Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 6, pp. 1099–1107
DOI : 10.47974/JIM-1609

Fuzzy partial T-transforms applied to a system of PDE

Roaa H. Hasan *
Department of Mathematics
Collage of Education for Girls
Kufa University
Kufa
Iraq

Murtadha A. Kadhim †
Kufa Technical Institute
Al-Furat Al-Awsat Technical University
Kufa
Iraq

Amal Mohammed Darweesh §
Department of Mathematics
Collage of Education for Girls
Kufa University
Kufa
Iraq

Abstract

Fuzzy partial integro-differential equations have widespread applications in the natural and applied sciences and engineering. The fuzzy T-transform transform is proposed as a new method for solving fuzzy partial differential equations in this research. Using very broad H-differentiability ideas, a formula for the first and second order fuzzy partial derivative has been derived. After an introduction to fuzzy partial T-transforms, fuzzy partial differential

This article has been corrected with minor changes. These changes do not impact the academic content of the article.

* *E-mail:* **Ruaah.shabaa@uokufa.edu.iq (Corresponding Author)**
ORCID: 0000-0002-2107-1639
† *E-mail:* **m.kuzai07@umail.umz.ac.ir, kuh.dr.mur@atu.edu.iq**
§ *E-mail:* **amalm.alhareezi@uokufa.edu.iq**

equations are developed. Finally, a working example has been presented to showcase the efficacy of the suggested approach.

Subject Classification: *03Cxx, 03-XX, 03Exx.*

Keywords: *T-transform, Fuzzy T-transform, Partial T-transform.*

1. Introduction

Differential equations may be used to simulate a wide variety of physical events. Building an accurate differential equation model for such problems, however, requires time-consuming and impracticable approaches. A fuzzy mathematical method seems to be the most appropriate here. For the last several decades, scientists and engineers have relied heavily on fuzzy differential equations (FDEs). The concept and mathematics of fuzzy sets were first introduced by L. A. Zadeh in [17]. The concepts of fuzzy derivatives and fuzzy integration were first explored [3,10, 13], subsequently [2,3,1,8, 14, 18], and finally [3,10, 19, 20]. Partial Differentia l Equations (PDEs) are a kind of mathematical equation that has many applications in the natural sciences, including chemistry and biology. A partial differential equation is a mathematical formula used when dealing with a large number of variables and their derivatives. When it comes to fixing issues in the real world, ordinary differential equations don't always deliver. This is because keeping track of all the moving parts need to observe an event may be rather challenging. For instance, modeling the flow of heat via a wire necessitates taking both spatial and temporal factors into account concurrently. It has been demonstrated that the partial differential equation has both analytical and numerical solutions by a number of authors [4, 6]. As fuzzy integrals simplify the original function, they may be utilized to solve linear partial differential equations [1,9]. This paper's parts are listed in the following order at the conclusion. Partition T-transform findings for first- and second-order fuzzy partial derivatives are presented in this study, along with some introductory concepts in fuzzy sets and fuzzy functions. A system of partial equations may be solved using these formulae.

2. Basic concepts

Definition 1: [7] A fuzzy number expressed in parametric form ω is a pair $(\underline{\omega}, \overline{\omega})$ of functions.

$\underline{\omega}(\vartheta)$, $\overline{\omega}(\vartheta)$, $0 \le \vartheta \le 1$, which satisfies the following requirements:

1. $\underline{\omega}(\vartheta)$ is a bounded left continuous that doesn't decrease. Function in (0, 1], and continuing right at 0.
2. $\overline{\omega}(\vartheta)$ is a right continuous at and abounded non-increasing continuous left function in (0, 1] at 0.
3. $\underline{\omega}(\vartheta) \le \overline{\omega}(\vartheta)$, $0 \le \vartheta \le 1$.

