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Simple method to extract minimal cut sets of a direct network

Roaa Aziz Fadhil *
Zahir Abdul Haddi Hassan †
Department of Mathematics
College of Education for Pure Sciences
University of Babylon
Babil
Iraq

Abstract

System planning, design, and control, network reliability validation, and evaluation are critical. The enumeration of minimal cut sets (MCs) in a network is usually more complicated than the search for minimal path sets (MPs). The purpose of this research is to offer an efficient accurate approach for enumerating directed networks' MCs. The method is concerned with determining the MCs depending on the set of minimal path nodes.

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Keywords: *Reliability, Network, Minimal cut, Minimal path.*

1. Introduction

The probability of a live connection between a source and a sink node is network reliability [3,16]. Furthermore, the problem of assessing network reliability is NP-hard [15], the use of assessing network reliability in different systems [1,2,5,6,8,10]. Minimal cut sets (MCs) or minimal path sets (MPs) are two ways to evaluate a system's reliability [3,13]. There are several ways to find a minimal path by edges [4,12] and by nodes [7,20], and the minimal cut sets can be obtained from the minimal paths in a variety of ways through a process known as inversion by using the inverse edge set of a network [9,11-16]. In this paper, we will find the MCs for a direct network using the set of nodes for paths by finding the inverse of the nodes using the definition of MCs in the algorithms [18, 19], using the relationship between the set and its complement.

* *E-mail:* roaaazizfadhil@gmail.com,
roaa.fadhil.pure256@student.uobabylon.edu.iq (Corresponding Author)
† *E-mail:* mathzahir@gmail.com

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2. Notation and Definitions

2.1 *Notation*

GN(V, X) be a network with a certain number of nodes V ={s, v_1 , v_2, ..., v_{i-1}, t} and the set of edges X, where *s* the source and *t* sink node.

e_{um} is an edge between nodes u and m , $e_{um} \neq e_{mu}$

Let ω_i be path where $\omega_i \subseteq$ V , $\overline{\omega_i} = V - \omega_i, i = 1, \dots n$, where n number of paths and

MC(ω) = { $e_{um} \in X \,|n \in \omega\,, m \in \bar{\omega}\}$,and

MC(t)= $\{e_{ut} \in X \,|u \in \bar{\omega}\}$

2.2 *Definitions*

Definition 2.2.1: Minimal path set MPs: If no node or branch intersection is traveled more than once along a path between the input and output, it is called a minimum path.

Definition 2.2.2: Cycle: A path that starts at a node and returns to it.

Definition 2.2.3: Minimal path node set MPN: They are the vertices that are connected by the edges that form the edges and form an ordered set.

For example: From the Figure 1 the path by nodes $\{ s, v_2, v_4, v_3, v_6, v_5, t\}$ while the path by edge $\{e_{s2}, e_{24}\, e_{43}, e_{36}, e_{56}\,, e_{6t}\}$ and path $\{s, v_2, v_4, v_5\,, v_3, v_6, t\}$ while path by edge $\{e_{s2}, e_{24}\, e_{45}, e_{53}, e_{36}\,, e_{6t}\}$, we note the difference in the path, although the points are similar in the two paths.

Definition 2.2.4: Minimal cut set MCs: is a set of system components that, when one fails, cause the system to fail, yet, if any other component in the set is working properly, the system will not fail.

3. Proposed Algorithm

In this section we will propose an algorithm to find all MCs by nodes of MPs for a directed network, using the definition MC in [18,19]. The steps that make up the algorithm are as follows:

Step 1: Delete the sink nodes t from each path $\omega = p - \{t\}$.

Step 2: Find complement for ω and {s},{t}.

Step 3: Find MCs.

Step 4: If the network contains a cycle, we delete the nodes that make up the cycle from the path and go back to step2.

Note 3.1: Delete those cut sets which contain all the symbols of some other cut set.

