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On $\bar{T}$-pure sub-acts over moniods

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Abstract

The notion of a $\bar{T}$-pure sub-act and so $\bar{T}$-pure sub-act relative to sub-act are introduced. Some properties of these concepts have been studied.

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Keywords: *$\bar{T}$-pure subact, $\bar{T}$-pure sub-act relative to sub-act, Share.*

1. Introduction

A right act $\ddot{E}_S$ over a monoid is a non-empty set $\ddot{E}$ with a map (put multiplicatively), $\cdot : \ddot{E} \times S \to \ddot{E}$ such that $\ddot{a}.1 = \ddot{a}$ and $\ddot{a}\ (s\ t) = (\ddot{a}\ s)\ t$, where $s, t \in S$ and for each $\ddot{a} \in \ddot{E}$. Put $\check{N}$ is a non-empty subset of $\ddot{E}_S$ is called S-sub-act if for all $s \in S$ and $n \in \check{N}$, then $ns \in \check{N}$. A mapping θ from S-act A_S into S –act B_S is said to be a homomorphism of acts over semi-group, if for all $a \in A_S$, $s \in S$ $\theta(a)s = \theta(as)$. We will denote $Hom(A_S, B_S)$ as the set of all homomorphisms of acts over a semi-group. An S-act congruence is said to be equivalence relation ρ' on an S-act $\ddot{E}_S$, if for $m', n' \in \ddot{E}_S$, $s' \in S$, $(m', n') \in \rho' \to (m's', n's') \in \rho'$ [1]. ρ' is a congruence on $\ddot{E}_S$, $\{ [m'] : m' \in \ddot{E}_S \} = \ddot{E}_S / \rho'$ is the quotient set turn into by $[m']s' = [m's']$. Let A' be act over S and B' be a sub-act of A', the Rees congruence $\rho'B'$ on A' is defined by $a'\rho'B'b'$ iff for all $b', a' \in A', b', a' \in B'$ or $a'=b'$. The Ress quotient of $A'/\rho'_{B'}$ is denoted by $\frac{A'}{B'}$ [2].

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2. $\overline{T}$-Pure Sub-Acts over Moniod

The notion of $\overline{T}$-pure sub-act is introduced in this part. Following [3], a submodule K of an R-module A is called a $\overline{T}$-pure R-submodule if, $K \cap J^2Z = J^2K$ for each J in R.

Definition 2.1: Let Ä be acted over Moniod S and let Ë be a sub-act of Ä is said to be a $\overline{T}$-pure sub-act of Ä if, $Ë \cap Ï^2Ä = Ï^2Ë$ for all ideal Ï in S.

Remark 2.2: Put Ż is an S-act and Ë is $\overline{T}$-pure sub-act of Ż

1. If Ö is $\overline{T}$-pure sub-act in Ë , then Ö is $\overline{T}$-pure subact in Ż
2. If È is a sub-act in Ż containing a sub-act Ë, then Ë is a $\overline{T}$-pure in È.
3. Put $\{ N_j , j \in J \}$ is a non-empty collection of sub-acts of Ż and K is an ideal of S then $K^2(\cup_{j\in J} N_j) = \cup_{j\in J}(K^2N_j)$.

Proof:

(1) Put $Ï \in S$, we have Ö is T_pure sub-act of Ë also Ë is $\overline{T}$-pure of Ż, hence $Ö \cap Ï^2Ë = Ï^2Ö$ and $Ë \cap Ï^2 Ż = Ï^2Ë$ but Ö is sub-act of S-act Ë, therefore $Ö \cap Ï^2 Ż \subseteq Ë \cap Ï^2 Ż = Ï^2Ë$ and hence $Ö \cap Ï^2 Ż \subseteq Ï^2Ë$ and therefore $Ö \cap Ï^2 Ż \subseteq Ï^2Ë \cap Ö = Ï^2Ö$.

(2) Assume that $Ĵ \in S$, since we have Ë is $\overline{T}$-pure in Ż, therefore $Ë \cap Ĵ^2 Ż = Ĵ^2Ë$. But È is a subact of Ż, thus $Ë \cap Ĵ^2 È \subseteq Ë \cap Ĵ^2 Ż = J^2Ë$, and therefore $Ë \cap Ĵ^2 È \subseteq Ĵ^2 Ë \cap Ĵ^2 È = Ĵ^2 Ë$. Since $Ĵ^2 Ë \subseteq Ë \cap Ĵ^2 È$, thus $Ë \cap Ĵ^2 È = Ĵ^2 Ë$.