Theorem 1: [15] *Let* $f : R \to \mathrm{E}$ *and it is portrayed by* $[\underline{f}(\varpi;\vartheta), \overline{f}(\varpi;\vartheta)]$. *For any definite* $\vartheta \in (0,1]$ *suppose that* $\underline{f}(\varpi;\vartheta)$ *&* $\overline{f}(\varpi;\vartheta)$ *are Riemann-integrable functions on [d, c] for every* $c \ge d$, *and consider two positive functions.* $\underline{\Im}_\vartheta$ *and* $\overline{\Im}_\vartheta$ *such that* $\int_d^c |\underline{f}w(\varpi;\vartheta)|\, d\varpi \le \underline{\Im}_\vartheta$ *and* $\int_d^c |\overline{f}w(\varpi;\vartheta)|\, d\varpi \le \overline{\Im_\vartheta}$ *for every* $c \ge d$. *Then,* $f(\varpi)$ *is inadequate fuzzy Riemann-integrable on* $[a,\infty)$ *Additionally, we have:* $\int_d^\infty f(\varpi)\, d\varpi = [\int_d^\infty \underline{f}w(\varpi;\vartheta)\, d\varpi, \int_d^\infty \overline{f}w(\varpi;\vartheta)\, d\varpi]$.

Definition 2: [12] Let $u : (d,c)\times(d,c) \to \mathrm{E}$, is side to be first order H-differentiableat $\delta_0 \in (d,c)$, with respect to δ, If such a thing exists

1. $\exists w(\delta_0 + h, t) \ominus u(\delta_0, t), \exists w(\delta_0, t) \ominus u(\delta_0 - h, t)$ the ensuing limit holds then $\lim_{h\to 0^+} \frac{w(\delta_0+h,t)\ominus w(\delta_0,t)}{h} = \lim_{h\to 0^+} \frac{w(\delta_0,t)\ominus w(\delta_0-h,t)}{h} = \frac{\partial}{\partial\delta} w(\delta_0, t)$ or $\frac{\partial}{\partial\delta} u(\delta_0, t)$

   $\frac{\partial}{\partial\delta} u(\delta_0, t)$ belongto E such that:

   in $\forall h > 0$ those who are small enough in

2. $\exists w(\delta_0, t) \ominus u(\delta_0 + h, t), \exists w(\delta_0 - h, t) \ominus u(\delta_0, t)$ then the following limit hold $\lim_{h\to 0^+} \frac{w(\delta_0,t)\ominus u(\delta_0+h,t)}{-h} = \lim_{h\to 0^+} \frac{w(\delta_0-h,t)\ominus u(\delta_0,t)}{-h} = \frac{\partial}{\partial\delta} w(\delta_0, t)$

   $\forall h > 0$ those who are small enough.

Definition 5: Assume $u = u(\varpi, t)$ a continuous If *t* is a real parameter and fuzzy-valued function, the function's fuzzy T-transform follows *u*. Denote by $\widetilde{T}_t(\varpi, \upsilon)$ is defined as follows:

$$\widetilde{T}_t(\varpi,\mathrm{s}) = T[u(\varpi,t)] = s\int_0^{\infty} e^{-(i\sqrt[3]{s}+\sin s)t}u(\varpi,t)dt = \lim_{\tau\to\infty} s\int_0^{\tau} e^{-(i\sqrt[3]{s}+\sin s)t}u(\varpi,t)dt,$$

$$\widetilde{T}_t(\varpi,\mathrm{s}) = \left[\lim_{\tau\to\infty} s\int_0^{\tau} e^{-(i\sqrt[3]{s}+\sin s)t}\underline{u}(\varpi,t)dt, \lim_{\tau\to\infty} s\int_0^{\tau} e^{-(i\sqrt[3]{s}+\sin s)t}\overline{u}(\varpi,t)dt\right].$$

when ever there are limits. The $\vartheta-$ cut representation of $\widetilde{Ta}_t(\varpi,\mathrm{s})$ is given as:

$$\widetilde{T}_t(\varpi,\mathrm{s};\vartheta) = T_t[w(\varpi,t;\vartheta)] = [\gamma(\underline{u}w(\varpi,t)), \gamma(\overline{u}w(\varpi,t))],$$

Theorem 2: *[5] Let* $u:(0,\infty)\times(0,\infty)\to E$ *be a function and denote by* $[u(\varpi,t)]^{\vartheta} = [\underline{u}(\varpi,t;\vartheta), \overline{u}(\varpi,t;\vartheta)]$. *Such that* $\underline{u}(\varpi,t;\vartheta)$ *and* $\overline{u}(\varpi,t;\vartheta)$ *are* 1^{st} *order differentiable functions with regard to t, then:*

1. If $u(\varpi,t)$ is the first form differentiable function, then:

$$\left[\frac{\partial}{\partial t}u(\varpi,t)\right]^{\vartheta} = \left[\frac{\partial}{\partial t}\underline{u}(\varpi,t;\vartheta), \frac{\partial}{\partial t}\overline{u}(\varpi,t;\vartheta)\right],$$

