Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 26 (2023), No. 4, pp. 613–617
DOI : 10.47974/JIM-1479

The topological space generated from the graphs by use of some relation

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Abstract

In this article, we study the topology and its properties generated from a simple undirected graph with no isolated vertices. We use a simple undirected graph to generate the topology, which connects event edges with the neighboring relationship between vertices, edges, and studies some properties of the graph.

Subject Classification: *18B30, 28C15.*

Keywords: *Graph, Special graph, Topological space.*

1. Introduction

2016 Nobel Prize in Physics was once awarded to three scientists for their discovery of phase transitions and the topological phases of matter. This tournament drew attention to the desire for more topology information. Topology is a branch of mathematics, the standards of which are now found not solely in almost all branches, and in a lot of real life, which includes all issues without delay or indirectly associated with continuity. Many topologists have proposed topological fashions in biology [5,6] and medicinal drug [2,9]. Topology is a branch of geometry, known as rubber sheet geometry. It's the science of dealing with the nature of things. It does not depend on space, whose potential it lets in make bigger and decrease, but does not cut things. If ω is a non-empty set, a collection σ of subsets of ω is stated to be a topology on ω if the following situation holds ω and ϑ belongs to σ , the finite intersection of any two sets in σ belongs to σ and the union of any quantity of sets in σ belongs to σ [8] graph theory [1, 3, 7]. Recently, it has ended up a subject in itself, as well as

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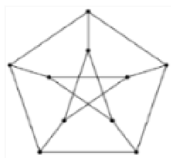
an essential mathematical device for distinctive topics such as operations research, chemistry sociology and genetics.

2. Preliminaries

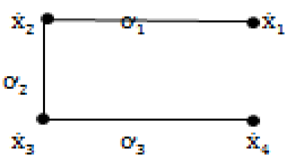
In this section, some groundwork definitions and theorems related to graph theory, approximate space, and topological space used in the work are introduced. [4] A graph is a triangular arrangement $(\alpha(\Omega), \gamma(\Omega), \psi_{\Omega})$, consisting of a nonempty collection $\alpha(\Omega)$ (vertex set) of vertices, a set $\gamma(\Omega)$ (edge set) of edges, and a mapping $\psi_{\bar{G}}: \gamma(\Omega) \to \alpha(\Omega) \times \alpha(\Omega)$ (an incidence function), it maps an edge to a pair of vertices called end vertices of the edge. Any two vertices in a graph are linked by an edge, we name the vertices are adjacent. If a vertex α is an endpoint of edge σ, we say they are incidents. A graph $\Omega = (\alpha(\Omega), \gamma(\Omega), \psi_{\Omega})$ is isomorphic to a graph $H = (\alpha(H), \gamma(H), \psi_H)$ denoted by $\Omega \cong H$ if there exist two bijective mappings $\theta: \alpha(\Omega) \to \alpha(H)$ and $\emptyset: \gamma(\Omega) \to \gamma(H)$, such that for any $\sigma \in \gamma(\Omega)$ $\quad \psi_{\Omega}(\sigma) = (Ɛ, Ʒ) \Leftrightarrow \psi_H(\emptyset(\sigma)) = (\theta(Ɛ), \theta(Ʒ)) \in \gamma(H)$. The pair $(\theta, \emptyset)$ of mappings is named an isomorphic mapping from Ω to H. The graph shown in the Figure is referred to as the Peterson graph.



Hypercube or n-cube, denoted by $Q_n = \left(\alpha\left(Q_n\right), \gamma\left(Q_n\right)\right)$, where $\alpha(Q_n) = \{Ɛ_1 Ɛ_2 \dots Ɛ_n : Ɛ_i \in \{0,1\}, i = 1,2, \dots, n\}$, $Ʒ = Ʒ_1 Ʒ_2 \dots Ʒ_n$ and $Ɛ = Ɛ_1 Ɛ_2 \dots Ɛ_n$ are two vertices connected by an undirected edge if they differ in precisely one coordinate i.e., $ƐƷ \in \gamma(Q_n) \leftrightarrow \sum_{i=1}^{n} |Ɛ_i - Ʒ_i| = 1$. Now, we define the relations:

- $R =\{$ the incident edges with any vertex in the graph$\}$, we can find the topological space of the graph by finding the basis, where B equals the intersection of the set of R, T equals the all-union of the sets in bases B.

Example 2.1: Find the topological space generated by the graph.



Solution:

$$R = \{\{\sigma_1\}, \{\sigma_3\}, \{\sigma_1, \sigma_2\}, \{\sigma_2, \sigma_3\}\}, B = \{\emptyset, \{\sigma_1\}, \{\sigma_2\}, \{\sigma_3\}, \{\sigma_1, \sigma_2\}, \{\sigma_2, \sigma_3\}\}$$

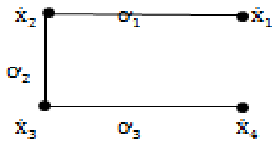
$$T = \{\emptyset, \gamma, \{\sigma_1\}, \{\sigma_2\}, \{\sigma_3\}, \{\sigma_1, \sigma_2\}, \{\sigma_1, \sigma_3\}, \{\sigma_2, \sigma_3\}\}$$

Theorem 2.1: *The topological space generated by any graph is discrete.*

Proof: Let, then the graph $\bar{G}$ is a connected graph and any vertex in $\bar{G}$ passes through one edge at least. Then any vertex intersection at least with one edge. From the intersection of each incident, we get any singleton set. Then T_γ is discrete.

