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The convergence of new iterations by resolvent $\mathcal{ZA}$-Jungck mappings

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Abstract

In this paper, new iterations with different steps such as resolvent Jungck ZSY, resolvent Jungck Modified Mann Z-, resolvent Jungck Petrusel, and the resolvent Jungck Ishikawa iterations are introduced. Also, the convergence and speediness of it are studied and shown that the resolvent Jungck ZSY- iteration scheme is faster than that of resolvent Jungck Modified Mann Z- iteration scheme. On the other hand, the resolvent Jungck ZSY- iteration is faster than the resolvent Jungck Picard iteration.

Subject Classification: *Primary 46B20, Secondary 46C50.*

Keywords: *Banach space, Iteration methods, Resolvene-Jungck mapping.*

1. Introduction

In the last period, scientists presented many studies and research that focused on finding the best and fastest algorithms, reaching the steadfast point that is considered a differential solution to nonlinear equations e.g. integral equations, partial differential equations, differential equations, variational inequalities, uniqueness theorem, existence theorem, etc. It is very important to develop some iterative diagrams. On the other hand, there are many authors to reach fixed point in different spaces in Hilbert space and Banach space, see ([1]-[10]). In 2008, Olatinwo and Imoru [1] discovered some convergence results for J-Mann and J-Ishikawa iterations. Now we will mention some basic priorities. An operator $A\colon D(A) \subset H \to 2^H$ is called monotone operator if $\forall x, y \in D(A)$ such that $\langle u - v, x - y\rangle \geq 0 \; for \; u \in A_x \; and \; v \in A_y$.And it's called maximal if its graph is not contained in the graph of another monotone operator.

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The resolvent operator of A denoted by $J_\lambda^A: H \to D(A)$ and defined by $J_\lambda^A = (I + \lambda A)^{-1}$ where λ is any positive number and also denote $A^{-1}0$ by the set of zeros of A.

2. Main Results

We introduced a new iterative scheme and proved its convergence and speediness of it by using resolvent $\mathcal{ZA}$-Jungck mappings. Now, we define

1. Let $\langle \eta_n \rangle$ be a sequence, in[0,1].The resolvent Jungck Modified Mann Z-iteration is defined as
$$\mathcal{S}\mathcal{O}_{n+1} = \mathcal{J}_{rn}\big((1-\eta_n)\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_n + \eta_n \mathcal{T}\mathcal{O}_n\big),$$
2. The resolvent Jungck ZSY- iteration is defined as:
$\mathcal{S}z_{n+1} = \mathcal{J}_{rn}\big((1-\eta_n)\,\mathcal{T}z_n + \eta_n \mathcal{J}_{rn}\mathcal{T}\mathcal{T}z_n\big)$, Where $\mathcal{S}, \mathcal{G}$ are commute mappings on $\mathcal{C}$and $\langle \eta_n \rangle$ are sequences in $[0,1]$
3. The resolvent Jungck Picard iteration $\mathcal{S}\hbar_{n+1} = \mathcal{J}_{rn}\mathcal{T}\hbar_n$
4. A self-mapping $\mathcal{T}: \mathsf{C} \to \mathsf{C}$ is called a resolvent $\mathcal{ZA}$-Jungck mapping if
$\|\mathcal{T}x - \mathcal{T}y\| \le \mathfrak{b}\psi\big(\mathfrak{a}_1\|\mathcal{S}x - \mathcal{S}y\| + \mathfrak{a}_2\|\mathcal{S}x - \mathcal{J}_{rn}y\|\big) + \min\Big\{\|\mathcal{J}_{rn}x - \mathcal{T}x\|, \frac{\|\mathcal{S}x - \mathcal{J}_{rn}x\| + \|\mathcal{J}_{rn}y - \mathcal{S}y\|}{2}, \|\mathcal{T}y - \mathcal{J}_{rn}y\|\Big\}.$
$\forall x, y \in \mathsf{C}, \mathfrak{a}_1, \mathfrak{a}_2, \mathfrak{b} \in [0,1], \mathfrak{a}_1 + \mathfrak{a}_2 \le 1$ and ψ is a comparison function
5. Property$\mathcal{W}$: The set of all Jungck mappings $\mathcal{S}$ satisfy $\|\mathcal{S}\mathcal{S}x - t\| \le \|\mathcal{S}x - t\|, t \in f(\mathcal{S})$. Now, we give new results

