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Interval valued fuzzy ideals of TM-algebra

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Abstract

We introduce the notion of interval value fuzzy ideal of TM-algebra as a generalization of a fuzzy ideal of TM-algebra and investigate some basic properties. Interval value fuzzy ideals and T-ideals are defined and several examples are presented. The relation between interval value fuzzy ideal and fuzzy T-ideal is studied.

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1. Introduction

In 2010 the notion of TM algebras was introduced by Megalai and Tamilarasi [5] as a generalization of BCK and BCI algebras. After that, many authors studied this structure in different ways; see [2, 4, and 6].

Jun and Dudek [3] studied the notion of Interval-valued fuzzy subalgebras /ideals in BCK-algebras. Mostafa et al. [7] presented an Interval-valued fuzzy KU-ideal in KU-algebras and discuss some interesting results. Also, some authors studied this concept in several papers; see [1, 8, 9, and 10]. In this work, the notion of interval value fuzzy ideal of TM-algebra as a generalization of a fuzzy ideal of TM-algebra and some basic properties are introduced. Interval value fuzzy ideals and T-ideals are defined and several examples are presented. The relation between interval value fuzzy ideal and fuzzy T-ideal is studied.

2. Preliminaries

We recall some basic definitions and theorems in this part.

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Definition 2.1[5]: Let $\aleph$ be a nonempty set with a constant "0" and a binary operation " *" satisfying the following:

$$(tm_1)\ \rho * 0 = \rho,$$
$$(tm_2)(\rho * \tau) * (\rho * \varepsilon) = \varepsilon * \tau, \forall\ \rho, \tau, \varepsilon \in \aleph\ .$$

Then $(\aleph, *\ ,0)$ is called a TM-algebra.

A binary operation $\leq$ in $\aleph$ is defined as follows $\rho \leq\ \tau \Leftrightarrow \rho * \tau = 0$.

Then the following axioms hold. $\forall\ \rho, \tau\ , \varepsilon \in \aleph$

a) $\rho * \rho\ = 0,$
b) $\rho * (\rho * \tau) = \tau,$
c) $(\rho * \varepsilon) * (\tau * \varepsilon) \leq \rho * \tau,$
d) $(\rho * \tau) * \varepsilon = (\rho * \varepsilon) * \tau,$
e) $\rho\ \leq\ \tau \Rightarrow \rho * \varepsilon \leq \tau * \varepsilon\ and\ \varepsilon * \tau \leq \varepsilon * \rho,$
k) If $\rho * \tau = 0$ and $\tau * \rho = 0$ imply $\rho = \tau$.

Definition 2.2 [5]: A TM-subalgebra of $\aleph$ is a non-empty subset S is satisfied $\rho * \tau \in S$ whenever $\rho, \tau \in S$.

Definition 2.3 [6]: An ideal ψ of $\aleph$ is a non-empty subset satisfied

i) $0 \in \psi,$
ii) $\rho * \tau \in \psi$ and $\tau \in \psi$ imply that $\rho \in \psi$. For any $\rho, \tau \in \psi$

Definition 2.4 [6]: A T-ideal E of $\aleph$ is a non-empty subset satisfied

i) $0 \in E$
ii) $\forall\ \rho, \tau, \varepsilon \in \aleph, (\rho * \tau) * \varepsilon \in E$and $\tau \in E$ imply $(\rho * \varepsilon) \in E$.

Definition 2.5 [6]: A fuzzy sub-algebra ϑ of $\aleph$ is $\vartheta(\rho * \tau) \geq min\{\vartheta(\rho), \vartheta(\tau)\}$, $\forall\ \rho, \tau \in \aleph$.

