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Sandwich results of meromorphic multivalent functions defined by multiplier transform

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Abstract

We investigate the characteristics of differential subordination for meromorphic multivalent functions and superordination linked to defined multiplier transforms for meromorphic multivalent functions. As a result, we get sandwich outcomes.

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1. Introduction

Let $\mathrm{K} = \mathrm{K}(\mathfrak{U})$ be the class of analytic functions in $\mathfrak{U} = \{z \in \mathbb{C}: |z| < 1\}$ let $\mathrm{K}[d, n]$ $(d \epsilon \mathbb{C}, n \in \mathbb{N})$ denotes subclass of functions $f(z) \in \mathrm{K}(\mathfrak{U})$:

$$f(z) = d + d_n z^n + d_{n+1} z^{n+1} + \cdots \quad (d \in \mathbb{C}), \tag{1.1}$$

and A_p be a subclass of K by form:

$$f(z) = z^{-p} + \sum_{k=0}^{\infty} d_k z^{k+1-p} \quad (p \in \mathbb{N}), \tag{1.2}$$

that is analytic in $\mathfrak{U}^* = \mathfrak{U}/\{0\}$.

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Let $f, F \in$ K. f is subordinate($f \prec F$) to F if $\omega(0) = 0$ a Schwarz function and $|\omega(z)| < 1$ such that $f(z) = F(\omega(z))$ $(z \in \mathfrak{U})$, then $f \prec \mathcal{J}$ (see [7]):

$$f(z) \prec \mathcal{J}(z) \leftrightarrow f(0) = \mathcal{J}(0) \text{ and } f(\mathfrak{U}) \subset \mathcal{J}(\mathfrak{U}).$$

Let $\ell, t \in$ K, and $\sigma(r, s, t, z): \mathbb{C}^3 \times \mathfrak{U} \to \mathbb{C}$ and t univalent in $\mathfrak{U}$. If ℓ analytic in $\mathfrak{U}$ and satisfies the second-order differential subordination

$$\sigma(\ell(z), z\ell'(z), z^2\ell''(z); z) \prec t(z), \tag{1.3}$$

if ℓ satisfies the second-order differential superordination

$$t(z) \prec \sigma(\ell(z), z\ell'(z), z^2\ell''(z); z), \tag{1.4}$$

then ℓ is called a solution of the differential superordination (1.4), if f is subordinate to F, then F is superordinate to f.

Miller and Mocanu[8] obtained conditions on t, j, and σ holds :

$$t(z) \prec \sigma(\ell(z), z\ell'(z), z^2\ell''(z); z) \Longrightarrow j(z) \prec t(z).$$

An analytic function $j(z)$ is called a subordinate of (1.4) if $j \prec \ell$ for all ℓ satisfies (1.4).

A subordinate $\tilde{j}(z)$ that satisfies $j \prec \tilde{j}$ for all subordinates $j(z)$ of (1.4) is called the best subordinate.

Ali et. al. [2] used Bulboaca's [6] results to derive the necessary criteria for a given normalized analytic function f(z) to meet $q_1(z) \prec \frac{f(z)}{zf'(z)} \prec q_2(z)$, where $q_1(z)$ and $q_2(z)$ univalent functions in U with $q_1(0) = q_2(0) = 1$. For further result (see[2]).

In 2017, Wang and Li [7] defined a new multiplier transform $\mathcal{Y}^{n,l}_{\gamma,p,\beta}$ for the function $f \in A_p$ by

$$\mathcal{Y}^{n,l}_{\gamma,p,\beta} := z^{-p} + \sum_{k=0}^{\infty} \frac{(\beta)_{k+1}}{(k+1)!} \left(\frac{\gamma}{\gamma + l(k+l)}\right)^n a_k z^{k+1-p}, z \in \mathfrak{U}^*, \tag{1.5}$$

where $n \geq 0, l \geq 0, p \in \mathbb{N}, and\ \beta, \gamma > 0,$

and $(\beta)_k$ is the Pochhammer symbol by

$$(\beta)_{\mathcal{K}} := \begin{cases} 1, & if\ k = 0, \\ \beta(\beta + 1) \ldots (\beta + k - 1), & if\ k \in \mathbb{N}. \end{cases}$$

That for all $f \in A_p$ we have

$$z\left(\mathcal{Y}^{n,l}_{\gamma,p,\beta} f\right)'(z) = \beta \mathcal{Y}^{n,l}_{\gamma,p,\beta+1} f(z) - (\beta + p)\mathcal{Y}^{n,l}_{\gamma,p,\beta} f(z), \ z \in U^*. \tag{1.6}$$

The principle question of present properties discovering adequate conditions

$$q_1(z) \prec \mathcal{Y}^{n,l}_{\gamma,p,\beta} f(z) \prec q_2(z),$$

whenever q_1 and q_2 are known univalent functions $\mathfrak{U}$ with $q_1(0) = q_2(0) = 1$. Many scholars in this sector were interested in the sandwich results of univalent and multivalent functions. For example, see[1-5].