2. If $u(\varpi,t)$ is the second form differentiable function then :

$$\left[\frac{\partial}{\partial t}w(\varpi,t)\right]^{\vartheta} = \left[\frac{\partial}{\partial t}\overline{u}w(\varpi,t;\vartheta), \frac{\partial}{\partial t}\underline{u}w(\varpi,t;\vartheta)\right].$$

3. Fuzzy partial derivative formulae for the first and second orders.

Theorem 3: *Let* $\mathrm{w}:(0,\infty)\times(0,\infty)\to \mathrm{E}$ *continuous fun. with fuzzy values, and* u_t *a portion of u its derivative with regard to t. suppose that* $se^{-(i\sqrt[3]{s}+\sin s)t}u(\varpi,t)$ *and* $se^{-(i\sqrt[3]{s}+\sin s)t}u_t(\varpi,t)$ *are inadequate Uncertain Riemann-integrable on* $[0,\infty]$, *then:*

1. Suppose w is a differentiable function of the first kind with respect to *t*.

$$T_t[u_t(\varpi,t)] = (i\sqrt[3]{s}+\sin s)T_t[u(\varpi,t)] \ominus Tu(\varpi,0),$$

2. If *u* is a differentiable function of the second kind with respect to *t*.

$$T_t[u_t(\varpi,t)] = -su(\varpi,0) \ominus (-i\sqrt[3]{s}+\sin s)T_t[u(\varpi,t)].$$

Proof: Since $u_t(\varpi,t)$ is a continuous fuzzy-valued function, then the following two situations apply:

Case 1: Since u is the first arbitrary form differentiable function $\vartheta \in [0,1]$, From Theorem 4/1:

$$T_t[wu_t(\varpi)] = \gamma_t[w\underline{u}_t(\varpi,t;\vartheta)], \gamma_t[w\overline{u}_t(\varpi,t;\vartheta)]. \tag{1}$$

From Theorem 3/1:

$$\begin{aligned}\gamma_t[\underline{u}_t(\varpi,\vartheta)] &= (i\sqrt[3]{s} + \sin s)\gamma_t[\underline{u}w(\varpi,t;\vartheta)] - s\underline{u}(\varpi,0;\vartheta),\\ \gamma_t[\overline{u}_t(\varpi,\vartheta)] &= (i\sqrt[3]{s} + \sin s)\gamma_t[\overline{u}w(\varpi,t;\vartheta)] - s\overline{u}(\varpi,0;\vartheta).\end{aligned} \tag{2}$$

Substitute (2) in (1), then:

$$\begin{aligned}T_t[u_t(\varpi)] = (i\sqrt[3]{s} + \sin s)\gamma_t[\underline{u}(\varpi,t;\vartheta)] - s\underline{u}(\varpi,0;\vartheta),\\ (i\sqrt[3]{s} + \sin s)\gamma_t[\overline{u}(\varpi,t;\vartheta)] - s\overline{u}(\varpi,0;\vartheta).\end{aligned}$$

By Theorem 1:

$$T_t[u_t(\varpi,t)] = (i\sqrt[3]{s} + \sin s)T_t[u(\varpi,t)] \ominus su(\varpi,0).$$

Case 2: Since u is a differentiable function in its 2nd form for any random $\vartheta \in [0,1]$ From Theorem 4/2:

$$T_t[wu_t(\varpi)] = \gamma_t[w\overline{u}_t(\varpi,t;\vartheta)], \gamma_t[w\underline{u}_t(\varpi,t;\vartheta)].$$

By same way, then:

$$T_t[u_t(\varpi,t)] = -su(\varpi,0) \ominus (-i\sqrt[3]{s} + \sin s)T_t[u(\varpi,t)].$$

Theorem 4: $u:(0,\infty)\times(0,\infty)\to \mathrm{E}$ *be continuous fuzzy-valued function Assume that,* u, u_t *are fuzzy partial derivatives of fuzzy T-transforms, which are continuous fuzzy-valued functions. with respect to t. about second order will be:*

1. If u and u_t are the first form differentiable functions with respect to t, then:

$$T_t[u_{tt}(\varpi,t)] = (i\sqrt[3]{s} + \sin s)^2 T_t[u(\varpi,t)] \ominus s(i\sqrt[3]{s} + \sin s)u(\varpi,0) \ominus su_t(\varpi,0),$$