Illustrative example:

Consider Figure 1 to find MCs, for the system network, we need to find MPs, it was found by an algorithm [4,7], we get MPs

$\{s, v_2, v_3, v_6, v_5, t\}, \{s, v_2, v_3, v_6, t\}, \{v_1, v_2, v_3, t\}, \{s, v_2, v_4, v_3, v_6, v_5, t\},$

$\{s, v_2, v_4, v_3, v_6, t\}, \{s, v_2, v_4, v_3, t\}, \{s, v_2, v_4, v_5, v_3, v_6, t\}, \{s, v_2, v_4, v_5, t\},$

$\{s, v_2, v_4, v_5, v_3, t\}$

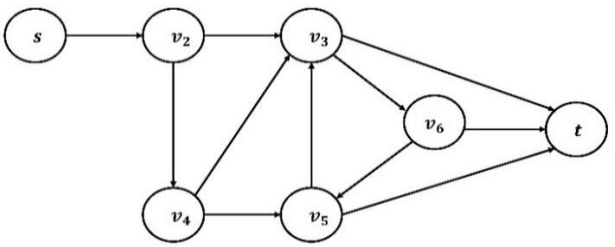


Figure 1

Direct network

In this example, we will apply the proposed algorithm and note that the network contains a cycle between the nodes $\{v_3, v_6, v_5\}$, three paths contain by step 4 and delete the nodes from paths, details of the method are shown in Table 2 The rest of the paths are shown in Table 1.

The delete sink node from all paths, and find complements for the path nodes, then find the MCs of edges by taking node from the paths set and node from the complements set, provided that the direction is from the path set to the complements set, except for the sink node by a node from the set of complements to sink node.

Table 1

Steps to find MCs for MCs that do not contain a cycle.

$w = p - \{t\}$ and{s},{t}	$\bar{w}$	MCs
{s}	$\{v_2, v_3, v_4, v_5, v_6, t\}$	$\{e_{s2}\}$
{t}	$\{s, v_2, v_3, v_4, v_5, v_6\}$	$\{e_{3t}, e_{5t}, e_{6t}\}$
$\{s, v_2, v_3, v_6\}$	$\{v_4, v_5, t\}$	$\{e_{24}, e_{65}, e_{3t}, e_{6t}\}$
$\{s, v_2, v_3\}$	$\{v_4, v_5, v_6, t\}$	$\{e_{24}, e_{36}, e_{3t}\}$
$\{s, v_2, v_4, v_3, v_6\}$	$\{v_5, t\}$	$\{e_{45}, e_{65}, e_{3t}, e_{6t}\}$
$\{s, v_2, v_4, v_3\}$	$\{v_5, v_6, t\}$	$\{e_{45}, e_{36}, e_{3t}\}$
$\{s, v_2, v_4, v_5, v_3\}$	$\{v_6, t\}$	$\{e_{36}, e_{5t}, e_{3t}\}$
$\{s, v_2, v_4, v_5\}$	$\{v_3, v_6, t\}$	$\{e_{23}, e_{43}, e_{53}, e_{5t}\}$

Table 2

Steps to find MCs for MPs that contain cycle.

path	$w = p - \{t\}$ with delete cycle	$\bar{w}$	MCs
$\{s, v_2, v_3, v_6, v_5, t\}$	$\{s, v_2\}$	$\{v_3, v_6, v_5, v_4, t\}$	$\{e_{23}, e_{24}\}$
$\{s, v_2, v_4, v_5, v_3, v_6, t\}$	$\{s, v_2, v_4\}$	$\{v_3, v_6, v_5, t\}$	$\{e_{23}, e_{45}, e_{43}\}$
$\{s, v_2, v_4, v_3, v_6, v_5, t\}$	$\{s, v_2, v_4\}$	$\{v_3, v_6, v_5, t\}$	$\{e_{23}, e_{45}, e_{43}\}$

We have three paths containing the points of the cycle. Although the path passes through each of these points once, we delete the points, delete the sink node, and find MCs of edges, the same way we used it for other paths, which are indicated in the Table 1.

4. Conclusion

This paper provides a new idea for locating all minimal cut sets for a direct complex network, which is an NP-hard problem. A new algorithm is proposed that is both easy and efficient. This algorithm is simple to use and does not take a lot of time to implement, knowing the paths of the directed network containing a cycle or not containing a cycle.

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