The next lemma is the foundation results in the theory of sandwich results of Meromorphic Multivalent functions.

2. Preliminaries

Definition 2.1: (see[9]): Let Φ be a set in $\mathbb{C}, q \in \Pi[a,n]$ and $q'(z) \neq 0$. The class of admissible functions $\Psi'_n[\Phi, q]$ consists of $\mathcal{T}: \mathbb{C}^3 \times \overline{\mathfrak{U}} \to \mathbb{C}$ that fulfill the following requirements :

$$\theta(i, g, j: \xi) \in \Phi$$

when

$$i = q(z), \quad g = \frac{zq'(z)}{m}, \ \mathcal{R}\left(\frac{j}{g}+1\right) \leq \frac{1}{m}\mathcal{R}\left(\frac{zq''(z)}{q'(z)}+1\right),$$

when $\xi \in \partial\mathfrak{U}$ and $m \geq n \geq 2$.

Theorem 2.1: (see[9]): *Let* $\mathcal{T} \in \Psi'_n[\Phi, q]$ *with* $q(0) = \alpha$. *If* $\ell \in \mathfrak{Q}(\alpha)$ *and*

$$\varphi(\ell(z), z\ell'(z), z^2\ell''(z); z)$$

is univalent in U, then

$$\Phi \subset \{\varphi(\ell(z), z\ell'(z), z^2\ell''(z); z): z \in \mathfrak{U}\}$$

implies that

$$q(z) \prec \ell(z) \quad (z \in \mathfrak{U}).$$

3. Subordination Results

We prove the following results.

Definition 3.1. A set Φ in $\mathbb{C}, \mathcal{S} \in Q_1 \cap \mathcal{H}_1$, and $\Phi_I[\Phi, \mathcal{S}]$ class of admissible functions consists of $\vartheta: \mathbb{C}^3 \times \mathfrak{U} \to \mathbb{C}$ fulfill the following requirements admissibility condition:

$$\vartheta(a, b, c; z) \notin \Phi \qquad (z \in \mathfrak{U})$$

whenever

$$a = \mathcal{S}(\tau), \ \ b = \frac{k\zeta\mathcal{S}'(\tau) + \mu\mathcal{S}(\tau)}{\mu}$$

$$Re\left(\frac{\mu(c-a)}{(c-a)} - 2\mu\right) \geq kRe\left(\frac{\tau q''(\tau)}{q'(\tau)}+1\right) \qquad (k \geq 1)$$

when $(\tau \in \partial\mathfrak{U}/E(\mathcal{S})$.

Theorem 3.1: *If the function* $f \in A_p$, $\vartheta \in \Phi_I[\Phi, \mathcal{S}]$ *satisfy the following condition:*

$$\left\{\vartheta\left(z^p\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z), z^p\mathcal{Y}^{n,I}_{\gamma,p,\beta}f(z), z^p\mathcal{Y}^{n,I}_{\gamma,p,\beta+1}f(z); z\right), z \in \mathfrak{U}\right\} \subset \Phi, \quad (3.2)$$

then

$$z^p\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z) \prec \mathcal{S}(z) \quad .$$

Proof: By $F(z)$ in $\mathfrak{U}$

$$F(z) = z^p\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z). \quad (3.3)$$

Differentiating (3.3) and (1.6), we get

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z) = \frac{1}{\beta}[\beta F(z) + zF'(z)]. \tag{3.4}$$

Furthermore compositions show that

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z) = \frac{1}{\beta(\beta+1)}[z^2F''(z) + 2\beta zF'(z)] + F(z), \tag{3.5}$$

the transformation from $\mathbb{C}^3$ to $\mathbb{C}$ defined by

$$a = i, \quad b = \frac{g+\beta i}{\beta}, \qquad c = \frac{j+2\beta g+\beta(\beta+1)i}{\beta(\beta+1)},$$

Let

$$\mathcal{T}(i,g,j;z) = \vartheta(a,b,c;z) = \vartheta(r, \frac{j+\beta i}{\beta}, \frac{j+2\mu g+\beta(\beta+1)i}{\beta(\beta+1)}). \tag{3.6}$$

Using (3.3), (3.6)-(3.9), from (3.6), we have

$$\mathcal{T}(F(z), zF'(z), z^2F''(z); z) = \vartheta\big(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\big). \tag{3.7}$$

Hence (3.2) leads to

$$\mathcal{T}(F(z), zF'(z), z^2F''(z); z) \in \Phi.$$

Note that

$$\frac{t}{s} + 1 = \frac{(\beta+1)(c-b)}{(b-a)} - (2(\beta+1)+1).$$

As a result, the admissibility of $\vartheta \in \Phi_I[\Phi, \mathcal{S}]$ in Definition (3.1) corresponds to the admissibility for $\mathcal{T}$ in Theorem (2.2).

