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Regularity in ideal grill topological space

Alyaa Yousef Khudayer Mutawek *
Department of Mathematics
College of Computer Science and Mathematics
University of Kufa
Kufa
Iraq

Raad Aziz Hussain Al-Abdulla †
Department of Mathematics
College of Science
University of Al-Qadisiyah
Al-Qadisiyah
Iraq

Abstract

Among the basic concepts that have a vital role in producing comprehensive topologies is a regular space. In this respect, a new definition is introduced in the current study of separation properties. The term $\mathcal{J}_{\mathcal{G}}$-regular is given to it. Then its important properties and connection with regular, $\mathcal{J}_{\mathcal{G}}$-compact and H-closed are presented.

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Keywords: *Compact, Ideals, Grills, $\mathcal{J}$-regular, G-regular.*

1. Introduction

In 1930, ideals in topological spaces were considered. The concept of ideal in topological spaces has been introduced by Kuratwaski [8]and Vaidyanathswamy[16]. In [7] Jankovic and Hamlett proposed a new topology from the topology space (Ꭓ, $\mathfrak{T}$) induced by an ideal on Ꭓ, described as follows: Let $\mathcal{I}$ be an ideal on a topological space (Ꭓ,$\mathfrak{T}$), the operator $(\,)^*: p(Ꭓ) \rightarrow p(Ꭓ)$ is called a local function of д concerning $\mathfrak{T}$ and $\mathcal{I}$ [16], given by $д^*(\mathcal{I}, \mathfrak{T}) = \{x: д \cap Ӳ \notin \mathcal{I}, \text{forevery open } Ӳ \text{ of } x\}$. The collection $\{Ѵ - д: Ѵ \in \mathfrak{T}, д \in \mathcal{I}\}$ is a

* *E-mail:* **alyaay.khudhair@uokufa.edu.iq (Corresponding Author)**
† *E-mail:* **raad.hussain@qu.edu.iq**

base for the topology $\mathfrak{T}^*(\mathcal{J})$on Ҳ which is strictly finer than $\mathfrak{T}$ in general. The concept of $\mathcal{J}$ -compact was studied by Newcomb[10] in 1967. Some contemporary researchers work in the field of ideal spaces for years in center, fuzzy, gem, frontier, and focal congested sets and we mention [1,2,9,12,17].
In 1947, Choquet [4] introduced the concept of the grill, following Choquet [4], a grill $\mathcal{G}$ on a space Ҳ is a collection of subsets of a non-empty set Ҳ such that:

(i) $\emptyset \notin \mathcal{G}$

(ii) д $\in \mathcal{G}$ and д $\subseteq \mathcal{L}$ then $\mathcal{L} \in \mathcal{G}$

(iii) д, $\mathcal{L} \subseteq$ Ҳ, and д $\cup \mathcal{L} \in \mathcal{G}$ then д $\in \mathcal{G}$ or $\mathcal{L} \in \mathcal{G}$.

A topological space (Ҳ $\mathfrak{T}$,) with a grill $\mathcal{G}$ on Ҳ is represented by (Ҳ, $\mathfrak{T}, \mathcal{G}$). Roy and Mukherjee [13] created a new topology on a topology space (Ҳ,$\mathfrak{T}$) resulting from a grill on Ҳ as shown below:

Suppose that $\mathcal{G}$ be a grill on a topology space (Ҳ, $\mathfrak{T}$), such that the operator $\phi_{\mathcal{G}}: p(\text{Ҳ}) \to p(\text{Ҳ})$ given by $\phi_{\mathcal{G}}(\text{д}) = \{x : \text{Ű} \cap \text{д} \in \mathcal{G}$, for every open Ű of $x\}$, where $\Psi_{\mathcal{G}} = \text{д} \cup \phi_{\mathcal{G}}(\text{д})$, hence induced a topology $\mathfrak{T}_{\mathcal{G}}$ on Ҳ which is strictly finer than $\mathfrak{T}$ in general. Then $\beta = \{\text{Ű} - \text{д}: \text{Ű} \in \mathfrak{T}\ and\ \text{д} \notin \mathcal{G}\}$ is a base for the topology $\mathfrak{T}_{\mathcal{G}}$. Roy and Mukherjee [14] defined a kind of compactness concerning the grill.

