

## Dispersive soft group space

Hanan Ali Hussein \*

*Department of Mathematics  
College of Education for Girls  
University of Kufa  
Kufa  
Iraq*

Sattar Hameed Hamzah <sup>†</sup>

*Department of Mathematics  
College of Education  
University of Al-Qadisyah  
Iraq*

Habeeb Kareem Abdullah <sup>§</sup>

*Department of Mathematics  
Kufa University  
Kufa  
Iraq*

---

### Abstract

The soft set theory proposed by Molodtsov is a recent mathematical approach for modeling uncertainty and vagueness. The main aim of this paper is to introduce the concepts of soft group space of dispersive soft group space and soft fixed point. Finally, the concept of dispersive soft group space was introduced and some properties related to it.

---

**Subject Classification:** (2010) 54A05, 54C08.

**Keywords:** Soft topological group, Soft group space, Soft orbit, Soft stabilizer and Soft periodic point.

### 1. Introduction

In 1999[4] the soft set theory introduced by Molodtsov. Although this theory has not had a long history, it has created a wide range of applications in

---

\* Orcid : 0000-0003-0003-190X

<sup>†</sup> Orcid : 0000-0003-1760-5092

<sup>§</sup> Orcid : 0000-0002-6057-8715

---

\* E-mail: hanana.hussein@uokufa.edu.iq (Corresponding Author)

<sup>†</sup> E-mail: sattar.hamzah@qu.edu.iq

<sup>§</sup> E-mail: habeebk.abdullah@uokufa.edu.iq

many disciplines. Especially, this theory has been studied by mathematicians in different ways such as algebraic, topological and categorical [1,2,3,6,7,8,9,10,11,12]. In this paper soft action is defined and studied, some important examples and properties are presented. Some concepts related to the soft action such as soft stabilizers, soft orbit are defined. Finally, the concept of dispersive soft group space is introduced.

## 2. Preliminaries

In this section, we review some main concepts and properties of soft topological group, soft action and soft group space for the sake of completeness. Let  $X$  be an initial universe set and let  $\hat{E}$  be a set of parameters. Let  $P(X)$  denotes the power set of  $X$ .

**Definition 2.1:** [5] Let  $(F, \hat{E})$  be a soft set over  $\mathbb{G}$  and  $T$  be a soft topology over  $\mathbb{G}$ , then  $(\mathbb{G}, T, F, \hat{E})$  is called a soft topological group (Stg) iff  $(F(\omega), T_{F(\omega)})$  is a topological group on  $F(\omega)$  for all  $\omega \in \hat{E}$ , where  $T_{F(\omega)}$  is the relativized topology on  $F(\omega)$  induced from  $T_\omega$ .

**Definition 2.2:** [5] Let  $(\mathbb{G}, T, F, \hat{E})$  be a Stg and let  $(H, \Gamma, \hat{E})$  be a Sts over  $X$ . A left soft (continuous) action of  $(\mathbb{G}, T, F, \hat{E})$  on  $(H, \Gamma, \hat{E})$  is a continuous map  $\varphi_\omega: F(\omega) \times H(\omega) \rightarrow H(\omega)$  such that for all  $\omega \in \hat{E}$ :

- (i)  $\varphi_\omega(e_\mathbb{G}, x) = x$ , for all  $x \in H(\omega)$
  - (ii)  $\varphi_\omega(g, \varphi_\omega(g^*, x)) = \varphi_\omega(gg^*, x)$ , for all  $g, g^* \in F(\omega)$  and  $x \in H(\omega)$ .
- In similar way, a right soft action is defined.

Note that the difference between the left and right soft action is not a trivial one, however there is a one to one correspondence between them as follows:

If  $\varphi_\omega$  is a left soft action of  $(\mathbb{G}, T, F, \hat{E})$  on  $(H, \Gamma, \hat{E})$ , then  $\phi_\omega: H(\omega) \times F(\omega) \rightarrow H(\omega)$  defined by  $\phi_\omega(x, g) = \varphi_\omega(g^{-1}, x)$  is a right soft action of  $(\mathbb{G}, T, F, \hat{E})$  on  $(H, \Gamma, \hat{E})$ , and similarly for right soft action. Thus, for every left soft action is a conjugate right soft action and vice versa. So, every theorem that is true of left soft action has a conjugate theorem for right soft action.

Because of this, we will usually use a left soft action.

The soft topological space  $(H, \Gamma, \hat{E})$  is called "Soft Group space" which is denoted by  $(S\mathbb{G}\text{-space})$ .