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z) \prec \mathcal{q}(z).$$

Corollary 3.1: *Suppose that $\mathcal{S}$ be a univalent function with $\mathcal{q}(0) = 1$ and $\Phi \subset \mathbb{C}$. Let $\vartheta \in \Phi_I[\Phi, \mathcal{S}_\sigma]$, $\mathcal{S}_\sigma(z) = \mathcal{S}(\sigma z)$. If $f \in A_p$ and $\mathcal{S}_\sigma$ for some $\sigma \in (0,1)$ satisfies the following conditions:*

$$\{\vartheta\big(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\big), z \in \mathfrak{U}\} \in \Phi,$$

then

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z) \prec \mathcal{S}(z) \quad (z \in \mathfrak{U}).$$

Proof: By using Theorem (3.1), we obtain

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z) \prec \mathcal{S}_\sigma(z).$$

The result of Corollary (3.1)

$$\mathcal{S}(z) \prec \mathcal{S}_\sigma(z), \quad (z \in \mathfrak{U}).$$

In the particular case when $\mathcal{S}(z)$ conformal transformation of $\mathfrak{U}$ onto Φ such that $\Phi \neq \mathbb{C}$ is simply connected domain, thus $\Phi = \mathcal{S}(\mathfrak{U})$, we denote the class $\Phi_I[\mathcal{S}(\mathfrak{U}), \mathcal{S}]$ by $\Phi_I[\mathrm{h}, \mathcal{S}]$. Theorem (3.2) and Theorem (3.3) are the immediate repercussion of Theorem (3.1) and Corollary (3.1).

Theorem 3.2: *Suppose that $\vartheta \in \Phi_I[h, \mathcal{S}]$. If $f \in A_p$ and $q \in Q_1$ satisfy the condition (3.2) and*

$$\vartheta\left(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\right) \prec \mathcal{S}(z), \qquad (3.8)$$

then

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z) \prec \mathcal{S}(z) \quad .$$

The following results show the differential subordination's best dominant (3.8).

Theorem 3.3: *Suppose that σ be a univalent function in $\mathfrak{U}$, and let $\vartheta: \mathbb{C}^3 \times \mathfrak{U} \to \mathbb{C}$), and $\mathcal{T}$ be given by(3.7). Let*

$$\mathcal{T}(\ell(z), z\ell'(z), z^2\ell''(z); z) = \sigma(z), \qquad (3.9)$$

has a solution $\mathcal{S}(z)$ with $\mathcal{S}(0)$=1 and satisfies (3.4). If $f \in A_{\mathrm{P}}$ satisfies (3.9) and

$$\vartheta\left(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\right),$$

is analytic in $\mathfrak{U}$, we get

$$z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z) \prec \mathcal{S}(z),$$

where $\mathcal{S}(z)$ is the best dominant.

Proof From Theorem (3.1), $\mathcal{S}$ is a dominant of (3.8). Since $\mathcal{S}$ satisfies (3.9) and (3.8) Hence $\mathcal{S}$ is best dominant.

Definition 3.2: Suppose that Φ be a set in $\mathbb{C}, \mathcal{S} \in Q_1 \cap H_1$. The class of $\Phi_I'[\Phi, \mathcal{S}]$ consist of ϑ satisfy the following :

$$\vartheta(a, b, c; z) \notin \Phi$$

whenever

$$a = q(\tau), \;\; b = \frac{\zeta q'(\tau) + m\beta q(\tau)}{m\beta}, \; Re\left(\frac{(\beta+1)(w-u)}{(v-u)} - 2(\beta+1)\right) \le \frac{1}{m} k Re\left(\frac{\tau \mathcal{S}''(\tau)}{\mathcal{S}'(\tau)} + 1\right),$$

where ($\tau \in \partial\mathfrak{U}/E(q)$ and $m \ge 1$).