2. If u is the first form differentiable function with respect to t and u_t second form differentiable function with respect to t, then:

$$T_t[u_{tt}(\varpi,t)] = -s(i\sqrt[3]{s} + \sin s)u(\varpi,0) \ominus (-i\sqrt[3]{s} + \sin s)^2 T_t[u(\varpi,t)] \ominus su_t(\varpi,0)$$

3. if is a first u_t-form differentiable function with respect to t and if is a second u-form differentiable function with respect to t, then:

$$T_t[u_{tt}(\varpi,t)] = -s(i\sqrt[3]{s}+\sin s)u(\varpi,0) \ominus (-i\sqrt[3]{s}+\sin s)^2 T_t[u(\varpi,t)] - su_t(\varpi,0),$$

4. If u and u_t are the second form differentiable functions with respect to t, then:

$$T_t[u_{tt}(\varpi,t)] = (i\sqrt[3]{s}+\sin s)^2 T_t[u(\varpi,t)] \ominus s(i\sqrt[3]{s}+\sin s)u(\varpi,0) - su_t(\varpi,0).$$

Proof: We will prove two cases as follows:

Case 1: Since u and u_t are the first form differentiable functions with respect to t, for any arbitrary $\vartheta \in [0,1]$, From Theorem 4/1:

$$T_t[u_{tt}(\varpi)] = \gamma_t[\underline{u}_{tt}(\varpi,t;\vartheta)], \gamma_t[\overline{u}_{tt}(\varpi,t;\vartheta)].$$

$$T[u_{tt}(\varpi)] = (i\sqrt[3]{s}+\sin s)^2 \gamma_t[\underline{u}(\varpi,t;\vartheta)] - s(i\sqrt[3]{s}+\sin s)\underline{u}(\varpi,0;\vartheta) - s\underline{u}_t(\varpi,0;\vartheta),$$
$$(i\sqrt[3]{s}+\sin s)^2 \gamma_t[\overline{u}(\varpi,t;\vartheta)] - s(i\sqrt[3]{s}+\sin s)\overline{u}(\varpi,0;\vartheta) - s\overline{u}_t(\varpi,0;\vartheta).$$

By Theorem 1:

$$T_t[u_{tt}(\varpi,t)] = (i\sqrt[3]{s}+\sin s)^2 T_t[u(\varpi,t)] \ominus s(i\sqrt[3]{s}+\sin s)u(\varpi,0) \ominus su_t(\varpi,0).$$

Case 2: Since u is the first form differentiable function with respect to t and u_t is a differentiable function in its second form for any random t, for any arbitrary $\vartheta \in [0,1]$, From Theorem 4/2:

$$T_t[u_{tt}(\varpi)] = \gamma_t[\overline{u}_{tt}(\varpi,t;\vartheta)], \ \gamma_t[\underline{u}_{tt}(\varpi,t;\vartheta)].$$

By Theorem 1:

$$T_t[u_{tt}(\varpi,t)] = -s(i\sqrt[3]{s}+\sin s)u(\varpi,0) \ominus (-i\sqrt[3]{s}+\sin s)^2 T_t[u(\varpi,t)] \ominus su_t(\varpi,0).$$

Example: Consider the following fuzzy system partial differential equation:

$$u_\varpi + w_t = 1; \text{ I.C. } (i) \ u(\varpi,0) = (2+\vartheta, 2-\vartheta), w(\varpi,0) = (2+\vartheta, 4-\vartheta)$$
$$w_\varpi + u_t = 0; \text{ B.C. } (ii) \ u(0,t) = (2+\vartheta, 4-\vartheta)\lim_{\varpi\to\infty} \text{exist}, w(\varpi,0)$$
$$= (\vartheta, 2-\vartheta)\lim_{\varpi\to\infty} \text{exist}.$$

Both sides of the final system equations are T-transformed.

$$T_t[u_\varpi(\varpi,t)] + T_t[w_t(\varpi,t)] = T_t[1],$$
$$T_t[w_\varpi(\varpi,t)] + T_t[u_t(\varpi,t)] = 0 .$$

We get the following cases:

Cases (1&3) or (2&4).

