}[\underline{\varphi}(\sigma;\phi)] = \cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\underline{\varphi}(\sigma;\phi)d\sigma,$$

$$\widehat{H}[\overline{\varphi}(\sigma;\phi)] = \cot\varepsilon\int_0^\infty e^{-\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$$

So: $\widehat{H}[\varphi(\sigma;\phi)] = H[\underline{\varphi}(\sigma;\phi)], \mathrm{H}[\overline{\varphi}(\sigma;\phi)]$

Theorem 5: *Assume that* $\varphi^{\backslash}(\sigma)$ *be continuous fuzzy-valued function and* $\varphi(\sigma)$ *the primitive of* $\varphi^{\backslash}(\sigma)$ *on* $[0,\infty)$, *then:*

1. $\widehat{H}[\varphi^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\widehat{H}[\varphi(\sigma)]\ominus\cot\varepsilon\varphi(0)$, *where* φ *is differentiable in the first form*
2. $\widehat{H}[\varphi^{\backslash\backslash}(\sigma)] = -\cot\varepsilon\varphi(0)\ominus\left(-\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)H[\varphi(\sigma)]$, *where* φ *is differentiable in the second form*

Proof: There are two scenarios that can arise due to the fact that $\varphi^{\backslash}(\sigma)$ is a continuous fuzzy-valued function.

Case 1: If φ is the first form, for any arbitrary $\phi\in[0,1]$,

$$\widehat{H}[\varphi^{\backslash}(\sigma)] = H[\underline{\varphi}^{\backslash}(\sigma,\phi)], H[\overline{\varphi}^{\backslash}(\sigma,\phi)]$$

From Theorem 1/1:

$$H[\underline{\varphi}^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\underline{\varphi}(\sigma,\phi)] - \cot\varepsilon[\underline{\varphi}(0,\phi)]$$
$$H[\overline{\varphi}^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\overline{\varphi}(\sigma,\phi)] - \cot\varepsilon[\overline{\varphi}(0,\phi)]$$

By Theorem 2: $\widehat{H}[\varphi^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\varphi(\sigma)] \ominus \cot\varepsilon\varphi(0)$

Case 2: If φ is the second form, for any arbitrary $\phi \in [0,1]$,

$$\widehat{H}[\varphi(\sigma)] = H[\underline{\varphi}(\sigma,\phi)], H[\overline{\varphi}(\sigma,\phi)]$$

From Theorem 1/1:

$$H[\overline{\varphi}^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\overline{\varphi}(\sigma,\phi)] - \cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)[\overline{\varphi}(0,\phi)]$$
$$H[\underline{\varphi}^{\backslash}(\sigma)] = \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\underline{\varphi}(\sigma,\phi)] - \cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)[\underline{\varphi}(0,\phi)]$$

By Theorem 2 $\widehat{H}[\varphi^{\backslash}(\sigma)] = -\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\varphi(0) \ominus \left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right) H[\varphi(\sigma)]$

Example 1 [11]: The plasma concentration of a medicine at any particular moment is used to calculate the appropriate dosage of a tablet or capsule taken orally. To illustrate, if we define $\eta^{\backslash}(t)$ to be the rate at which a medication from an oral dosage is absorbed by the body, then the human body's $\eta(t)$ response to a unit input of drug is a function of time. It will look like this, mathematically speaking: $\eta^{\backslash}(t) = \frac{1}{t}\eta(t)$, $\eta(0) = [\underline{\eta}(0;\vartheta), \overline{\eta}(\vartheta;0)]$

There are two cases:

1. **If $\eta(t)$ is the frst form then:**
   fuzzy H-transform for both sides of original equation, we get: $\widehat{H}[\eta^{\backslash}(t)] = \widehat{H}[\eta(t)]$ Last equation becomes:

$$\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)H[\underline{\eta}(t;\phi)]-\cot\varepsilon\underline{\eta}(0;\phi)=H[\underline{\eta}(t;\phi)],$$

$$\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)H[\overline{\eta}(t;\phi)]-\cot\varepsilon\overline{\eta}(0;\vartheta)=H[\overline{\eta}(t;\phi)],$$