Lemma 2.1: *Let $\mathcal{T}$ be a resolvent $\mathcal{ZA}$-Jungck mapping on C. Suppose that $\langle \mathcal{S}\mathcal{O}_n \rangle$ is resolvent Jungck Modified Mann Z- iteration scheme in C with $f(\mathcal{S}) \cap f(\mathcal{T}) \cap A^{-1}(0) \neq \emptyset$. Then $\lim_{n\to\infty}\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|$ exists, for all $n \in N$.*

Proof: Let $\mathcal{P}$ be a fixed point of $\mathcal{S}, \mathcal{G}$ and $A^{-1}(0)$ then the following inequalities hold

$$\begin{aligned}
\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\| &\le \|(1-\eta_n)\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_n + \eta_n\,\mathcal{G}\mathcal{O}_n - \mathcal{P}\| \\
&\le (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\|\mathcal{G}\mathcal{O}_n - \mathcal{P}\| \\
&\quad (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\mathfrak{b}\psi\big(a_1\|\mathcal{S}\mathcal{O}_n - \mathcal{S}\mathcal{P}\| + a_2\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{P}\|\big) + \\
&\le \min\left\{\|\mathcal{J}_{rn}\mathcal{O}_n - \mathcal{G}\mathcal{O}_n\|, \frac{\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{O}_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{G}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\} \\
&\le (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + a_2\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|) \\
&\le (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\mathfrak{b}\psi(\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|)
\end{aligned}$$
$$\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\| \le \|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| \le \ldots\ldots\ldots\ldots \le \|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|$$

So, $\{\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|\}$ is decreasing and bounded,for each $t \in f(\mathcal{S}) \cap f(\mathcal{G}) \cap A^{-1}(0)$ this implies that $\lim_{n\to\infty}\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|$ exists.

Theorem 2.2: *Let $\mathcal{T}$ be a resolvent $\mathcal{ZA}$-Jungck self-mapping on C.and $\langle \mathcal{S}\mathcal{O}_n \rangle$ be a resolvent Jungck Modified Mann Z- iteration scheme. Then the resolvent*

Jungck Modified Mann Z- iteration converges to a unique common fixed point of $\mathcal{S}$ *,* $\mathcal{T}$ *and* $A^{-1}(0)$.

Proof: By Lemma (1), we have
$\|\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\| \leq ((1-\eta_n)+\eta_n \mathfrak{b})\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|$
$\|\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\| \leq \prod_{j=0}^{n}(1-\eta_j(1-\mathfrak{b})\|\mathcal{S}\mathcal{O}_0-\mathcal{P}\|$
$\|\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\| \leq e^{-(1-\mathfrak{b})\sum_{j=0}^{\infty}\eta_j}$.

Since $\eta_j \in [0,1]$ and $\mathfrak{b} \in [0,1]$ and $\sum_{j=0}^{\infty}\eta_j=\infty$. Then $e^{-(1-\mathfrak{b})\sum_{j=0}^{\infty}\eta_j} \to 0$ as $n\to\infty$. $\lim_{n\to\infty}\|\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\|=0$. Therefore, $\langle \mathcal{S}\mathcal{O}_n\rangle$ converges strongly to $\mathcal{P}$.

Now, Let there exist another $\mathcal{P}^* \in C$ such that $\mathcal{T}\mathcal{P}^*=\mathcal{J}_{rn}\mathcal{P}^*=\mathcal{S}\mathcal{P}^*=\mathcal{P}^*$.

$$0 \leq \|\mathcal{P}-\mathcal{P}^*\|=\|\mathcal{T}\mathcal{P}-\mathcal{T}\mathcal{P}^*\|$$
$$\leq \mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{P}-\mathcal{S}\mathcal{P}^*\|+a_2\|\mathcal{S}\mathcal{P}-\mathcal{J}_{rn}\mathcal{P}^*\|)+$$
$$\min\left\{\|\mathcal{J}_{rn}\mathcal{P}-\mathcal{T}\mathcal{P}\|,\frac{\|\mathcal{S}\mathcal{P}-\mathcal{J}_{rn}\mathcal{P}^*\|+\|\mathcal{J}_{rn}\mathcal{P}^*-\mathcal{S}\mathcal{P}^*\|}{2},\|\mathcal{T}\mathcal{P}^*-\mathcal{J}_{rn}\mathcal{P}^*\|\right\}$$
$$\leq \mathfrak{b}(a_1\|\mathcal{S}\mathcal{P}-\mathcal{S}\mathcal{P}^*\|+a_2\|\mathcal{S}\mathcal{P}-\mathcal{J}_{rn}\mathcal{P}^*\|)\leq \mathfrak{b}\|\mathcal{S}\mathcal{P}-\mathcal{S}\mathcal{P}^*\| \quad \Rightarrow \quad \mathcal{P}=\mathcal{P}^*$$

