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Numerical optimization software for solving stochastic optimal control

Ahmed Hasan Alridha *
Abbas Musleh Salman †
Ekhlas Annon Mousa §
General Directorate of Education of Babylon
Ministry of Education
Iraq

Abstract

Stochastic optimal control is a branch of control theory that deals with uncertain system parameters. One of the requirements for an accurate description of systems is a complete knowledge of the structure or values of the basic parameters of the system, which if achieved, the optimization process will also have accurate results. In this paper, we highlight a method for optimizing control under the control of a stochastic uncertainty approach that involves the use and dependence on uncertain parameters for systems whether they are distributed by a distribution function or within other sets of potential values by using the python language. Finally, the numerical results were implemented in python using the GEKKO package.

Subject Classification: *16Y60.*

Keywords: *Stochastic optimal control, Optimization approach, Python language.*

1. Introduction

Many applications of management science require stochastic optimal control, and control involves dynamic systems or systems that change over time. Depending on whether the time is constantly changing or discretely, they are referred to as continuous or discrete-time systems. Optimal control theory is an area of mathematics focused on determining the best strategies for controlling a dynamic system. J. L. Doob, considered a pioneer in this discipline, was the first to analyze stochastic processes

* *E-mail:* `amqa92@yahoo.com (Corresponding Author)`
† *E-mail:* `abbas.mmm2019 @gmail.com`
§ *E-mail:* `ekhlasanoon@yahoo.com`

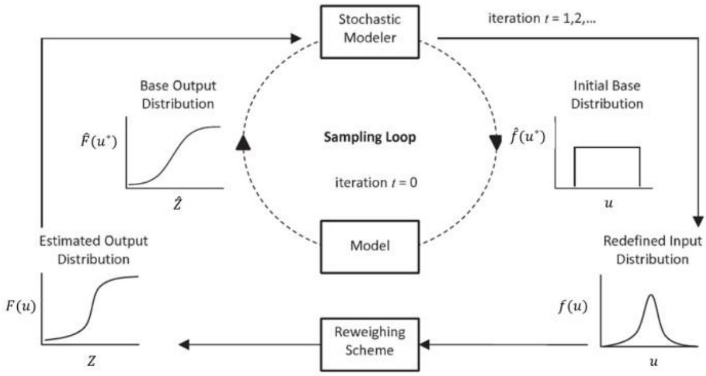

Figure 1

Optimization under uncertainty: a reweighting approach.

with mathematical rigor (see Doob (1942, 1953), Getoor (2009)). Many applications of management science require stochastic optimal control, and control involves dynamic systems or systems that change over time [1,2,6,7] . Depending on whether the time is constantly changing or discretely, they are referred to as continuous or discrete time systems. Optimal control theory is an area of mathematics focused on determining the best strategies for controlling a dynamic system. Stochastic processes are defined as the evolution of variables over ε_2^2 me t, x_t using any probability formula. As a result, for certain time values $t_1, t_2, t_3, \ldots, t_1$ one may cal z_1 late the 2 likelihood that the values of the relevant variables, $x_1, x_2, x_3, \ldots, x_n,$ fall within certain rang 1. When time t_l arrives and we observe the (1) real value of $x_1,$ we can v this information to condition the probability of future events. Brownian motions, such as the one-dimensional stochastic walk process, a stochastic process in which the value of the variable x at time t depends only and exclusively on the value at time $t-1$, as well as an uncertain term with zero mean and unit variance, are among the most commonly used stochastic processes. Therefore, the pursuit was vigorously for the aim of improvement by using one of the programming languages with the GEKKO package, so that the optimization strategy is implemented in the shadow of uncertainty, as well as the possibility of expanding the system to meet the intended goal [3,4,8]as shown in Figure 1.

2. Stochastic Optimal Control

The most widely utilized strategy in the literature is stochastic-based approaches (Straub and Grossmann, 1992; Terrazas-Moreno et. al., 2011).

The deterministic mathematical model is turned into a stochastic model in which the uncertainties are treated as random variables (Shapiro, 2012 and Shapiro et al, 2013). A situation known as "load following" is widespread in grid systems. In response to shifting demand dynamics, the optimizer attempts to balance supply and demand. When ramping limits are imposed on a single generator, an attempt is made to respond to a single load with perfect foresight. To minimize overproduction, a generator must initially ramp up production to keep supply and demand in balance. The underlying model or process parameters that describe a system are rarely known accurately. A sub-optimal solution results from optimizing a system with an erroneous model. If the process model has uncertain parameters specified by a distribution function or another set of probable process values, this information can be directly used in the optimal control problem to optimize under uncertainty. Our starting station will be to choose a simplified type of system and then try to expand it and implement optimal control of its output. Our starting stop would be to choose a simplified type of system and then try to extend it and implement optimal control of its output. We will consider a system with an uncertain parameter F that is uniformly distributed among specific values where the process model will be as follows:

$$x\frac{dv}{dt} = -v + Fb$$

This is a straightforward first-order dynamic model that connects the variable input p (MV) to the desired output v. (CV). If v is the controlled variable (CV) with the target value and p is the manipulated variable (MV) with a range of integers. Considering that the rate of change of MV is confined to a specified time. Furthermore, by constructing (x times) instances of the model that randomly select the value of F, a random optimal control solution can be established [5,9,10,11]. We will implement the solution approach by creating two models using the GEKKO package for the Python programming language.