**Definition 2.3:** [5] Let  $(\mathbb{G}, T, F, \hat{E})$  be a Stg, then for all  $x \in H(\omega)$  the set  $Orb_\omega(x) = \{\varphi_\omega(g, x): g \in F(\omega)\}$  for all  $\omega \in \hat{E}$  is called the soft orbit of  $x$ .

**Definition 2.4:** Let  $(H, \Gamma, \hat{E})$  be a  $S\mathbb{G}$ -space, for any point  $x \in H(\omega)$ , then for all  $\omega \in \hat{E}$ :

- (i)  $J_\omega(x) = \{\psi \in H(\omega) : \text{there is a net } \{g_\beta\}_{\beta \in \Omega} \text{ in } F(\omega) \text{ and there is a net } \{\chi_\beta\}_{\beta \in \Omega} \text{ in } H(\omega) \text{ with } g_\beta \rightarrow \infty \text{ and } \chi_\beta \rightarrow x \text{ such that } \varphi_\omega(g_\beta, \chi_\beta) \rightarrow \psi\}$  is called first soft prolongation limit set of  $H(\omega)$ .
  - (ii)  $\Lambda_\omega(x) = \{\psi \in H(\omega) : \text{there is a net } \{g_\beta\}_{\beta \in \Omega} \text{ in } F(\omega) \text{ with } g_\beta \rightarrow \infty \text{ such that } \varphi_\omega(g_\beta, x) \rightarrow \psi\}$  is called soft limit set of  $H(\omega)$ .
- It is clear that the set  $\Lambda_\omega(x)$  is a subset of  $J_\omega(x)$  for all  $\omega \in \hat{E}$ .

**Definition 2.5:[5]** Let  $(H, \Gamma, \hat{E})$  be a  $S\mathbb{G}$ -space, then the set  $Stab_\omega(x) = \{g \in F(\omega) : \varphi_\omega(g, x) = x\}$  for all  $\omega \in \hat{E}$  is called the soft stabilizer of  $x$ .

### 3. Dispersive soft group space

In this section, we will describe the concept of dispersive soft group space and exemplify it. In addition, we will establish some characterizations about it.

**Definition 3.1:** A  $S\mathbb{G}$ -space  $(H, \Gamma, \hat{E})$  is called Dispersive soft group space (briefly  $DS\mathbb{G}$ -space) if for each two points  $x, \psi \in H(\omega)$ , there are open neighborhoods  $M, N$  of  $x$  and  $\psi$  respectively such that the set  $((M, N)) = \{g \in F(\omega) : \varphi_\omega(g, M) \cap N \neq \emptyset\}$  is relatively compact in  $F(\omega)$ .

**Example 3.2:** Let  $\mathbb{G} = (Z_4, +_4)$  with discrete soft topology  $T$ ,  $\hat{E} = \{\omega_1, \omega_2\}$  and let  $(F, \hat{E})$  be a soft set over  $\mathbb{G}$  such that  $F(\omega_1) = Z_4$ ,  $F(\omega_2) = \{0\}$ , let  $(X, \Gamma)$  be a discrete  $S\tau_s$  over  $Z_4$  such that  $H(\omega_1) = H(\omega_2) = Z_4$ . If we defined  $\varphi_{\omega_1} : F(\omega_1) \times H(\omega_1) \rightarrow H(\omega_1)$  by  $\varphi_{\omega_1}(x, \psi) = x +_4 \psi$  and  $\varphi_{\omega_2} : F(\omega_2) \times H(\omega_2) \rightarrow H(\omega_2)$  by  $\varphi_{\omega_2}(x, \psi) = \psi$ .

Then  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space.

**Theorem 3.3:** Let  $(H, \Gamma, \hat{E})$  be a  $S\mathbb{G}$ -space, then the following statements are equivalents:

- (i)  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space.
- (ii)  $J_\omega(x) = \emptyset$  for all  $x \in H(\omega)$ .
- (iii)  $\psi \notin J_\omega(x)$  for all  $x, \psi \in H(\omega)$ .

**Proof:**

(i)  $\rightarrow$  (ii)

Suppose  $J_\omega(x) \neq \emptyset$ , then there is  $\psi \in H(\omega)$  such that  $\psi \in J_\omega(x)$ .