(3) Clear.

2.3 *Proposition*

Suppose that Ż is an S-act and Q is $\overline{T}$-pure sub-act in Ż

1. If $\mathcal{Q}$ is $\overline{T}$-pure subact and $\mathcal{P}$is subact in $\mathcal{Q}$, then $\frac{\mathcal{Q}}{\mathcal{P}}$ is $\overline{T}$-pure sub-act in $\frac{Z}{P}$.
2. Let $\mathcal{Q}, \mathcal{P}$, be sub-acts in Ż, If $\mathcal{P}$ is $\overline{T}$-pure sub-act in Ż and $\frac{\mathcal{Q}}{P}$ is $\overline{T}$-pure sub-act in $\frac{\dot{Z}}{P}$, then $\mathcal{Q}$ is $\overline{T}$-pure in Z.
3. If $\mathcal{Q}$ and $\mathcal{P}$ are $\overline{T}$-pure sub-acts of Ż then so is $\mathcal{Q} \cup \mathcal{P}$.

Proof:

(1) Put $J \in S$, so we have Q is $\overline{T}$-pure in Z, then $Q \cap J^2 Z = J^2 Q$. So $\frac{\mathcal{Q}}{\mathcal{P}} \cap J^2\left(\frac{\dot{Z}}{\mathcal{P}}\right) = \frac{\mathcal{Q}}{\mathcal{P}} \cap \frac{J^2\dot{Z}+\mathcal{P}}{\mathcal{P}} = \frac{J^2\mathcal{Q}+L\cap(\mathcal{Q}\cap J^2\dot{Z})+\mathcal{P}}{\mathcal{P}} = \frac{J^2\mathcal{Q}+}{\mathcal{P}}\mathcal{P} = J^2\left(\frac{\mathcal{Q}}{\mathcal{P}}\right)$.

(2) Clearly.

(3) Let J ∈ S. Since $\mathcal{Q}$ and $\mathcal{P}$ are pure in M, then $J^2\mathcal{Q} = \mathcal{Q} \cap J^2\dot{Z}$ and $J^2\mathcal{P} = \mathcal{P} \cap J^2\dot{Z}$. So by remark(1.2) we have $(\mathcal{Q} \cup \mathcal{P}) \cap J^2\dot{Z} = (\mathcal{Q} \cap J^2\dot{Z}) \cup (\mathcal{P} \cap J^2\dot{Z}) = J^2\mathcal{Q} \cup J^2\mathcal{P} = J^2(\mathcal{Q} \cup \mathcal{P})$, and hence $\mathcal{Q} \cup \mathcal{P}$ is $\overline{\top}$-pure.

Definition 2.4: Let Ë be an act over semi-group, Ë is called posses the $\overline{\top}$-pure intersection property, in so much as the intersection of any two $\overline{\top}$-pure sub-acts is again $\overline{\top}$-pure sub-act.

Proposition 2.5: Let Ë be an S-act

1. If Ë owns the $\overline{\top}$-pure sub-act intersection property, then each $\overline{\top}$-pure sub-act in Ë has the $\overline{\top}$-pure sub-act intersection property.
2. Put N is $\overline{\top}$-pure sub-act in Ë. Ë has $\overline{\top}$-puresub-act crossing property, if and only if, $\frac{Ë}{N}$ has $\overline{\top}$-pure sub-act intersection property.

Proof:

(1) Clearly.