Theorem 2.3: *Let* $\mathcal{T}$ *be a resolvent* $\mathcal{ZA}$*-Jungck self-mapping on* C *and* $\langle\mathcal{S}\mathcal{O}_n\rangle$ *be a resolvent Jungck Modified Mann Z- iteration scheme in* C*. Then* $f(\mathcal{S})\cap f(\mathcal{T})\cap A^{-1}(0)\neq\phi$ *if and only if* $\lim_{n\to\infty}\|\mathcal{T}\mathcal{O}_n-\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\|=0$

Proof: Since $f(\mathcal{S})\cap f(\mathcal{T})\cap A^{-1}(0)\neq\emptyset \Rightarrow \exists\mathcal{P}\in f(\mathcal{S})\cap f(\mathcal{T})\cap A^{-1}(0)\Rightarrow \mathcal{P}=f(\mathcal{S})=f(\mathcal{T})=A^{-1}(0)$.By Lemma (1), we have, $\lim_{n\to\infty}\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|=\lambda$

$\lim_{n\to\infty}\sup\|\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\| \leq \lim_{n\to\infty}\sup\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|$.

$$\lim_{n\to\infty}\sup\|\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}-\mathcal{P}\|\leq\lambda \qquad (1)$$

Now, to prove that $\lim_{n\to\infty}\sup\|\mathcal{T}\mathcal{O}_n-\mathcal{P}\| \leq\lambda$

$$\lim_{n\to\infty}\sup\|\mathcal{T}\mathcal{O}_n-\mathcal{P}\|\leq \lim_{n\to\infty}\sup \mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|+a_2\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|)$$
$$+\min\left\{\|\mathcal{J}_{rn}\mathcal{O}_n-\mathcal{T}\mathcal{O}_n\|,\frac{\|\mathcal{S}\mathcal{O}_n-\mathcal{J}_{rn}\mathcal{O}_n\|+\|\mathcal{J}_{rn}\mathcal{P}-\mathcal{S}\mathcal{P}\|}{2},\|\mathcal{T}\mathcal{P}-\mathcal{J}_{rn}\mathcal{P}\|\right\}$$

$$=\lim_{n\to\infty}\sup \mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|+a_2\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|)\leq\lim_{n\to\infty}\sup\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|=\lambda$$

We get, $\lim_{n\to\infty}\sup\|\mathcal{S}\mathcal{O}_n-\mathcal{P}\|\leq\lambda$ (2)

Now, by Lemma(1), we have

$$\lambda=\lim_{n\to\infty}\sup\|\mathcal{S}\mathcal{O}_{n+1}-t\|=\lim_{n\to\infty}\sup\left\|\mathcal{J}_{rn}\left((1-\eta_n)\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_n+\eta_n\mathcal{T}\mathcal{O}_n\right)-\mathcal{P}\right\|$$

$$\leq \lim_{n\to\infty} \sup[(1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n \mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n - \mathcal{S}\mathcal{P}\| + a_2\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{P}\|)$$

$$+\min\left\{\|\mathcal{J}_{rn}\mathcal{O}_n - \mathcal{G}\mathcal{O}_n\|, \frac{\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{O}_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{G}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\}$$

$$\leq \lim_{n\to\infty} \sup[(1-\ \eta_n(1-\mathfrak{b})\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|]$$

$$\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\| \leq \lim_{n\to\infty} \sup\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| = \lambda \qquad (3)$$

By (1),(2),(3),we get $\lim_{n\to\infty}\|\mathcal{T}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\| = 0$.