Then, there is a net  $\{g_\beta\}_{\beta \in \Omega}$  in  $F(\omega)$  and a net  $\{\chi_\beta\}_{\beta \in \Omega}$  in  $H(\omega)$  such that  $g_\beta \rightarrow \infty$  and  $\chi_\beta \rightarrow x$  such that  $\varphi_\omega(g_\beta, \chi_\beta) \rightarrow \psi$ .

Since  $(H, \Gamma, \hat{E})$  be  $DS\mathbb{G}$ -space, then there are open neighborhoods  $M, N$  of  $x$  and  $\psi$  respectively such that  $((M, N))$  is relative compact in  $F(\omega)$ .

Since  $\chi_\beta \rightarrow x$  and  $\varphi_\omega(g_\beta, \chi_\beta) \rightarrow \psi$ , then there is  $\beta_0 \in \Omega$  such that  $\chi_\beta \in M$  and  $\varphi_\omega(g_\beta, \chi_\beta) \in N$  for all  $\beta \geq \beta_0$ , hence  $\varphi_\omega(g_\beta, \chi_\beta) \in \varphi_\omega(g_\beta, M)$

Therefore  $g_\beta \in ((M, N))$

Then  $\{g_\beta\}$  has a cluster point  $g \in F(\omega)$  and this is contradiction.

Then  $J_\omega(x) = \emptyset$ .

(ii)  $\rightarrow$  (iii)

Clear

(iii)  $\rightarrow$  (i)

Suppose that  $(H, \Gamma, \hat{E})$  be not a  $DS\mathbb{G}$ -space, then there are two points  $x, y \in H(\omega)$  such that for all open neighborhoods  $M$  of  $x$  and  $N$  of  $y$ , the set  $((M, N))$  isn't relative compact in  $F(\omega)$ .

Thus, there is a net  $\{g_\beta\}_{\beta \in \Omega}$  in  $F(\omega)$  such that  $g_\beta \rightarrow \infty$ .

Since  $\varphi_\omega(g_\beta, M) \cap N \neq \emptyset$  for all  $\beta \in \Omega$ .

Then there is  $x_\beta \in M$  such that  $\varphi_\omega(g_\beta, x_\beta) \in N$  for all  $\beta \in \Omega$

Hence  $\{x_\beta\}$  and  $\{\varphi_\omega(g_\beta, x_\beta)\}$  are nets in  $H(\omega)$  such that  $x_\beta \rightarrow x$  and  $\varphi_\omega(g_\beta, x_\beta) \rightarrow y$ .

Then  $y \in J_\omega(x)$  and this contradiction.

Hence  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space.

**Corollary 3.4:** If  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then  $\Lambda_\omega(x) = \emptyset$ , for all  $x \in H(\omega)$  and  $\omega \in \hat{E}$ .

**Proof:** Since  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then by theorem (4.4.3)(ii)  $J_\omega(x) = \emptyset$ , for all  $x \in H(\omega)$ .

Since  $\Lambda_\omega(x) \subseteq J_\omega(x)$  for all  $x \in H(\omega)$  see (4.2.14).

Then  $\Lambda_\omega(x) = \emptyset$  for all  $x \in H(\omega)$  and  $\omega \in \hat{E}$ .

**Remark 3.5:** The converse of corollary (3.4) isn't be true, consider the following example:

**Example 3.6:** Let  $\hat{E} = \{\omega\}$ ,  $\mathbb{G} = (Q, +)$  with relative usual topology  $T$ , let  $(F, \hat{E})$  be a soft set over  $\mathbb{G}$  such that  $F(\omega) = Q$ . If  $X = \mathbb{G}$

If we defined  $\varphi_\omega: F(\omega) \times H(\omega) \rightarrow H(\omega)$  by  $\varphi_\omega(g, x) = g + x$ .

Let  $y \in \Lambda_\omega(x)$ , then there is a net  $\{g_\beta\}_{\beta \in \Omega}$  in  $Q$  with  $g_\beta \rightarrow \infty$  such that  $\varphi_\omega(g_\beta, x) \rightarrow y$ .

Then  $g_\beta + x \rightarrow \infty$  this contradiction, then  $\Lambda_\omega(x) = \emptyset$ .

But  $Q$  isn't compact, then  $(H, \Gamma, \hat{E})$  isn't  $DS\mathbb{G}$ -space.

**Definition 3.7:** Let  $(H, \Gamma, \hat{E})$  be a  $S\mathbb{G}$ -space. A point  $x \in H(\omega)$  is called a **fixed point** if  $Orb_\omega(x) = Stab_\omega(x) = \{x\}$  for all  $\omega \in \hat{E}$ .