Definition 2.6 [6]: A fuzzy ideal ϑ of $\aleph$ is satisfied

$(F_1)\ \vartheta(0) \geq \vartheta(\rho), \forall\ \rho \in \aleph\ .$

$(F_2)\ \forall \rho, \tau \in \aleph, \vartheta(\rho) \geq min\{\vartheta(\rho * \tau), \vartheta(\tau)\}.$

Definition 2.7 [6]: A fuzzy T-ideal ϑ of $\aleph$ is satisfied

$(FI_2)\ \vartheta(0) \geq \vartheta(\rho), \forall\ \rho \in \aleph.$

$(FI_2)\ \forall\ \rho, \tau, \varepsilon \in \aleph, \vartheta(\rho * \varepsilon) \geq min\{\vartheta((\rho * \tau) * \varepsilon), \vartheta(\tau)\}.$

3. Interval Value Fuzzy T-Ideals in TM-Algebra

Definition 3.1: An interval-valued fuzzy subset over TM-algebra $\aleph$ is an object having the form $V = \{< \rho, \tilde{\vartheta}(\rho) >: \rho \in \aleph\} = (\rho, \vartheta^L, \vartheta^U)$, where $\tilde{\vartheta}: \aleph \longrightarrow$ D[0,1], such that D[0, 1] is denote the family of all closed subintervals of [0,1], ϑ^L and

ϑ^U are two fuzzy sets in $\aleph$ such that $\vartheta^L(\rho) \leq \vartheta^U(\rho)$, for all $\rho \in \aleph$. Hence the interval-valued fuzzy subset V is given by $V =< \rho, \tilde{\vartheta}(\rho) >$ and in short, we write (IVFS).

Definition 3.2: In a TM-algebra $(\aleph, * , 0)$, the set $V =< \rho, \tilde{\vartheta}(\rho) >$ is named an (IVF) sub-TM-algebra $\aleph$, if

$$\tilde{\vartheta}(\rho * \tau) \geq rmin\{\tilde{\vartheta}(\rho), \tilde{\vartheta}(\tau)\}, \forall \rho, \tau \in \aleph. \text{ where } \tilde{\vartheta}: \aleph \rightarrow D[0,1]$$

Example 3.3: Let $\aleph = \{0,1,2,3\}$ be a set and $*$ defined in the following table

*	0	1	2	3
0	0	0	3	2
1	1	0	3	2
2	2	2	0	3
3	3	3	2	0

Then $(\aleph, * , 0)$ is a TM-algebra. Define $\tilde{\vartheta}(\rho)$ as follows

$$\tilde{\vartheta}(\rho) = \{[0.1,0.7] \quad \text{if} \quad \rho = 0,1 \ [0.2,0.4] \quad if \quad \rho = 2,3 \}.$$

It is easy to check that $V =< \rho, \tilde{\vartheta}(\rho) >$is an (IVF) sub-TM-algebra.

Proposition 3.4: If $V =< \rho, \tilde{\vartheta}(\rho) >$ is an (IVF) sub-TM-algebra of $\aleph$ $\aleph$, then for all $\rho \in \aleph$, $\tilde{\vartheta}(0) \geq \tilde{\vartheta}(\rho)$.

Proof: For all $\rho \in \aleph$, we have $\rho * \rho = 0$, then $\tilde{\vartheta}(0) = \tilde{\vartheta}(\rho * \rho) \geq \{\tilde{\vartheta}(\rho), \tilde{\vartheta}(\rho)\} = rmin\{[\vartheta^L(\rho), \vartheta^U(\rho)], [\vartheta^L(\rho), \vartheta^U(\rho)]\} = rmin\{[\vartheta^L(\rho), \vartheta^U(\rho)]\} = \tilde{\vartheta}(\rho)$.

Definition 3.5: In a TM-algebra $(\aleph, * , 0)$, the set $V =< \rho, \tilde{\vartheta}(\rho) >$ is named an (IVF) ideal if

(i_1) $\tilde{\vartheta}(0) \geq \tilde{\vartheta}(\rho), \forall \rho \in \aleph$,

(i_2) For all $\rho, \tau \in \aleph, \tilde{\vartheta}(\rho) \geq rmin\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\}$, where $\tilde{\vartheta}: \aleph \rightarrow D[0,1]$.