$(\mathrm{C}, \langle \mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\rangle) \Rightarrow r(\mathrm{C}, \langle \mathcal{S}\mathcal{O}_n\rangle) = r(\mathcal{T}\mathcal{P}, \langle \mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\rangle)$

Now, $r(\mathcal{T}\mathcal{P}, \langle \mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\rangle) = lim_{n\to\infty} sup\ \left\|\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1} - \mathcal{T}\mathcal{P}\right\|$

$$\leq \lim_{n\to\infty}\sup[\ \mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + a_2\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\|) +$$

$$\min\left\{\|\mathcal{J}_{rn}\mathcal{O}_n - \mathcal{T}\mathcal{O}_n\|, \frac{\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{O}_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{T}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\}$$

$$\leq \lim_{n\to\infty} sup\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| = r(\mathcal{P}, \langle \mathcal{S}\mathcal{O}_n\rangle) = r(\mathrm{C}, \langle \mathcal{S}\mathcal{O}_n\rangle)$$

$r(\mathrm{C}, \langle \mathcal{S}\mathcal{O}_n\rangle) = r(\mathcal{T}\mathcal{P}, \langle \mathcal{J}_{rn}\mathcal{S}\mathcal{O}_{n+1}\rangle) \Longrightarrow \mathcal{T}\mathcal{P} \in A(\mathrm{C}, \langle \mathcal{S}\mathcal{O}_n\rangle) \Rightarrow A(\mathrm{C}, \langle \mathcal{S}\mathcal{O}_n\rangle)$ is singleton$\Rightarrow \mathcal{P} = \mathcal{T}\mathcal{P} \Rightarrow \mathcal{P} \in f(\mathcal{T}) \Rightarrow f(\mathcal{T}) \neq \emptyset$. In the same way, we get, $\mathcal{P} \in f(\mathcal{S})$ & $\mathcal{P} \in A^{-1}(0) \Longrightarrow f(\mathcal{S}) \cap f(\mathcal{T}) \cap A^{-1}(0) \neq \emptyset$.

Lemma 2.4: *Let $\mathcal{T}$ be a resolvent $\mathcal{ZA}$-Jungck self-mapping on C . Suppose that $\langle \mathcal{S}z_n\rangle$ be a resolvent Jungck ZSY- iteration scheme in C. If $f(\mathcal{S}) \cap f(\mathcal{T}) \cap A^{-1}(0) \neq \emptyset$, then $\lim_{n\to\infty}\left\|\mathcal{S}z_n - \mathcal{P}\right\|$ exists.*

Theorem 2.5: *Let $(X, \|.\|)$ be a normed space, and $\mathcal{T}$ be a resolvent $\mathcal{ZA}$-Jungck mapping on C if $\mathcal{P} \in f(\mathcal{S}) \cap f(\mathcal{T}) \cap A^{-1}(0) \neq \phi$ where $0 < \mathfrak{r} \leq \eta_n < 1$, then the resolvent Jungck ZSY- iteration scheme is faster than the resolvent Jungck Modified Mann Z- iteration scheme.*

Proof: *Let $\mathcal{P} \in f(\mathcal{S}) \cap f(\mathcal{T}) \cap A^{-1}(0)$ the resolvent Jungck Modified Mann Z- iteration scheme, we have*

$$\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\| = \left\|\mathcal{J}_{rn}\big((1-\eta_n)\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_n + \eta_n\mathcal{T}\mathcal{O}_n\big) - \mathcal{P}\right\|$$

$$\leq (1-\eta_n)\|\mathcal{J}_{rn}\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\|\mathcal{G}\mathcal{O}_n - \mathcal{P}\|$$

$$\leq (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\|\mathcal{G}\mathcal{O}_n - \mathcal{P}\|$$

$$\leq (1-\eta_n)\|\mathcal{S}\mathcal{O}_n - \mathcal{P}\| + \eta_n\mathfrak{b}\psi(a_1\|\mathcal{S}\mathcal{O}_n - \mathcal{S}\mathcal{P}\| + a_2\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{P}\|) +$$

$$\min\left\{\|\mathcal{J}_{rn}\mathcal{O}_n - \mathcal{G}\mathcal{O}_n\|, \frac{\|\mathcal{S}\mathcal{O}_n - \mathcal{J}_{rn}\mathcal{O}_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{G}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\}.$$

$$\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\| \leq ((1-\ \mathfrak{r}(1-\mathfrak{b}))^n\|\mathcal{S}\mathcal{O}_0 - \mathcal{P}\|$$