$$\frac{\partial}{\partial \varpi} T_t[u(\varpi,t)] + (i\sqrt[3]{s}+\sin s)T_t[w(\varpi,t)] \ominus sw(\varpi,0) = \frac{s}{i\sqrt[3]{s}+\sin s},$$
$$\frac{\partial}{\partial \varpi} T_t[w(\varpi,t)] + (i\sqrt[3]{s}+\sin s)T_t[u(\varpi,t)] \ominus su(\varpi,0) = 0\ .$$

Then:

$$\frac{\partial}{\partial \varpi} \gamma_t[\underline{u}(\varpi,t)] + (i\sqrt[3]{s}+\sin s)\gamma_t[\underline{w}(\varpi,t)] - s\underline{w}(\varpi,0) = \frac{s}{i\sqrt[3]{s}+\sin s},$$
$$\frac{\partial}{\partial \varpi} \gamma_t[\overline{u}(\varpi,t)] + (i\sqrt[3]{s}+\sin s)\gamma_t[\overline{w}(\varpi,t)] - s\overline{w}(\varpi,0) = \frac{s}{i\sqrt[3]{s}+\sin s}.$$
$$\frac{\partial}{\partial \varpi} \gamma_t[\underline{w}(\varpi,t)] + (i\sqrt[3]{s}+\sin s)\gamma_t[\underline{u}(\varpi,t)] - s\underline{u}(\varpi,0) = 0\ ,$$
$$\frac{\partial}{\partial \varpi} \gamma_t[\overline{w}(\varpi,t)] + (i\sqrt[3]{s}+\sin s)\gamma_t[\overline{u}(\varpi,t)] - s\overline{u}(\varpi,0) = 0.$$

Solving the remaining partial equations of the system from the given beginning conditions:

$$\underline{u}(\varpi,t;\vartheta) = (2+\vartheta)k(t-\varpi) - (2+\vartheta)k(t-\varpi) + (2+\vartheta),$$
$$\overline{u}(\varpi,t;\vartheta) = (4-\vartheta)\,k(t-\varpi) - (2-\vartheta)k(t-\varpi) + (2-\vartheta).$$
$$\underline{w}(\varpi,t;\vartheta) = t + (2+\vartheta) + (2+\vartheta)k(t-\varpi) - (2+\vartheta)k(t-\varpi),$$
$$\overline{w}(\varpi,t;\vartheta) = t + (4-\vartheta) + (4-\vartheta)k(t-\varpi) - (2-\vartheta)k(t-\varpi).$$

Cases (1&4) or (2&3).

$$\frac{\partial}{\partial \varpi} T_t[u(\varpi,t)] - sw(\varpi,0) \ominus (i\sqrt[3]{s}+\sin s)T_t\ [w(\varpi,t)] = \frac{s}{i\sqrt[3]{s}+\sin s},$$
$$\frac{\partial}{\partial \varpi} T_t[w(\varpi,t)] - su(\varpi,0) \ominus (-i\sqrt[3]{s}+\sin s)T_t\ [u(\varpi\ ,t)] = 0\ .$$

Then:

$$\underline{u}(\varpi,t;\vartheta) = (2+\vartheta) + \vartheta k(t-\varpi) - tk(t-\varpi) - (2+\vartheta)k(t-\varpi),$$
$$\overline{u}(\varpi,t;\vartheta) = (2-\vartheta) + (2-\vartheta)k(t-\varpi) - tk(t-\varpi) - (4-\vartheta)k(t-\varpi).$$
$$\underline{w}(\varpi,t;\vartheta) = \vartheta k(t-\varpi) - tk(t-\varpi) - (2+\vartheta)k\ \ (t-\varpi) + t + (4-\vartheta),$$
$$\overline{w}(\varpi,t;\vartheta) = (2-\vartheta)k(t-\varpi) - tk(t-\varpi) - (4-\vartheta)k\ \ (t-\varpi) + t + (2+\vartheta).$$

4. Conclusion

We apply these theorems to solve a partial system problem, and we also get the fuzzy derivatives of the fuzzy partial T-transform at the generalized order.

References

[1] Abdul Rahman, N.A.; Ahmad, M.Z. Fuzzy Sumudu transform for solving fuzzy partial differential equations. *J. Nonlinear Sci. Appl.* 9(5), 3226-3239 (2016).