Example 3.6: Let $\aleph = \{0,1,2,3\}$ be a set an Example3.3. Then we define $\tilde{\vartheta}(\rho)$ as follows $\tilde{\vartheta}(\rho) = \{[0.2,0.8]$ if $\rho = 0,1,2$ $[0.1,0.5]$ if $\rho = 3 \}$.

It is easy to check that $V =< \rho, \tilde{\vartheta}(\rho) >$ is an (IVF) ideal.

Proposition 3.7: Let $V =< \rho, \tilde{\vartheta}(\rho) >$ be (IVF) ideal of $\aleph$. If the inequality $\rho \leq \tau$ holds in $\aleph$, then $\tilde{\vartheta}(\rho) \geq \tilde{\vartheta}(\tau)$.

Proof: Let $\rho, \tau \in \aleph$, such that $\rho \leq \tau$. Then $\rho * \tau = 0$, and we have

$$\tilde{\vartheta}(\rho) \geq rmin\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\}\tilde{\vartheta}(\rho) \geq rmi\{\tilde{\vartheta}(0), \tilde{\vartheta}(\tau)\} \geq \tilde{\vartheta}(\tau).$$

Theorem 3.8: *Every (IVF) ideal of a TM-algebra* $(\aleph,*,0)$ *must be (IVF) sub-TM-algebra.*

Proof: Let $V =< \rho, \tilde{\vartheta}(\rho) >$ be (IVF) ideal of TM-algebra $\aleph$.

Since $\rho * \tau \leq \rho$, $\forall \rho, \tau \in \aleph$, it follows from Proposition 3.7 that

$$\tilde{\vartheta}(\rho * \tau) \geq \tilde{\vartheta}(\rho) \geq \ rmin\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\} \geq rmin\{\tilde{\vartheta}(\rho), \tilde{\vartheta}(\tau)\}$$

Then $V =< \rho, \tilde{\vartheta}(\rho) >$ is an (IVF) sub-TM-algebra of a TM-algebra $\aleph$.

In the next example, it is easy to show that the reverse of the previous theorem is not true

Example 3.9: Let $\aleph = \{0, a, b, c\}$ be a set. Define the operations $*$ by the following table.

*	0	a	b	c
0	0	a	b	c
a	0	0	0	c
b	0	a	0	c
c	0	0	0	0

Then $(\aleph,*,0)$ is a TM-algebra. Define $\tilde{\vartheta}(\rho)$ as follows:

$$\tilde{\vartheta}(\rho) = \{[0.3,0.9] \quad \text{if} \quad \rho = 0, c \quad [0.6,0.7] \quad \text{if} \quad \rho = a \quad [0.2,0.3] \quad \text{if} \quad \rho = b$$

Then $V =< \rho, \tilde{\vartheta}(\rho) >$ is an (IVF) sub-TM-algebra, but not an (IVF) ideal since

$$\tilde{\vartheta}(b) = [0.2\,, 0.3] \leq \{\tilde{\vartheta}(a * b)\,,\ \tilde{\vartheta}(a)\} \ = [0.6\,, 0.7].$$

Theorem 3.10: *An (IVFS)* $V =< \rho, \tilde{\vartheta}(\rho) >$ *is an (IVF) ideal of* $\aleph$ *if and only if* $\vartheta^L(\rho)$ *and* $\vartheta^U(\rho)$ *are (IVF) ideals of* $\aleph$.

Proof: Suppose V is an (IVF) ideal of $\aleph$ and for any $\rho, \tau \in \aleph$, we have $\left[\vartheta^L(\rho), \vartheta^U(\rho)\right] = \tilde{\vartheta}(\rho) \geq rmin\left\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\right\} = rmin\{[\vartheta^L(\rho * \tau), \vartheta^U(\rho * \tau)], [\vartheta^L(\tau), \vartheta^U(\tau)]\}$

$= [\{\vartheta^L(\rho * \tau), \vartheta^L(\tau)\}\,, \{\vartheta^U(\rho * \tau), \vartheta^U(\tau)\}\,]$, then $\vartheta^L(\rho) \geq min\{\vartheta^L(\rho * \tau), \vartheta^L(\tau)\}$ and $\vartheta^U(\rho) \geq min\{\vartheta^U(\rho * \tau), \vartheta^U(\tau)\}$. Hence $\vartheta^L(\rho)$ and $\vartheta^U(\rho)$ are (IVF) ideals of $\aleph$.