Hamlet and Jankovic [6] established the definition of $\mathcal{J}$ -regular space. Roy and Mukherjee [14] formulated the concept of $\mathcal{G}$-regular space.

In this research, we introduce a new definition of ideal grill spaces, namely $\mathcal{J}_{\mathcal{G}}$-regular space. Also, we give useful characterizations of $\mathcal{J}_{\mathcal{G}}$-regular space.

2. Preliminaries

Definition 2.1[10]: A subset Ҕ of ideal space (Ҳ, $\mathfrak{T}, \mathcal{J}$) is called $\mathcal{J}$ -compact if for every open cover $\{\mathfrak{H}_{\tilde{\alpha}}: \tilde{\alpha} \in \Omega\}$ of Ҕ, there exists a finite subsets Ω_0 of Ω such that Ҕ $- \bigcup_{\tilde{\alpha}\in\Omega_0} \mathfrak{H}_{\tilde{\alpha}} \in \mathcal{J}$.The space(Ҳ,$\mathfrak{T}, \mathcal{J}$) is called $\mathcal{J}$ -compact if Ҳ is $\mathcal{J}$ -compact a subset. So (Ҳ, $\mathfrak{T}$) is compact if and only if (Ҳ,$\mathfrak{T}$,$\{\emptyset\}$) is $\{\emptyset\}-$ compact.

Definition 2.2[14]: A grill space (Ҳ,$\mathfrak{T}$,$\mathcal{G}$) is called $\mathcal{G}-$compact if every open cover $\{\text{Ư}_{\tilde{\alpha}}: \tilde{\alpha} \in \Lambda\}$ of Ҳ, there exists a finite subset Λ_0 of Λ such that Ҳ $- \bigcup_{\tilde{\alpha}\in\Lambda_0} \text{Ư}_{\tilde{\alpha}} \notin \mathcal{G}$.

Remark 2.3 [10]: If (Ҳ, $\mathfrak{T}, \mathcal{J}$) is an ideal space and $\mathfrak{K}$ is a subset of Ҳ, then ($\mathfrak{K}$,$\mathfrak{T}_{\mathfrak{K}}, \mathcal{J}_{\mathfrak{K}}$) where $\mathfrak{I}_{\mathfrak{K}}$is the relative topology on $\mathfrak{K}$ and $\mathcal{J}_{\mathfrak{K}} = \{\text{I}_{\mathfrak{K}} \cap \text{ʝ}: \text{ʝ} \in \mathcal{J}\} is\ an\ ideal$ topological subspace.

Lemma 2.4[10]: *Let f:$X \to Y$ be function then :*

i. If $\mathfrak{F}$: (X,T, $\mathcal{J}$)→ (Y, ΰ)is one to one function where $\mathcal{J}$ is ideal on X, then $\mathfrak{F}$ $(\mathcal{J}) = \{\mathfrak{F}(д): д \in \mathcal{J}\}$ *is an ideal on* Y .

ii. If $\mathfrak{F}$: (X, T)→ (Y, ΰ, ɟ)is one to function where ɟ is ideal on Y, then $\mathfrak{F}^{-1}$(ɟ) = {$\mathfrak{F}^{-1}$(B):B ∈ ɟ} *is an ideal on* X .

Proposition 2.5 [5]: Let f: X → Y be function then:

i. If f:(X,T,$\mathcal{G}$)→ (Y, ΰ) is one to one where $\mathcal{G}$ is a grill on X, then $f(\mathcal{G}) = \{f(\mathcal{D}): \mathcal{D} \in \mathcal{G}\}$ is an grill on Y,

ii. If f: (X,T)→ (Y,ΰ, $\mathcal{G}'$)is one to function where $\mathcal{G}'$is a grill on Y, then $f^{-1}(\mathcal{G}') = \{f^{-1}$(B):B ∈ $\mathcal{G}'$ } is a grill on X.