[2] Cano Y.C. Flores H. R. On new solutions of fuzzy differential Equations. *Chaos, Solitons & Fract.* 38(1): 112-119 (2008). https://doi.org/10.1016/j.chaos.2006.10.043.

[3] A. M. Wazwaz "Linear and non-linear equation: method and application", Springer, London (2011).

[4] B. Bede, I. J. Rudas, nd A. L.Bencsik, "First Order Linear Fuzzy Differential Equations Under Generalized Differentiability," *Information Sciences*, 177(7), 1648-1662 (2007).

[5] C. Duraisamy , Usha B, "Another Approach to Solution of Fuzzy Differential Equations," *Applied Mathematical Sciences*, 4(16): 777-790 (2010), 692; https://doi.org/10.3390/math8050692.

[6] Gündogdu, H. Ömer G.F, Solving nonlinear partial differential equations by using Adomian decomposition method modified decomposition method and Laplace decomposition method, *MANAS Journal of Engineering*, 5(1), 1-13 (2017).

[7] Haydar A. K. Hassan R. H. Generalization of Fuzzy Laplace Transforms of Fuzzy Fractional Derivatives about the General Fractional Order $n-1 < \beta < n$. *Mathematical Problems in Engineering*. 1-13 (2016). https://doi.org/10.1155/2016/6380978.

[8] Rodríguez-López R. Comparison results for fuzzy differential equations. *Inform Sci.* 178(6): 1756-1779 (2008). https://doi.org/10.1016/j.ins.2007.10.011.

[9] Salahshour, S. Allahviranloo T. Applications of fuzzy Laplace transforms, *Soft Comput.* 17(1), 145-158 (2013). https://doi.org/10.1007/s00500-012-0907-4.

[10] Seikkala S. On the fuzzy initial value problem, *Fuzzy Sets and Systems.* 24(3): 319-330 (1987). https://doi.org/10.1016/0165-0114(87)90030-3.

[11] Sleibi A. Alkiffai A.N, Solving Ordinary Differental Equations Using Fuzzy Transformation, Msc. these Kufa University, 2020.

[12] Ullah S. Farooq M. Ahmad L. Abdullah S. Application of fuzzy Laplace transforms for solving fuzzy partial Volterra integro-differential Equations, arXiv (2018). https://doi.org/10.48550/arXiv.1405.1895.

[13] Vignesh R. Thanga Rajeswari K. Venkatesh S. C. Natarajan, Alaeiyan M. On some degree-based irregularity indices of certain networks. *Journal of Discrete Mathematical Sciences & Cryptography*. 24(2), 617-628 (2021). https://doi.org/10.1080/09720529.2021.1892269.

[14] Vignesh R. Desikan K. Rajeswari K. T. Shanmugavelan S. Natarajan C. On certain degree-based irregularity indices of some nanotubes. *Journal of Discrete Mathematical Sciences & Cryptography*. 24(2), 603-615 (2021). https://doi.org/10.1080/09720529.2021.1891694.

[15] Wu H.-C. The improper fuzzy Riemann integral and its numerical integration, *Inform. Sci*. 111(1-4), 109-137 (1998). https://doi.org/10.1016/S0020-0255(98)00016-4.

[16] Wiley A. Sons, Applied Engineering Analysis, Tai-Ran Hsu, John (2018).

[17] Zadeh L. A. Fuzzy sets, Inform, Cont. 8(3): 338-353 (1965). https://doi.org/10.1016/S0019-9958(65)90241-X.

[18] Zahid Z. Asif F. Jalil A. Khalaf M. & Farahani M.R. On leap indices of CNCk[n] by using line operator on its subdivision. *Journal of Discrete Mathematical Sciences & Cryptography*. 24(2), 343-352 (2021). https://doi.org/10.1080/09720529.2021.1882157.

[19] Zhang X. Ahmed F.S. Y. Li, L. Yan, Baig A.Q. Farahani M.R. A family of fifth-order convergent methods for solving nonlinear equations using variational iteration technique. *Journal of Information and Optimization Sciences*. 39(3), 673-694 (2018). https://doi.org/10.1080/02522667.2018.1443628.

[20] Zhang X. Razzaq A. Ali K. Rizvi S.T. R. Farahani M.R. A new approach to find eccentric indices of some graphs. *Journal of Information and Optimization Sciences*. 41(4), 2020, 865-877. https://doi.org/10.1080/02522667.2020.1744304.