(2) (⇒) Assume that $\frac{K}{N}, \frac{F}{N}$ are two $\overline{\top}$-pure sub-acts in $\frac{Ë}{N}$ and put J is an ideal of S. We have to show that $\left(\frac{K}{N} \cap \frac{F}{N}\right) \cap J^2\left(\frac{Ë}{N}\right) = J^2\left(\frac{K}{N} \cap \frac{F}{N}\right)$. We claim that every one of K , F is $\overline{\top}$-pure sub-act in Ë. Put J ∈ S and suppose that y $\in K \cap J^2Ë$. Therefore $\frac{K}{N}$ is $\overline{\top}$-pure sub-act in $\frac{Ë}{N}$, hence $\frac{K}{N} \cap J^2\frac{Ë}{N} = J^2\frac{K}{N}$, therefore $\frac{K}{N} \cap \frac{J^2Ë+N}{N} = \frac{J^2K+N}{N}$, then $\frac{K \cap (J^2Ë+N)}{N} = \frac{J^2K+N}{N}$, thus $K \cap (J^2Ë + N) = J^2K + N$. Since $y \in K \cap J^2Ë \subseteq K \cap (J^2Ë + N)$, thus $y \in (K \cap J^2Ë) + N$. Let y = x + w, such that $x \in (K \cap J^2Ë)$ and $w \in N$. Thus w=y–x $\in N \cap J^{2Ë} = J^2N \subseteq J^2K$ and therefore K is $\overline{\top}$-pure sub-act in M. Since M is an act over semi-group has the $\overline{\top}$-pure sub-act intersection property, therefore K∩F is $\overline{\top}$-pure in Ë. Hence $(K \cap F) \cap J^2Ë = J^2(K \cap F)$ Now, let $y \in (\frac{K}{N} \cap \frac{F}{N}) \cap J^2\frac{Ë}{N}$, therefore y=x+N , when x $\in J^2Ë$ also y=k+N=f+N, when $k \in K, f \in F$. Thus $y - k \in N \subseteq K, y - f \in N \subseteq F$ also we get $x \in K \cap F$. Hence $x \in (K \cap F) \cap J^2Ë = J^2(K \cap F)$.Thus $y = x + N \in J^2\left(\frac{K \cap F}{N}\right) = J^2\left(\frac{K}{N} \cap \frac{F}{N}\right) \subseteq J^2\left(\frac{K}{N} \cap \frac{F}{N}\right)$.

(⇐) On the other side, Put E and F are $\overline{\top}$-pure sub-act in Ë, suppose that N is an R-sub-act of E and N is a sub-act in F, therefore $\frac{E}{N}$, $\frac{F}{N}$ are $\overline{\top}$-pure sub-acts. Since $\frac{M}{N}$ has the $\overline{\top}$-pure sub-act intersection property, then $\frac{E}{N} \cap \frac{F}{N} = \frac{E \cap F}{N}$ is $\overline{\top}$-pure sub-act in $\frac{Ë}{N}$. Hence, F ∩E is $\overline{\top}$-pure sub-act in Ë .

Theorem 2.6: *Put Ż is an S-act, Ż owns $\overline{\mathbf{T}}$-pure sub-act intersection property if and only if* $(J^2K \cap J^2F) = J^2(K \cap F)$ *for each ideal J of S and all $\overline{\mathbf{T}}$-pure sub-acts K , F sub-act in Ż.*

Proof: Put Ż owns the $\overline{\mathbf{T}}$-pure sub-act intersection property then for all $\overline{\mathbf{T}}$-pure sub-act of Ż, K also F, $K \cap F$ is $\overline{\mathbf{T}}$-pure sub-act. Put J ∈S, therefore $(K \cap F) \cap J^2Z = J^2(K \cap F)$. Clearly that $J^2(K \cap F) \subseteq (J^2K \cap J^2F)$. But $J^2K \cap J^2F) \subseteq K \cap (F \cap J^2Z) = (K \cap F) \cap J^2Z = J^2(K \cap F)$.
Then $J^2K \cap J^2F \cap J^2Z = J^2(K \cap F)$. On the other side, put A, and B are $\overline{\mathbf{T}}$-pure sub-act of Ż and J is an ideal of S. Thus A∩B ∩ J^2Z = A ∩(B ∩ J^2Z) = A ∩ J^2 B. Similarly A ∩ B ∩ J^2Z = B ∩ J^2 A. But A, and B are $\overline{\mathbf{T}}$-pure sub-acts of Ż. Then A ∩ B ∩ J^2 Z ⊆ J^2 A ∩ J^2B = J^2 (A ∩ B).

Proposition 2.7: Put Ż is an S–act and { N_j , j ∈ J } is a non-empty collection of sub-acts of Ż. If Ż = $\cup_{j\in J}N_j$, then N_j is a $\overline{\mathbf{T}}$-pure sub-acts of Ż for all j ∈J .