**Example 3.8:** Let  $\mathbb{G} = (\mathcal{R}, .)$  with discrete topology,  $\hat{E} = \{\omega_1, \omega_2\}$ . Let  $(F, \hat{E})$  be a soft set over  $\mathbb{G}$  defined by  $F(\omega_1) = F(\omega_2) = H(\omega_1) = H(\omega_2) = \{1\}$ , and defined  $\varphi_\omega: F(\omega) \times F(\omega) \rightarrow F(\omega)$  by  $\varphi_\omega(g, h) = h$ , for all  $\omega \in \hat{E}$ , then  $(H, \Gamma, \hat{E})$  is  $S\mathbb{G}$ -space has fixed point  $\{1\}$ .

**Definition 3.9:** Let  $(\mathbb{G}, \Gamma, F, \hat{E})$  be a Stg, a subset  $U$  of  $F(\omega)$  is called **Syndetic** if there is a compact subset  $V$  of  $F(\omega)$  such that  $F(\omega) = \varphi_\omega(U, V)$  for all  $\omega \in \hat{E}$ .

**Definition 3.10:** Let  $(H, \Gamma, \hat{E})$  be a  $DS\mathbb{G}$ -space, a point  $x \in H(\omega)$  is called **periodic point** if  $Stab_\omega(x)$  is syndetic in  $F(\omega)$  for all  $\omega \in \hat{E}$ .

**Example 3.11:** If  $(\mathbb{G}, T, F, \hat{E})$  be a compact Stg, then every point in  $H(\omega)$  is periodic.

**Theorem 3.12:** If  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then:

- (i) There is no fixed point.
- (ii) There is no periodic point.

**Proof:**

- (i) Suppose that  $x \in H(\omega)$  be a fixed point.  
 Since  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then there are neighborhoods  $M, N$  of  $x$  in  $H(\omega)$  such that  $((M, N))$  is relatively compact in  $F(\omega)$ .  
 Since  $x$  is a fixed point, then  $((M, N)) = F(\omega)$ .  
 Then  $F(\omega)$  is compact for all  $\omega \in \hat{E}$ , but this is contradiction.  
 Then  $(H, \Gamma, \hat{E})$  has no fixed point.
- (ii) Suppose that  $x \in H(\omega)$  be a periodic point.  
 Then,  $Stab_\omega(x)$  is syndetic in  $F(\omega)$ .  
 That is there is a compact subset  $U$  of  $F(\omega)$  such that  $F(\omega) = \varphi_\omega(Stab_\omega(x), U)$ .  
 Since  $Stab_\omega(x)$  is compact from (4.4.15(ii)), thus  $F(\omega)$  is compact for all  $\omega \in \hat{E}$ , this is contradiction.  
 Then  $(H, \Gamma, \hat{E})$  has no periodic point.

**Theorem 3.13:** If  $(H, \Gamma, \hat{E})$  be a  $DS\mathbb{G}$ -space, let  $(K, \hat{E})$  be a soft closed subgroup of  $(F, \hat{E})$  over  $\mathbb{G}$  and let  $Y$  be an open subspace of  $H(\omega)$  which is an invariant under  $K(\omega)$ . Then  $Y$  is  $DK$ -Space.

**Proof:** Let  $x, y \in Y$ , then  $x, y \in H(\omega)$ .

Since  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then there are open neighborhoods  $M$  and  $N$  of  $x$  and  $y$  respectively in  $H(\omega)$ , such that  $((M, N))$  is relatively compact in  $F(\omega)$ .

Let  $U = M \cap Y$  and  $V = N \cap Y$ .

Since  $Y$  is an open subspace of  $H(\omega)$ , then we have  $U$  and  $V$  to be open neighborhoods of  $x$  and  $y$  respectively in  $Y$ .

Since  $((U, V)) \subseteq ((M, N))$ , then  $((U, V))$  is relatively compact in  $F(\omega)$ .

Since  $K(\omega)$  is a closed subgroup of  $F(\omega)$ , then  $((U, V))$  is relatively compact in  $K(\omega)$ .

Then  $Y$  is  $DK$ -space.