So: $H[\underline{\eta}(t;\vartheta)]=\dfrac{\cot\varepsilon}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)-1}\underline{\eta}(0;\vartheta), H[\overline{\eta}(t;\vartheta)]=\dfrac{\cot\varepsilon}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)-1}\overline{\eta}(0;\vartheta)$

Now we use inverse H-Transform for last equations:

$$\underline{\eta}(t;\vartheta)=e^{\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\sigma}\underline{\eta}(0;\vartheta),\overline{\eta}(t;\vartheta)=e^{\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\sigma}\overline{\eta}(0;\vartheta)$$

2. **If $\eta(\sigma)$ is second form, then:** By using FH-Transform for both sides of original equation, we get: $\widehat{H}[\eta^{\backslash}(\sigma)]=\widehat{H}[\eta(\sigma)]$ Last equation becomes:

$$\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)H[\overline{\eta}(t;\phi)]-\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\overline{\eta}(0;\vartheta)=H[\overline{\eta}(t;\phi)],$$
$$\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)H[\underline{\eta}(t;\phi)]-\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)\underline{\eta}(0;\phi)=H[\underline{\eta}(t;\phi)]$$

By solve above equation:

$$H[\underline{\eta}(\mathrm{t};\vartheta)]=\frac{\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}\overline{\eta}(0;\vartheta)-\frac{\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta)$$

$$H[\overline{\eta}(\mathrm{t};\vartheta)]=\frac{\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta)-\frac{\cot\varepsilon\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)}{\left(\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}\right)^2-1}\underline{\eta}(0;\vartheta)$$

Now we use inverse H-Transform for last equations:

$$\underline{\eta}(0;\vartheta)=\cosh\ \frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}t\overline{\eta}(0;\vartheta)-\sinh\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}t\underline{\eta}(0;\vartheta),\overline{\eta}(0;\vartheta)$$
$$=\cosh\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}t\underline{\eta}(0;\vartheta)-\sinh\frac{\sqrt[3]{2}}{\sqrt[2]{\cot\varepsilon}}t\overline{\eta}(0;\vartheta)$$

5. System Analysis and Design:

In this part, various image processing approaches were used to achieve Automatic Border (Types of Diseases) Identification, and then it will be classified with theoffuzzy H-transform. The developed system works on RGB color images with size 128 x 128 pixel. The system is built with four primary processes these are explained in Figure (1).

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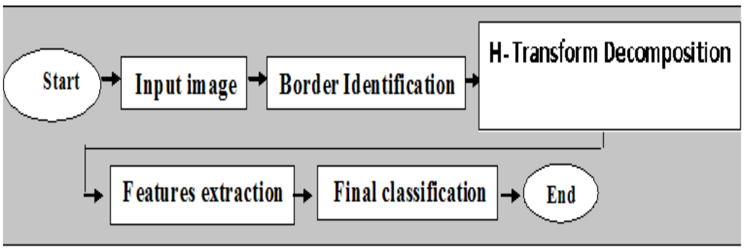


Figure 1

Main stages for the Automatic Border (Leukemia) Identification

Clustering approaches were presented as a means of distinguishing lymphoblasts from normal blood cells. Once an image has been pre-processed, feature extraction can be performed to gain insight into the image. There is no other purpose served by the pre-processing processes. It provides no essential details about the image. Since the image isn't immediately clear after acquisition, contrast augmentation is required. After an RGB image has been transformed to the HSI color model, a clustering approach is used to create individual segments. To clean up an image, a median filter is applied. Once features have been extracted, images can be sorted using clustering methods.