Proof: Put K is an ideal of S. By remark(1.2) we get $K^2Ż = K^2 \cup_{j\in J}N_j = \cup_{j\in J}(K^2 N_j)$ Now, let i∈ J then, $K^2Ż \cap N_i = \cup_{j\in J}(K^2 N_j) \cap N_i = (K^2 N_j \cap N_j) \cup (\cup_{i\neq j\in J} N_i \cap N_j) = K^2 N_j \cup Θ = K^2 N_j$. Hence, N_j is a $\overline{\mathbf{T}}$-pure sub-acts of Ż for all j in J .

3. $\overline{\mathbf{T}}$-Pure Sub-Acts Relative to Sub-Act.

Definition 3.1: Put Z is an S-act and Q is a sub-act of Z. A sub-act L of Z is said to be $\overline{\mathbf{T}}$-pure relative to sub-act Q (res. $\overline{\mathbf{T}}$-Q-pure) if for each ideal J of S, L ∩ J^2 Z = J^2 L ∪ (Q ∩ (L ∩ J^2 Z)) = (J^2 L ∪ Q) ∩ (L ∩ J^2 Z).

Remark 3.2: Put Z is an S-act and Q is a sub-act of Z

1. Put L is $\overline{\mathbf{T}}$-Q-pure sub-act of Z. If M is a $\overline{\mathbf{T}}$-Q-pure sub-act of L, then M is $\overline{\mathbf{T}}$-Q-pure sub-act
2. Put L is $\overline{\mathbf{T}}$-Q-pure sub-act of Z. if M is a sub-act of Z containing L, then L is a $\overline{\mathbf{T}}$-Q-pure
3. Put L is $\overline{\mathbf{T}}$-Q-pure sub-act of Z. If M is a sub-act of L and so M is sub-act of Q, then L/M is Q/M -pure sub-act of Z/M.
4. Put L and M are sub-acts of Z, If M is $\overline{\mathbf{T}}$-Q-pure sub-act of Z and L/M is $\overline{\mathbf{T}}$-Q/M -pure sub-act of Z/M, then L is $\overline{\mathbf{T}}$-Q-pure sub-act of.

Proof:

(1) Put J ∈ S, because L is $\overline{\mathbf{T}}$-Q-pure of Z also M is $\overline{\mathbf{T}}$-Q-pure in L, therefore L ∩J^2 Z = (J^2 L ∩Q)∩(L ∩J^2 Z) and M ∩J^2 L = (J^2 L ∪Q)∩(L ∩J^2 A) but M is sub-act of Z, therefore M ∩J^2Z ⊆ L ∩J^2 Z = (J^2 L ∪Q)∩(L ∩J^2 Z) and hence M ∩J^2 Z ⊆ M ∩[(J^2 L ∪Q)∩(L ∩J^2 Z)] = [M ∩(J^2 L ∪Q)]∩(L ∪J^2Z) = M ∩[(J^2 L ∪Q)∩(L ∩J^2 Z)] = [(M ∩J^2 L)∪(M ∩Q)]∩(L ∩J^2 Z) = [(M ∩J^2 L)∩ (L ∩J^2Z)]∪[(M∩Q)∩(L∩J^2Z)]=[(M∩(J^2L∩L)∩J^2Z)]∪[(M∩Q)(L∩ J^2Z)] =

$[M\cap(J^2L\cap J^2Z)]\cup[Q\cap(M\cap J^2Z)] = (M\cap J^2L)\cup[Q\cap(M\cap J^2Z)] = (M\cap J^2L)\cup[(M\cap J^2Z)\cap Q] = [(M \cap J^2 L)\cup(M \cap J^2Z)]\cap[(M \cap J^2L)\cup Q] \subseteq [(M \cap J^2L)\cup(M \cap J^2Z)] = [(J^2M \cup Q) \cap (M \cap J^2L)] \cup (M \cap J^2Z) = [[(J^2Z\cup Q) \cap (M \cap J^2L)] \cup M] \cap [[(J^2M \cup Q) \cap (M \cap J^2L)] \cup J^2M] \subseteq [(J^2M \cup Q) \cup J^2M] \cap [(M \cap J^2L) \cup J^2M] = (J^2 M \cup Q)\cap[(M \cap J^2L)\cup J^2M] = [(J^2M \cup Q)\cap(M \cap J^2L)]\cup[(J^2M \cup Q)\cap J^2M] = (J^2M \cup Q) \cap [(M \cap J^2L) \cup J^2M] \subseteq (J^2M \cup Q) \cap [(M \cup J^2M) \cap (J^2L \cup J^2M)] = (J^2M \cup Q)\cap(M \cap J^2L) \subseteq [(J^2M \cup F)\cap(M \cap J^2Z)]$. Since $(J^2M \cup F)\cap(M \cap J^2Z) \subseteq M \cap J^2Z$, then $M \cap J^2Z = (J^2M \cup F) \cap (M \cap J^2Z)$.