**Corollary 3.14:** *If  $(H, \Gamma, \hat{E})$  be a  $DS\mathbb{G}$  – space and  $Y$  be an open subspace of  $H(\omega)$  which is an invariant under  $F(\omega)$ , then  $Y$  is  $D\mathbb{G}$ -space.*

**Corollary 3.15:** *If  $(H, \Gamma, \hat{E})$  be a  $DS\mathbb{G}$ -space and  $(K, \hat{E})$  be a soft closed subgroup of  $(F, \hat{E})$ , then  $H(\omega)$  is  $DK$  –space.*

**Theorem 3.16:** *Let  $(I_G, f_\omega)$  be a morphism from  $S\mathbb{G}$ -space  $(H_1, \Gamma_1, \hat{E})$  into  $S\mathbb{G}$ -space  $(H_2, \Gamma_2, \hat{E})$  such that  $f_\omega: H_1(\omega) \rightarrow H_2(\omega)$  be homeomorphism for all  $\omega \in \hat{E}$ . If  $(H_1, \Gamma_1, \hat{E})$  is  $DS\mathbb{G}$ -space, then so is  $(H_2, \Gamma_2, \hat{E})$ .*

**Proof:** Let  $\psi_1, \psi_2 \in H_2(\omega)$ .

Since  $f_\omega$  is surjective, then there are  $x_1, x_2 \in H_1(\omega)$  such that  $f_\omega(x_1) = \psi_1, f_\omega(x_2) = \psi_2$ .

Since  $(H_1, \Gamma_1, \hat{E})$  is  $DS\mathbb{G}$ -space and  $x_1, x_2 \in H_1(\omega)$ , then there are open neighborhoods  $M$  and  $N$  of  $x_1$  and  $x_2$  respectively in  $H_1(\omega)$  such that  $((M, N))$  is relatively compact in  $F_1(\omega)$ .

Since  $f_\omega$  are homeomorphism for all  $\omega \in \hat{E}$ , then  $f_\omega(M), f_\omega(N)$  be open neighborhoods of  $\psi_1, \psi_2$  respectively in  $H_2(\omega)$ .

Since  $f_\omega$  be injective and equivariant functions for all  $\omega \in \hat{E}$ , then  $g \in ((M, N)) \leftrightarrow \varphi_\omega(g, M) \cap N \neq \emptyset$

$$\leftrightarrow f_\omega(\varphi_\omega(g, M) \cap N) \neq \emptyset$$

$$\leftrightarrow f_\omega(\varphi_\omega(g, M)) \cap f_\omega(N) \neq \emptyset$$

$$\leftrightarrow \varphi_\omega(g, f_\omega(M)) \cap f_\omega(N) \neq \emptyset$$

$$\leftrightarrow g \in ((f_\omega(M), f_\omega(N))).$$

$$\text{Hence } ((M, N)) = ((f_\omega(M), f_\omega(N))).$$

Since  $((M, N))$  is relatively compact in  $F_1(\omega)$ .

Then  $((f_\omega(M), f_\omega(N)))$  is relatively compact in  $F_2(\omega)$

Thus  $(H_2, \Gamma_2, \hat{E})$  is a  $DS\mathbb{G}$ -space.

**Theorem 3.17:** *If  $(H, \Gamma, \hat{E})$  and  $(K, \hat{F}, \hat{E})$  are two  $S\mathbb{G}$ -spaces, if  $(H, \Gamma, \hat{E})$  is a  $DS\mathbb{G}$ -space, then so is  $(H \times K, \Gamma_{prod}, \hat{E})$ .*

**Proof:** Let  $(H, \Gamma, \hat{E})$  be a  $DS\mathbb{G}$ -space.

Let  $(h_1, k_1), (h_2, k_2) \in H(\omega) \times K(\omega)$ .

Then  $h_1, h_2 \in H(\omega)$  and  $k_1, k_2 \in K(\omega)$ .

Since  $(H, \Gamma, \hat{E})$  is  $DS\mathbb{G}$ -space, then there are open neighborhoods  $M, N$  of  $h_1$  and  $h_2$  respectively in  $H(\omega)$  such that  $((M, N))$  is relatively compact in  $F(\omega)$ .

Then  $M \times K(\omega)$  and  $N \times K(\omega)$  are open neighborhoods of  $(h_1, k_1), (h_2, k_2)$  respectively in  $H(\omega) \times K(\omega)$ .

Since  $((M, N)) = ((M \times K(\omega), N \times K(\omega)))$

Then  $((M \times K(\omega), N \times K(\omega)))$  is relatively compact in  $\hat{F}(\omega)$ .

Then  $(H \times K, \Gamma_{prod}, \hat{E})$  is  $DS\mathbb{G}$  –space.