Theorem 2.6[15]: *If $G_i \in \mathcal{G}$ for all $i \in I$ then* $\bigcup_{i \in I} G_i \in \mathcal{G}$.

Definition 2.7[3]: A subset $\mathfrak{M}$ of ideal grill space (X, $\mathfrak{T}, \mathcal{J}, \mathcal{G}$) is called ideal grill compact (briefly $\mathcal{J}_{\mathcal{G}}$-compact) if for every open cover $\{\breve{G}_{\tilde{\alpha}}: \tilde{\alpha} \in \Lambda\}$ of $\mathfrak{M}$, there exists a finite subset $\{\breve{G}_{\tilde{\alpha}_j}: j = 1,2, \ldots, n\}$ such that $\mathfrak{M} - \bigcup_{i=1}^{n} \breve{G}_{\tilde{\alpha}_j} \in \mathcal{J}$ and $\bigcup_{i=1}^{n} \breve{G}_{\tilde{\alpha}_j} \in \mathcal{G}$ .The space(X, T) is called be $\mathcal{J}_{\mathcal{G}}$-compact if X is $\mathcal{J}_{\mathcal{G}}$-compact.

Remark 2.8[3]:

(i) Every compact space is $\mathcal{J}_{\mathcal{G}}$-compact for any ideal grill.

(ii) Every $I_{\mathcal{G}}$-compact space is $\mathcal{J}$-compact and $\mathcal{G}$-compact.

Remark 2.9[3]:

(i) A space(X, $\mathfrak{T}, \mathcal{J}, \mathcal{G}$) is called an ideal grill topological space such that $\mathcal{J}$ *and* $\mathcal{G}$ are ideal grill on X and denoted by$\mathcal{J}_{\mathcal{G}}$ –topological space.

(ii) It is clear that (X, $\mathfrak{T}$)is compact iff $\{\emptyset\}_{p(X)-\{\emptyset\}}$ − compact space such that $\mathcal{J}$ =$\{\emptyset\}$ and $\mathcal{G} = p(X) - \{\emptyset\}$.

Definition 2.10[11]: A space (X, $\mathfrak{T}$) is called quasi H- closed (QHC), if for every open cover $\mathfrak{E} = \{\mathfrak{E}_{\dot{\rho}}: \dot{\rho} \in \Lambda\}$ of X, there is a finite subset $\{\mathfrak{E}_{\dot{\rho}_i}: i = 1,2, \ldots, m\}$ such that $X = \bigcup_{i=1}^{n} \mathfrak{E}_{\dot{\rho}_i}$. A space (X, $\mathfrak{T}$) is said to be H- closed if its Hausdorff and (QHC).

Theorem 2.11[14]: *Let (X, $\mathfrak{T}$) be grill space such that $\mathfrak{T} - \{\emptyset\} \subseteq \mathcal{G}$. If (X, $\mathfrak{T}$) is $\mathcal{G}$ −compact, then (X, $\mathfrak{T}$) is QHC.*

Definition 2.12[6]: A space ($\mathfrak{X}$, $\mathfrak{T}$, $\mathcal{J}$) is called $\mathcal{J}$ -regular, if for each closed set $\mathfrak{H}$ with $c \notin \mathfrak{H}$, there exist open subsets $\mathfrak{S}$ and $\mathfrak{N}$ of $\mathfrak{X}$ such that $c \in \mathfrak{S}$, $\mathfrak{H} - \mathfrak{N} \in \mathcal{J}$and $\mathfrak{S} \cap \mathfrak{N} = \emptyset$ .If $\mathcal{J} = \{\emptyset\}$ then $\mathcal{J}$ -regular and regular are identical.

Definition 2.13[14]: A space ($\mathfrak{X}$,$\mathfrak{T}$, $\mathcal{G}$) is called $\mathcal{G}$ -regular, if for each closed set with $\mathfrak{d} \notin \mathfrak{H}$, there exist disjoint open sets $\mathfrak{S}$ and $\mathfrak{N}$ such that $\mathfrak{d} \in \mathfrak{S}$ and $\mathfrak{H} - \mathfrak{N} \notin \mathcal{G}$.