(2) Put $J \in S$, because L is $\overline{\mathsf{T}}$-pure in A, thus $L \cap J^2 A = (J^2L \cup Q) \cap (L \cap J^2Z)$. But M sub-act of Z, therefore, $L \cap J^2M \subseteq L \cap J^2Z = (J^2L \cup Q) \cap (L \cap J^2Z)$, and hence $L \cap J^2M \subseteq [(J^2L \cup Q) \cap (L \cap J^2Z)] \cap J^2M = (J^2L \cup Q) \cap [(L \cap J^2Z) \cap J^2M]$. Since $(J^2L \cup Q) \cap (L \cap J^2M) \subseteq L \cap J^2M$, then $L \cap J^2M = (J^2L \cup Q) \cap (L \cap J^2M)$.

(3) Put $J \in S$, because L is $\overline{\mathsf{T}}$-Q-pure sub-act in Z, therefore $L\cap J^2 Z = (J^2 L \cup Q) \cap (L \cap J^2 Z)$. So $L/M \cap J^2(Z/M) = L/M \cap (J^2 Z \cup M)/M = [L \cap (J^2 Z \cup M)]/M = [(L \cap J^2 Z) \cup (L \cap M)]/M = [(J^2 Z \cup Q) \cap (L \cap J^2 Z)] \cup M/M = ([(J^2L\cup Q) \cup M] \cap [(L \cap J^2 Z) \cup M])/M = [(J^2L \cup Q) \cup M]/M \cap [(L \cap J^2 Z) \cup M])/M = [(J^2L \cup Q)(M \cup M)]/M \cap [(L \cap J^2 Z) \cup M])/M = [(J^2L \cup M) \cup (Q \cup M)]/M \cap [(L \cap J^2 Z) \cup M])/M = J^2(L/M) \cup Q/M \cap [(L \cup M) \cap (J^2 Z \cup M)]/M = [J^2(L/M) \cup Q/M] \cap (L \cup M)/M \cap (J^2 Z \cup M)/M = [J^2(L/M) \cup Q/M] \cap [L/M \cap J^2(Z/M)]$.

(4) Clear.

Definition 3.3: S-act Z is called owns $\overline{\mathsf{T}}$-pure relative to sub-act intersection property (resp. $\overline{\mathsf{T}}$-Q−PIP) if we have the intersection of any two $\overline{\mathsf{T}}$-Q-pure sub-acts is again $\overline{\mathsf{T}}$-Q-pure.

Proposition 3.4: Put Z is an S-act.

1. If Z owns the $\overline{\mathsf{T}}$-Q−P IP, then all $\overline{\mathsf{T}}$-Q-pure sub-act of Z owns the $\overline{\mathsf{T}}$-Q − P IP.

2. Put L is $\overline{\mathsf{T}}$-Q-pure sub-act of Z also L is sub-act of Q. Z owns $\overline{\mathsf{T}}$-Q−P IP if and only if Z/L owns $\overline{\mathsf{T}}$-Q/L − P IP.

Proof:

(1) It is evident.

(2) Put M/L, N/L are $\overline{\mathsf{T}}$-Q/L- pure sub-acts of Z/L and also assume $J \in S$. Prove that $(M/L \cap N/L) \cap J^2(Z/L) = [J^2(M/L \cap N/L) \cup Q/L] \cap [(M/L \cap N/L) \cap J^2(Z/L)]$. We demand that all of M also N is $\overline{\mathsf{T}}$-Q-pure in Z. For prove, put K is an ideal of S and put $x \in M \cap K^2 Z$. Because M/L is $\overline{\mathsf{T}}$-Q/L -pure in Z/L, then $M/L \cap K^2(Z/L) = [K^2(M/L) \cup Q/L] \cap [(M/LN \cap K^2(Z/L)]$, thus $M/L \cap (K^2 Z \cup L)/L = [(K^2 M \cup L)/L \cup Q/L] \cap [(M/L \cap (K^2 Z \cup L)/L]$, and this