## References

- [1] H. Aktas, N. Cagman, soft sets and soft groups, *Inform. Sci.*, 77(13), 2726-2735 (2007). <https://doi.org/10.1016/j.ins.2006.12.008>.
- [2] A. O. Atagun, A. Sezgin, soft substructure of rings, fields and modules, *Comput and Math. With Appl.*, 61(3), 592-601 (2011). <https://doi.org/10.1016/j.camwa.2010.12.005>.
- [3] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, *Comput. And Math. With Appl.*, 45(4-5), 555-562 (2003). [https://doi.org/10.1016/S0898-1221\(03\)00016-6](https://doi.org/10.1016/S0898-1221(03)00016-6).
- [4] H. Aktas, N.Çağman, soft set and soft groups, *Inform. Sci.*, 177(13), 2726-2735 (2007). <https://doi.org/10.1016/j.ins.2006.12.008>.
- [5] Oguz G., Soft topological transformation groups, *Mathematics*, 8(9), 1545 (2020). <https://doi.org/10.3390/math8091545>.
- [6] S. Sardar, S. Gupta, Soft category theory an introduction, *Journal of Hyperstructures*, 2(2), 118-135 (2013).
- [7] M. Shabir, M. Naz, On soft topological spaces, *Comput. and Math. With Appl.*, 61(7), 1786-1799 (2011). <https://doi.org/10.1016/j.camwa.2011.02.006>.
- [8] T. Shah, S. Shaheen, Soft topological group and rings, *Ann.Fuzzy Math. Inform.*, 7(5), 725-743 (2014). Corpus ID: 198182320.
- [9] S.A. Abbas, S.N. Al-Kafaji, A.H. Hussain, E.K. Monajeeb, M.S. Rasheed, Novel of soft sets and soft topologies in soft ideal spaces, *Journal of Interdisciplinary Mathematics*, 23(4), 791-802 (2020). <https://doi.org/10.1080/09720502.2019.1706857>.
- [10] S. Rathee, R. Girdhar, K. Dhingra, On soft  $\omega$ -interior and soft  $\omega$ -closure in soft topological spaces, *Journal of Interdisciplinary Mathematics*, 23(6), 1223-1239 (2020). <https://doi.org/10.1080/09720502.2020.1815399>.
- [11] M.A. Hasan, N.M.A. Abbas, S.M. Kalil, On soft  $\alpha^*$ -open sets and soft contra  $\alpha^*$ -continuous mappings in soft topological spaces, *Journal of Interdisciplinary Mathematics*, 24(3), 729-734 (2021). <https://doi.org/10.1080/09720502.2020.1861786>.
- [12] R.N. Majeed, Generalized closed fuzzy soft sets in Čech fuzzy soft closure spaces, *Journal of Interdisciplinary Mathematics*, 22(8), 1433-1441 (2020). <https://doi.org/10.1080/09720502.2019.1700931>.

- [13] S. Ediz, İ. Çiftçi, M. Cancan, M. R. Farahani, On  $k$ -total distance degrees and  $k$ -total Wiener polarity index. *Journal of Information and Optimization Sciences*. 42(7), 1469-1477 (2021). <https://doi.org/10.1080/02522667.2021.1896652>.
- [14] F. Afzal, Shah Rukh, D. Afzal, M. R. Farahani, M. Cancan, S. Ediz. A study of non-commutative Von-Neumann regular rings. *Journal of Information and Optimization Sciences*. 42(7), 1425-1436 (2021). <https://doi.org/10.1080/02522667.2021.1896650>.
- [15] M. Shoaib Sardar, M. Alaeiyan, M. R. Farahani, M. Cancan, S. Ediz. Resistance distance in some classes of rooted product graphs obtained by Laplacian generalized inverse method. *Journal of Information and Optimization Sciences* 42(7), 1447-1467 (2021). <https://doi.org/10.1080/02522667.2021.1899210>.
- [16] M. Nadeem, S. Ahmad, M. K. Siddiqui, M. A. Ali, M. R. Farahani, A. J. M. Khalaf. On some applications related with algebraic structures through different well known graphs. *Journal of Discrete Mathematical Sciences and Cryptography*. 24(2), 451-471 (2021). <https://doi.org/10.1080/09720529.2021.1885806>.
- [17] A. Alsinai, H. M. U. Rehman, Y. Manzoor, M. Cancan, Z. Taş, M. R. Farahani, Sharp upper bounds on forgotten and SK indices of cactus graph, *Journal of Discrete Mathematical Sciences and Cryptography*, 1-22 (2022). <https://doi.org/10.1080/09720529.2022.2027605>.