3. $\mathcal{J}_{\mathcal{G}}$-Regular space

Definition 3.1: Let $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$ be $\mathcal{J}_{\mathcal{G}}$-topological space. A space ($\mathfrak{X}$, $\mathfrak{T}$) is called $\mathcal{J}_{\mathcal{G}}$-regular if for any closed set $\mathfrak{L}$ in $\mathfrak{X}$ with $d \notin \mathfrak{L}$, there exist disjoint open sets $\mathfrak{M}$ and $\mathfrak{K}$ such that $d \in \mathfrak{M}$, $\mathfrak{L} - \mathfrak{K} \in \mathcal{J}$, and $\mathfrak{L} - \mathfrak{K} \notin \mathcal{G}$.

Example 3.2: Every discrete space is $\mathcal{J}_{\mathcal{G}}$-compact space for any ideal $\mathcal{J}$ and grill $\mathcal{G}$.

Remark 3.3:
(i) Every regular space is $\mathcal{J}_{\mathcal{G}}$-regular for any ideal $\mathcal{J}$ and grill.
(ii) Every $\mathcal{J}_{\mathcal{G}}$-regular space is $\mathcal{J}$-regular resp. $\mathcal{G}$-regular.
(iii) Every $\mathcal{J}_{\mathcal{G}}$-regular space iff $\mathcal{J}$-regular and $\mathcal{G}$-regular .
(iv) If $\mathcal{J}$ =$\{\emptyset\}$ and $\mathcal{G} = p(\mathfrak{X}) - \{\emptyset\}$, then the concepts of regular space and $\mathcal{J}$ -regular correspond.

The opposite of Remark (3.3, i, ii) is not generally true, as shown in the following examples.

Example 3.4: Let$(\mathfrak{X}, \mathfrak{T})$ be co-finite space on $\mathcal{J}$ infinite set. Then $(\mathfrak{X}, \mathfrak{T})$ is $(\mathcal{J}_f)_{\mathcal{G}_\infty}$regular space, but not regular space.

Example 3.5: Let $\mathfrak{X} = \{c, d, \mathfrak{d}\}$,$\mathfrak{T} = \{\emptyset, \mathfrak{X}, \{c\}, \{d\}\}$ be topological space. Then ($\mathfrak{X}$, T) is $\mathcal{J}$-regular space and it's not $(\mathcal{J}_f)_{\mathcal{G}_\mathfrak{d}}$-regular.

Proposition 3.6: Every subspace $\mathfrak{S}$ of $\mathcal{J}_{\mathcal{G}}$- regular space$(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$ is $(\mathcal{J}_{\mathfrak{S}})_{\mathcal{G}_{\mathfrak{S}}}$-regular.

Proof: Suppose that $\mathfrak{S}$ be a subset of a space $\mathfrak{X}$ and $\mathfrak{d} \in \mathfrak{S}$ .Let $\mathfrak{N}$ be a closed subset of $\mathfrak{S}$ such that $\mathfrak{d} \notin \mathfrak{N}$, there exists a closed set $\mathfrak{E}$ in $\mathfrak{X}$ with $\mathfrak{N} = \mathfrak{S} \cap \mathfrak{E}$ and $\mathfrak{d} \notin \mathfrak{E}$. Given $\mathfrak{X}$ is $\mathcal{J}_{\mathcal{G}}$- regular space, then there exist disjoint open sets $\mathfrak{B}, \mathfrak{K}$ such that $\mathfrak{d} \in \mathfrak{B}, \mathfrak{E} - \mathfrak{K} \in \mathcal{J}$, and $\mathfrak{E} - \mathfrak{K} \notin \mathcal{G}$. Suppose that $\mathfrak{H} = \mathfrak{E} - \mathfrak{K}$, then $\mathfrak{E} \subseteq \mathfrak{H} \cup \mathfrak{K}$ and $\mathfrak{S} \cap \mathfrak{E} \subseteq \mathfrak{S} \cap (\mathfrak{H} \cup \mathfrak{K})$, thus $\mathfrak{N} - (\mathfrak{S} \cap \mathfrak{K}) \subseteq \mathfrak{S} \cap (\mathfrak{E} - \mathfrak{K})$.