Table 2

Main stages for the Automatic Border (Leukemia) Identification

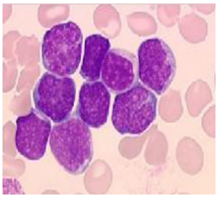
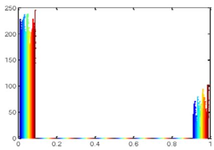
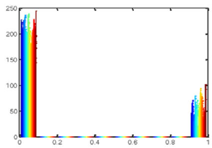
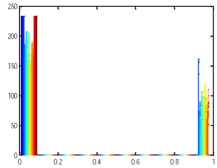
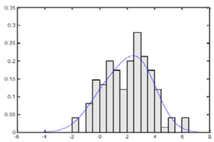
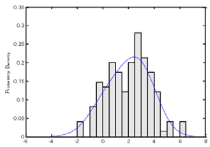
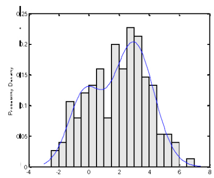
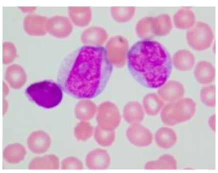
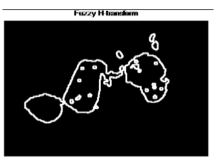
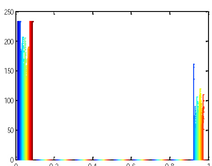
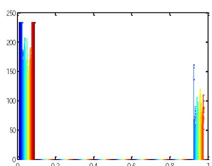
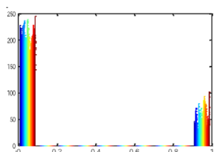
Case one	The original image	fuzzy H-transform	fuzzy H-transform
Edge detection			
histogram			
Density function			

Table 3

Case one for the Automatic Border (Leukemia) Identification

Case two	The original image	fuzzy H-transform	fuzzy H-transform
Edge detection			
histogram			

Contd...

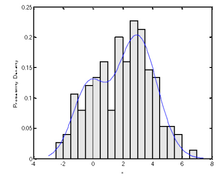
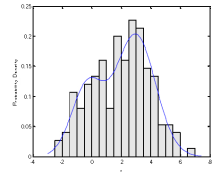
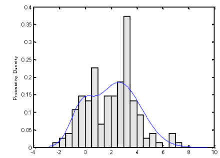
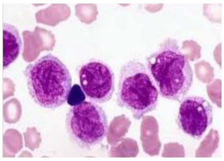
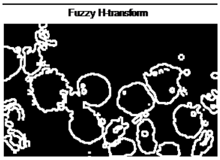
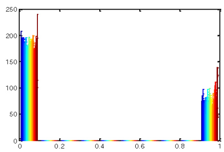
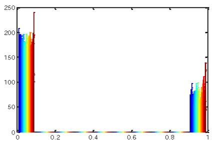
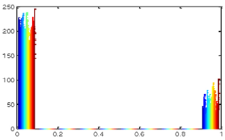
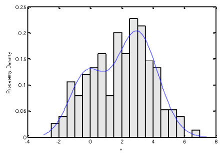
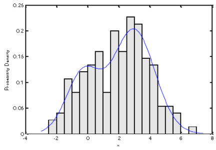
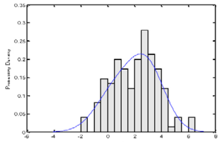
Density function			

Table 4

Case two for the Automatic Border (Leukemia) Identification

Case three	The original image	fuzzy H-transform	fuzzy H-transform
Edge detection			
histogram			
Density function			

Conclusion:

This research proposes the methodology of classifying Leukemia disease and dividing it into four stages using a new technique called (*H*-transform), where the data set consists of 30 images. It gives excellent results, as it gives preference to choosing the optimal solution for the proposed conversion, unlike the rest of the transformations, which do not provide such solutions. The conversion efficiency was also tested by distinguishing through statistical parameters, as is evident in the tables (Table 2, Table 3, Table 4) for each case, adding to the speed of results in terms of time and calculations.

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