The resolvent Jungck ZSY- iteration scheme. We have

$$\|\mathcal{S}z_{n+1} - \mathcal{P}\| = \left\|\mathcal{J}_{rn}\left((1-\eta_n)\mathcal{T}z_n + \eta_n\mathcal{J}_{rn}\mathcal{T}\mathcal{T}z_n\right) - \mathcal{P}\right\|$$

$$\leq (1-\eta_n)\mathfrak{b}\psi\left(a_1\left\|\mathcal{S}z_n - \mathcal{S}\mathcal{P}\right\| + a_2\left\|\mathcal{S}z_n - \mathcal{J}_{rn}\mathcal{P}\right\|\right) +$$

$$\min\left\{\|\mathcal{J}_{rn}z_n - \mathcal{T}z_n\|, \frac{\|\mathcal{S}z_n - \mathcal{J}_{rn}z_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{T}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\} + \eta_n\|\mathcal{T}\mathcal{T}z_n - \mathcal{P}\| \leq (1-\eta_n)\,\mathfrak{b}\psi\left(a_1\left\|\mathcal{S}z_n - \mathcal{P}\right\| + a_2\left\|\mathcal{S}z_n - \mathcal{P}\right\|\right) + \eta_n\|\mathcal{T}\mathcal{T}z_n - \mathcal{P}\|$$

$$\leq \left((1-\eta_n)\,\mathfrak{b}\right)\|\mathcal{S}z_n - \mathcal{P}\| + \eta_n\mathfrak{b}\psi\left(a_1\left\|\mathcal{S}\mathcal{T}z_n - \mathcal{S}\mathcal{P}\right\| + a_2\left\|\mathcal{S}\mathcal{T}z_n - \mathcal{J}_{rn}\mathcal{P}\right\|\right) +$$

$$\min\left\{\|\mathcal{J}_{rn}\mathcal{T}z_n - \mathcal{T}z_n\|, \frac{\|\mathcal{S}\mathcal{T}z_n - \mathcal{J}_{rn}\mathcal{T}z_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{T}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\}$$

$$\leq \left((1-\eta_n)\,\mathfrak{b}\right)\|\mathcal{S}z_n - \mathcal{P}\| + \eta_n\mathfrak{b}\mathfrak{b}\psi(a_1\|\mathcal{S}z_n - \mathcal{S}\mathcal{P}\| + a_2\|\mathcal{S}z_n - \mathcal{J}_{rn}\mathcal{P}\|) +$$

$$\min\left\{\|\mathcal{J}_{rn}z_n - \mathcal{T}z_n\|, \frac{\|\mathcal{S}z_n - \mathcal{J}_{rn}z_n\| + \|\mathcal{J}_{rn}\mathcal{P} - \mathcal{S}\mathcal{P}\|}{2}, \|\mathcal{T}\mathcal{P} - \mathcal{J}_{rn}\mathcal{P}\|\right\}$$

$$\leq \left((1-\eta_n)\,\mathfrak{b}\right) + \eta_n\mathfrak{b}^2)\left\|\mathcal{S}z_n - \mathcal{P}\right\| \leq \mathfrak{b}(1-\eta_n(1-\mathfrak{b})\left\|\mathcal{S}z_n - \mathcal{P}\right\|$$

$$\frac{\left\|\mathcal{S}z_{n+1} - \mathcal{P}\right\|}{\|\mathcal{S}\mathcal{O}_{n+1} - \mathcal{P}\|} \leq \frac{\mathfrak{b}^n((1-\mathfrak{r}(1-\mathfrak{b}))^n\left\|\mathcal{S}z_0 - \mathcal{P}\right\|}{((1-\mathfrak{r}(1-\mathfrak{b}))^n\|\mathcal{S}\mathcal{O}_0 - \mathcal{P}\|} \to 0 \quad as \quad n \to \infty.$$

3. Discussion and Conclusion

A new iteration of different steps is introduced, an affinity and velocity for it were studied and it was found that the resolvent Jungck modified Mann Z- the scheme is faster than that of the resolvent Jungck modified Mann Z. Also, the resolvent Jungck ZSY iteration is faster than that of resolvent Jungck Picard.

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