Then $\mathfrak{N} - (\mathfrak{S} \cap \mathfrak{K}) \in \mathcal{J}_{\mathfrak{S}}$ and $\mathfrak{N} - (\mathfrak{S} \cap \mathfrak{K}) \notin \mathcal{G}_{\mathfrak{S}}$ and $\mathfrak{d} \in \mathfrak{S} \cap \mathfrak{V}$ .Therefore $\mathfrak{S}$ is $(\mathcal{J}_{\mathfrak{S}})_{\mathcal{G}_{\mathfrak{S}}}$-regular.

Through the properties and definitions of continuous and open functions and the bijective property and definition (3.1), we get the following fact.

Proposition 3.7: If $\pounds : (\mathfrak{X}, \mathrm{T}, \mathcal{J}, \mathcal{G}) \to (\Upsilon, \mathrm{T})$ is a continuous, open, and bijective function. Then $\mathfrak{X}$ is $\mathcal{J}_{\mathcal{G}}$-regular space, then $\mathfrak{Y}$ is $\pounds(\mathcal{J})_{\pounds(\mathcal{G})}$-compact space.

Proof: Let $\mathscr{b} \in \Upsilon$ and $\mathfrak{L}$ be closed subset of Υ such that $\mathscr{b} \notin \mathfrak{L}$, thus $\pounds^{-1}(\mathfrak{L})$ is a closed set in $\mathfrak{X}$ and $\pounds^{-1}(\mathscr{b}) \notin \pounds^{-1}(\mathfrak{L})$ .By $\mathfrak{X}$ is $\mathcal{J}_{\mathcal{G}}$-regular space, then there exist disjoint open sets $\mathfrak{M}$ and $\mathfrak{K}$ such that $\pounds^{-1}(\mathscr{b}) \in \mathfrak{M}$, $\pounds^{-1}(\mathfrak{L}) - \mathfrak{K} \in \mathcal{J}$ and $\pounds^{-1}(\mathfrak{L}) - \mathfrak{K} \notin \mathcal{G}$. So $\pounds(\mathfrak{M})$ and $\pounds(\mathfrak{K})$ are disjoint open sets in $\mathscr{b}$ and $\mathscr{b} \in \pounds(\mathfrak{M})$, then $\pounds(\pounds^{-1}(\mathfrak{L}) - \mathfrak{K}) \in \pounds(\mathcal{J})$ and $\pounds(\pounds^{-1}(\mathfrak{L}) - \mathfrak{K}) \notin \pounds(\mathcal{G})$.Thus$\mathfrak{L} - \pounds(\mathfrak{K}) \in \pounds(\mathcal{J})$ and $\mathfrak{L} - \pounds(\mathfrak{K}) \notin \pounds(\mathcal{G})$. Hence Υ is$\pounds(\mathcal{J})_{\pounds(\mathcal{G})}$-compact space.

Theorem 3.8: *Let* $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$*be* $\mathfrak{T}_2 - \mathcal{J}_{\mathcal{G}}$*-topological space. If* $(\mathfrak{X}, \mathfrak{T})$*is* $\mathcal{J}_{\mathcal{G}}$*-compact then it is* $\mathcal{J}_{\mathcal{G}}$*-regular.*

Proof: It is simple to prove by using the properties of $\mathfrak{T}_2$ $-$space and definitions of $\mathcal{J}_{\mathcal{G}}$-compact space.

Theorem 3.9: *Let* $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$*be* $\mathcal{J}_{\mathcal{G}}$ *−Topological space such that* $\mathfrak{T} - \{\emptyset\} \subseteq \mathcal{G}$. *If (X, $\mathfrak{T}$) is* $\mathcal{J}_{\mathcal{G}}$ *−compact, then (X, $\mathfrak{T}$) is* QHC.