Conversely, let $\vartheta^L(\rho)$ and $\vartheta^U(\rho)$ be (IVF) ideals of $\aleph$ and $\rho, \tau \in \aleph$, then

$$\tilde{\vartheta}(\rho) = [\vartheta^L(\rho), \vartheta^U(\rho)] \geq [min\{\vartheta^L(\rho * \tau), \vartheta^L(\tau)\}, \{\{\vartheta^U(\rho * \tau), \vartheta^U(\tau)\}]$$
$$= rmin\{[\vartheta^L(\rho * \tau), \vartheta^U(\rho * \tau)], [\vartheta^L(\tau), \vartheta^U(\tau)]\} = rmin\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\}.$$
Hence V is an (IVF) ideal of $\aleph$.

Definition 3.11: In a TM-algebra $(\aleph, * ,0)$, the set $V =< \rho, \tilde{\vartheta}(\rho) >$ is called an (IVF) T-ideal if $\forall \rho, \tau, \varepsilon \in \aleph$,

(i_1) $\tilde{\vartheta}(0) \geq \tilde{\vartheta}(\rho), \forall \rho \in \aleph$,

(i_2) $\tilde{\vartheta}(\rho * \varepsilon) \geq rmin\{\tilde{\vartheta}((\rho * \tau) * \varepsilon), \tilde{\vartheta}(\tau)\}$. Where $\tilde{\vartheta}: \aleph \rightarrow D[0,1]$

Example 3.12: Let $\aleph = \{0,1,2,3\}$ be a set an Example 3.3. Then we define $\tilde{\vartheta}(\rho)$ as follows $\tilde{\vartheta}(\rho) = \{[0.3,0.9]$ if $\rho = 0,1,2$ $[0.2,0.6]$ if $\rho = 3$ $\}$.

It is easy to check that $V =< \rho, \tilde{\vartheta}(\rho) >$ is an (IVF) T-ideal.

Proposition 3.13: Let $V =< \rho, \tilde{\vartheta}(\rho) >$ be (IVF) ideal of $\aleph$. If the inequality $\rho * \tau \leq \varepsilon$ holds in $\aleph$, then $\tilde{\vartheta}(\rho) \geq rmin\{\tilde{\vartheta}(\tau), \tilde{\vartheta}(\varepsilon)\}$.

Proof: Suppose $\rho * \tau \leq \varepsilon$ that in $\aleph$. Then $(\rho * \tau) * \varepsilon = 0$ and by Definition 3.11, we have that

$$\tilde{\vartheta}(\rho * \tau) \geq \{\tilde{\vartheta}((\rho * \varepsilon) * \tau), \tilde{\vartheta}(\varepsilon)\} \geq rmin\{\tilde{\vartheta}((\rho * \tau) * \varepsilon), \tilde{\vartheta}(\varepsilon)\}$$
$$= \{\tilde{\vartheta}(0), \tilde{\vartheta}(\varepsilon)\} = \tilde{\vartheta}(\varepsilon) \quad \dots\dots\dots (1)$$

Now, $\tilde{\vartheta}(\rho * 0) = \tilde{\vartheta}(\rho) \geq \{\tilde{\vartheta}((\rho * \tau) * 0), \tilde{\vartheta}(\tau)\}$

$$= rmin\{\tilde{\vartheta}(\rho * \tau), \tilde{\vartheta}(\tau)\} \geq rmin\{\tilde{\vartheta}(\varepsilon), \tilde{\vartheta}(\tau)\} \text{ (by using(1))}$$

i.e. $\tilde{\vartheta}(\rho) \geq rmin\{\tilde{\vartheta}(\tau), \tilde{\vartheta}(\varepsilon)\}$.

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