- $R =$ {the adjacent edges with any edge in the graph}, B is the intersection of the set R, T equals the all-union of the sets in bases B.

Example 2.2: Find the topological space generated by the graph.



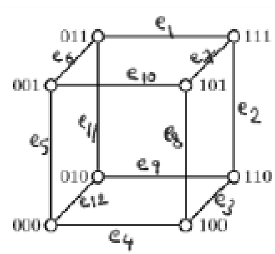
Solution:

$$R = \{\{\sigma_2\}, \{\sigma_1, \sigma_3\}\},\ B = \{\emptyset, \{\sigma_2\}, \{\sigma_1, \sigma_3\}\}, T = \{\emptyset, \gamma, \{\sigma_2\}, \{\sigma_1, \sigma_3\}\}$$

Theorem 2.2: *The topological space generated by the graphs ($Q_n, n \geq 3, P, C_n$) is discrete.*

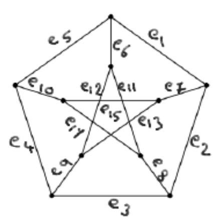
Proof:

- $Q''_n, n \geq 3$



Let n=3, $\{e_1\} = \{e_1, e_3, e_7, e_9\} \cap \{e_1, e_5, e_{10}, e_{11}\}$
In the same way, can be found $\{e_i\}$, because the shape is regular. Then T_γ is discrete. In the same way, can be proven that generated of $Q_n, n \geq 3$ is discrete.

- P

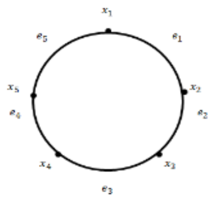


$$\{e_1\} = \{e_1, e_3, e_7, e_8\} \cap \{e_1, e_4, e_6, e_{10}\}$$

In the same way can be found $\{e_i\}$, then is discrete, since containing all singleton set.

- C_n

  Let $n = 5$



the intersection of each adjacent we get a singleton set. Then T_γ is discrete.

$$\{e_1\} = \{e_1, e_3\} \cap \{e_1, e_4\}$$

In the same way, can be found $\{e_i\}$, because the shape is regular. Then T_γ is discrete. In the same way, prove that the topological space generated of C_n is discrete.

Theorem 2.3: *Let Ω be a graph, if the minimum degree $\delta(\Omega) \geq 5$, then the topological space generated by the graph is discrete.*

Proof: By contradiction, suppose $\delta(\Omega) \geq 5$ and T_γ is not discrete. Since $\delta(\Omega) \geq 5$, then Ω is a connected graph and any vertex in Ω passes through 5 edges at least. Then any edge adjacent to at least 4 edges. From the intersection of each adjacent, we get a singleton set. Then T_γ is discrete.

Theorem 2.4: *If the graph is k-regular, $k \geq 5$. Then the topological space generated by the graph is discrete.*

Proof: By theorem (2.3)T_γ is discrete

Theorem 2.5: *If* $\Omega_1 \cong \Omega_2$ *then* $T_{\gamma_1} \cong T_{\gamma_2}$

Proof: Let $\Omega_1 \cong \Omega_2$, wanted to prove that $T_{\gamma_1} \cong T_{\gamma_2}$i.e. we wanted to prove there is a bijection map $f: \gamma_1 \to \gamma_2$, and f is continuous and f^{-1} continuous.
(we re-concede the second graph in proportion to the first graph).

Now, since $\Omega_1 \cong \Omega_2$, $\exists\, \theta: \alpha(\Omega_1) \to \alpha(\Omega_2), \aleph: \gamma(\Omega_1) \to \gamma(\Omega_2), \forall e \in \gamma(\Omega_1) \to \aleph(\emptyset(\sigma)) = (\theta(Ƹ), \theta(Ʒ))$

Now suppose $f: \gamma_1 \to \gamma_2$ define as $f(\sigma) = \emptyset(\sigma) = \sigma'$

f is well defined since: $\forall \sigma_i \in \gamma_1, \exists \sigma'_i \in \gamma_2 \ni \sigma'_i = f(\sigma_i), \quad i = 1,2,3,\ldots$

And $\sigma_i = \sigma_j \rightarrow \sigma'_i = \sigma'_j$, $i = 1,2,3,\ldots, j = 1,2,3,\ldots$

f is continuous since is $\emptyset$ a bijection map than any open set in T_{γ_2} the inverse of the set in T_{γ_1}. f^{-1} is continuous since is $\emptyset$ a bijection map than any open set in T_{γ_1} the inverse of the set in T_{γ_2}

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Received November, 2021

Revised March, 2022