2.1 *Mathematical Model Style I*

In this model, suppose we have a system with an uncertain parameter F that's evenly distributed between 1.0 and 3.0. Therefore, the process model will be as follows:

$$x\frac{dv}{dt} = -v + Fb$$

When x is the controlled variable with a goal value of 50, and v is the randomly selected value of F that will have a value of 15 within that model. The manipulated variable (MV) is a variable that can be modified between 0 and 80. The variable input b (MV) is linked to the desired output v in this dynamic model (CV). The MV can only vary at a rate of 50 times per second. Create 20 instances of the model that randomly determine the value of F to find the stochastic optimal control solution. Starting with a condition of 0, the purpose is to optimize this set of models to obtain the target CV value of 50. When executing the GEKKO package using Python, we will get the results according to the following Table 1:

Table 1

Model I

	Scaled	**Unscaled**
Objective	3.9329377189406669e+06	3.9329377189406669e+06
Dual infeasibility	3.8562575355172157e-10	3.8562575355172157e-10
Constraint violation	5.5024429457262158e-11	5.5024429457262158e-11
Complementarity	9.8147574881265305e-10	9.8147574881265305e-10
Overall NLP error	1.8894188659594727e-10	9.8147574881265305e-10

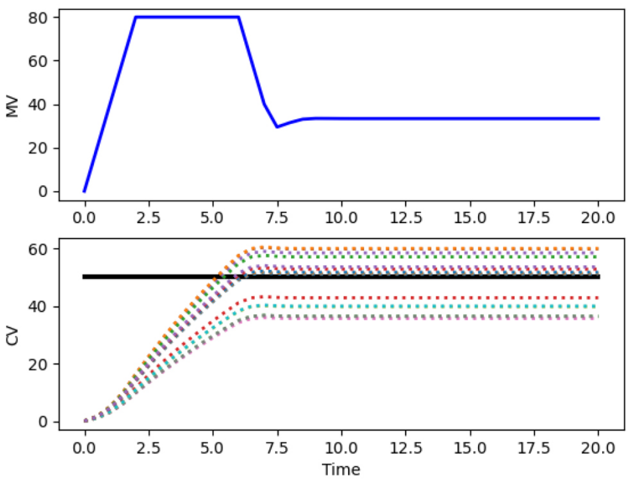

Figure 2

Model I

It is clear from the above table that the optimal time to solve the problem of Stochastic Optimal Control **is** by employing the new model from the prior objective function, as shown in Figure 2.

The solution was found.

Solution time: 1.33870000000752 sec

Objective: 3932937.71893339

2.2 *Mathematical Model Style II*

In this model, we will extend the distribution in the lag process where it will be uniformly between 1.0 and 4.0. We also modify the value of F by $b(t-\mu)$ to be in the following form:

$$3\frac{dv}{dt} = -v + b(t-\mu)$$

Where v is the controlled variable (CV) with a goal range of 50.0 to 48.0, b is the manipulated variable (MV) with a range of 0 to 80 and is the delay. This is a straightforward first-order dynamic model that connects the variable input p (MV) to the desired output v. (CV). The MV can only vary at a rate of 48 times per second. we seek to create three instances of the model that randomly determine the delay to get the stochastic optimal control solution. we will be starting with a condition of 0, and optimizing this set of models to reach the goal CV value of between 48 and 50.

Table 2

Model I

	Scaled	**Unscaled**
Objective	8.4222039644808447e+03	8.4222039644808447e+03
Dual infeasibility	2.1316282072803006e-14	2.1316282072803006e-14
Constraint violation	7.1054273576010019e-15	7.1054273576010019e-15
Complementarity	2.6353813503763706e-09	2.6353813503763706e-09
Overall NLP error	2.6353813503763706e-09	2.6353813503763706e-09

It is clear from the above Table 2, that the optimal time to solve the problem of Stochastic Optimal Control by employing the new model from the prior objective function, as shown in Figure 3

The solution was found.

Solution time: 0.421400000021094 sec

Objective: 8422.20400233562

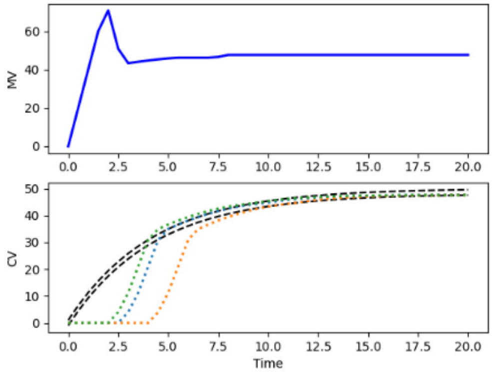

Figure 3

Model II

3. Conclusion

Describing the system by process parameters may be scarce in many cases. In this case, the optimizing process is completely inaccurate, because the description of the basics of the system is incomplete, as it sometimes described by the distribution function or possibly within another set of process values. Therefore, it is permissible to use this information directly in case of uncertainty in the problem of optimal control for optimization. Two models are described, one is simple and the other is extended within specific values, and the process of optimization has been done for optimal control using the Python language. The numerical results showed a comprehensive and constructive description that could be relied upon for the improvement of other systems, whose parameters were described uncertainly.

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