implies that $[M\cap(K^2\ Z\cup L)]/L = [(K^2\ M\cup L)/L\cup Q/L]\cap[(M/L\cap(K^2Z\cup L)/L] = [[(K^2M\cup L)\cup(Q\cup L)]\cap[(M\cap(K^2\ Z\cup L)]]/L$, therefore $M\cap(K^2Z\cup L) = [(K^2M\cup Q)\cup L]\cap[(M\cap K^2Z)\cup(M\cap L)] = [(K^2M\cup Q)\cup L]\cap[(M\cap K^2Z)\cup L] = [(K^2M\cup Q)\cap[(M\cap K^2Z)]\cup L$, and hence $(M\cap I\ K^2Z)\cup L = [(K^2M\cup Q)\cap[(M\cap K^2Z)]\cup L$. Since $x\in M\cap K^2Z\subseteq(M\cap K^2Z)\cup L$, then $x\in[(K^2M\cup Q)\cap[(M\cap K^2Z)]\cup L$, thus $x\in(K^2M\cup Q)\cap[(M\cap K^2Z)$ or $x\in L$, if $x\in(K^2M\cup Q)\cap(M\cap K^2Z)$, then $M\cap K^2Z = (K^2M\cup Q)\cap(M\cap K^2Z)$, therefore M is $\overline{\top}$-Q -pure, if $x\in L$ thus $x\in(L\cap K^2Z)\cup L = [(K^2M\cup Q)\cap(M\cap K^2Z)]\cup L$, hence M is $\overline{\top}$-Q -pure of Z. Because Z owns the $\overline{\top}$-Q − P IP, therefore $M\cap N$ is $\overline{\top}$-Q - pure in Z. Thus $(M\cap N)\cap J^2\ Z = [J^2(M\cap N)\cup Q]\cap[(M\cap N)\cap J^2Z]$. Now, put Rees congruence $\rho L\in[M/L\cap N/L]\cap J^2\ (Z/L)$, then Rees congruence $\rho L\in J^2\ (Z/L)$ (Rees congruence on K^2Z) and $\rho L\in M/L$ (Rees congruence on M) and $\rho L\in N/L$ (Rees congruence on N), therefore Rees congruence ρL on M also N thus $\rho L\in MN$, but $(M\cap N)\cap J^2\ Z = [J^2\ (M\cap N)\cup Q]\cap[(M\cap N)\cap J^2Z]$, therefore $(M/L\cap N/L)\cap K^2\ (Z/L) = [K^2\ (M/L\cap N/L)\cup Q/L]\cap[(M/L\cap N/L)$. Conversely, let P and L be $\overline{\top}$-Q - pure sub-acts of Z, let N be a sub-act of P and N be a sub-act of L then P/N and L/N is $\overline{\top}$-Q/N-pure sub-acts of Z/N. Since Z/N has $\overline{\top}$-Q /N−P IP, thus $P/N\cap L/N = (P\cap L)/N$ is Q/N- pure sub-act of Z/L. Thus $P\cap L$ is $\overline{\top}$-Q- pure sub-act of Z.

References

[1] C. Miller and N. Skuc, An introduction to presentations of monoid acts: quotients and subacts, arXiv:1709.08916v [math.GR], 26Spt (2017).

[2] M. Kilp, U. Knauer and A. Mikhalev, Monoids, acts and categories, Walter de Gruyter (2000).

[3] Shaymaa Abdul kareem Muheyaddin, New Types Of Regular Fuzzy Modules And Pure Fuzzy Submodules, ph.d thesis (2017).

[4] Shubham Sharma & Ahmed J. Obaid. Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox, *Journal of Interdisciplinary Mathematics*, 23:4, 843-849 (2020), DOI: 10.1080/09720502.2020.1727611.

[5] Ratnakar Das, Jibitesh Mishra, Sujogya Mishra & P. K. Pattnaik. Design of mathematical model for the prediction of rainfall, *Journal of Interdisciplinary Mathematics*, 25:3, 587-613 (2022), DOI: 10.1080/09720502.2021.2016853.

[6] Nadeem Akhtar, M. M. Sufyan Beg & Md. Muzakkir Hussain. Sparse two level topic model for extraction of general summary words, *Journal of Interdisciplinary Mathematics*, 23:1, 303-310 (2020), DOI: 10.1080/09720502.2020.1721925.

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