Theorem 3.4: *Suppose that $\vartheta \in \Phi_I'[\Phi, \mathcal{S}]$. If the function $f \in A_p$ with $\mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z) \in Q_1$, if $\mathcal{S} \in \mathcal{H}_1$ with $\mathcal{S}'(z) \neq 0$ and*

$$\vartheta\left(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\right)$$

is a univalent function in $\mathfrak{U}$, then

$$\Phi \subset \left\{\vartheta\left(z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta} f(z), z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta+1} f(z); z\right), z \in \mathfrak{U}\right\}, \qquad (3.10)$$

indicates that

$$\mathcal{S}(z) \prec z^p \mathcal{Y}^{n,I}_{\gamma,p,\beta-1} f(z).$$

Proof: From (3.7) and (3.10), we get

$$\Phi \subset \{\mathcal{T}(F(z), zF'(z), z^2F''(z); z)\}.$$

According to equation (3.6), for $\vartheta \in \Phi_I'[\Phi, \mathcal{S}]$, in Definition(3.2) is similar to the requirement for $\mathcal{T}$ in Definition (2.1). Hence $\mathcal{T} \in \Psi_2[\Phi, \mathcal{S}]$ and by Theorem (2.1), we obtain

$$\mathcal{S}(z) \prec z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z).$$

If $\Phi \neq \mathbb{C}$, Then, for some $\Phi = \omega(\mathfrak{U})$ conformal mapping of $\mathfrak{U}$ onto Φ. In this instance, the following two outcomes are direct results of Theorem (3.4).

Theorem 3.5: *Suppose that $\vartheta \in \Phi_I'[\omega(\mathfrak{U}), \mathcal{S}]$ and let ω be analytic in $\mathfrak{U}$. If the function $f \in A_p$ with $z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z) \in Q_1$ and if $q \in \mathcal{H}_1$ and*

$$\vartheta\left(z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta+1}^{n,I} f(z); z\right)$$

is univalent in $\mathfrak{U}$, thus

$$\omega(z) \prec \vartheta\left(z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta+1}^{n,I} f(z); z\right), \qquad (3.11)$$

indicates that

$$\mathcal{S}(z) \prec z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z) \qquad (z \in \mathfrak{U}).$$

Theorem 3.6: *Suppose that ω be a univalent function in $\mathfrak{U}$, and let ϑ and $\mathcal{T}$ be given by (3.6). Let*

$$\mathcal{T}(\mathcal{S}(z), z\mathcal{S}'(z), z^2\mathcal{S}''(z); z) = \omega(z), \qquad (3.12)$$

has a solution $\mathcal{S}(z) \in Q_1$. If the function $f \in A_p$ with $z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z) \in Q_1$ and if $\mathcal{S} \in \mathcal{H}_1$ with $\mathcal{S}'(z) \neq 0$ and

$$\vartheta\left(z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta+1}^{n,I} f(z); z\right),$$

is univalent in $\mathfrak{U}$, then

$$\omega(z) \prec \vartheta\left(z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta}^{n,I} f(z), z^p \mathcal{Y}_{\gamma,p,\beta+1}^{n,I} f(z); z\right)$$
$$\mathcal{S}(z) \prec z^p \mathcal{Y}_{\gamma,p,\beta-1}^{n,I} f(z),$$

and $\mathcal{S}(z)$ is the best subordinate.

Proof: We can see that q is a subordinate of $\mathcal{S}$ from Theorems (3.3) and (3.9), (3.11). Since $\mathcal{S}$ is a solution of (3.12), it is also a solution of (3.11), and therefore $\mathcal{S}$ will be subordinate to all subordinates. As a result, $\mathcal{S}$ is the best subordinate. This concludes Theorem's proof (3.6).

We obtain Theorem (3.7) Sandwich Theorem by combining Theorem(3.1) and Theorem (3.4).

Theorem 3.7: *Let t_1 and $\mathcal{S}_1$ be an analytic function in $\mathfrak{U}$, let t_2 be a univalent function in $\mathfrak{U}$ and $\mathcal{S}_2 \in Q_1$ with $\mathcal{S}_1(0) = \mathcal{S}_2(0) = 1$ and $\vartheta \in \Phi_{I_p}[t_2, \mathcal{S}_2] \cap \Phi'_{I_p}[t_1, \mathcal{S}_1]$. If the function $f \in A_p$ with $\frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-2}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)} \in Q_1 \cap \mathcal{H}_1$ and the function $\vartheta\left(\frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-2}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)}, \frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta}f(z)}, \frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta+1}f(z)}; z\right)$ is univalent in $\mathfrak{U}$, then*

$$t_1(z) \prec \vartheta\left(\frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-2}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)}, \frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta}f(z)}, \frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta+1}f(z)}; z\right) \prec t_2(z)$$

implies that

$$\mathcal{S}_1(z) \prec \frac{\mathcal{Y}^{n,I}_{\gamma,p,\beta-2}f(z)}{\mathcal{Y}^{n,I}_{\gamma,p,\beta-1}f(z)} \prec \mathcal{S}_2(z) \qquad (z \in \mathfrak{U}).$$

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