Proof: By Remark (2.8, ii) and Theorem (2.11)

Corollary 3.10: *Let* $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$*be*$\mathfrak{T}_2 - \mathcal{J}_{\mathcal{G}}$ *-topological space such that* $\mathfrak{T} - \{\emptyset\} \subseteq \mathcal{G}$.*If* $(\mathfrak{X}, \mathfrak{T})$*is* $\mathcal{J}_{\mathcal{G}}$*-compact, then* $(\mathfrak{X}, \mathfrak{T})$*is H-closed and* $\mathcal{J}_{\mathcal{G}}$*-regular.*

Proof: By Theorem (3.9) and Theorem (3.8)

Main Theorem

Theorem 3.11: *Let* $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$*be* $\mathcal{J}_{\mathcal{G}}$ *−topological space such that (X, $\mathfrak{T}$) is H-closed. If (X, $\mathfrak{T}$) is*$\mathcal{J}_{\mathcal{G}}$*-regular space, then it is* $\mathcal{J}_{\mathcal{G}}$*-compact.*

Proof: Let $\mathfrak{V} = \{\mathfrak{V}_x : x \in \Delta\}$ is an open cover of $\mathfrak{X}$, $\forall x \in \mathfrak{X}$, there exist $\mathfrak{V}_x \in \mathfrak{V}$ such that $x \in \mathfrak{V}_x$ .Since $\mathfrak{X} - \mathfrak{V}_x$ is closed set and $x \notin \mathfrak{X} - \mathfrak{V}_x$, then there exist disjoint open sets $\mathfrak{S}_x$ and $\mathfrak{N}_x$such that $x \in \mathfrak{S}_x$, $(\mathfrak{X} - \mathfrak{V}_x) - \mathfrak{N}_x \in \mathcal{J}$ and$(\mathfrak{X} - \mathfrak{V}_x) - \mathfrak{N}_x \notin \mathcal{G}$.Suppose that $\mathfrak{A}_x = (\mathfrak{X} - \mathfrak{V}_x) - \mathfrak{N}_x$, but $\mathrm{cl}(\mathfrak{S}_x) \cap \mathfrak{N}_x = \emptyset$, then $\mathrm{cl}(\mathfrak{S}_x) \subseteq \mathfrak{A}_x \cup \mathfrak{V}_x$.So $\{\mathfrak{S}_x : x \in \mathfrak{X}\}$ is an open cover of the H-closed space $\mathfrak{X}$, then there exist finite subsets $\left\{\mathfrak{S}_{x_{\alpha}} : \alpha = 1,2,\dots,n\right\}$ such that $\mathfrak{X}=\bigcup_{\alpha=1}^{n} cl(\mathfrak{S}_{x_{\alpha}})$. Thus $\mathfrak{X} - \bigcup_{\alpha=1}^{n} \mathfrak{B}_{x_{\alpha}} \subseteq \bigcup_{\alpha=1}^{n} \mathfrak{A}_{x_{\alpha}}$, then $\mathfrak{X} - \bigcup_{\alpha=1}^{n} \mathfrak{B}_{x_{\alpha}} \in \mathcal{J}$ and $\mathfrak{X} - \bigcup_{\alpha=1}^{n} \mathfrak{B}_{x_{\alpha}} \notin \mathcal{G}$ .So $\bigcup_{\alpha=1}^{n} \mathfrak{B}_{x_{\alpha}} \in \mathcal{G}$, hence $\mathfrak{X}$ is $\mathcal{J}_{\mathcal{G}}$-compact space.

Proposition 3.12: If $\mathfrak{f} :(\mathfrak{X}, \mathrm{T}) \rightarrow (\Upsilon, \mathrm{T}, \mathcal{J}, \mathcal{G})$ is a continuous, bijective, and open function such that $(\Upsilon, \mathrm{T}, \mathcal{J}, \mathcal{G})$ is $\mathcal{J}_{\mathcal{G}}$-regular space ,then$(\mathfrak{X}, \mathrm{T}, \mathfrak{f}^{-1}(\mathcal{J}), \mathfrak{f}^{-1}(\mathcal{G}))$ is $\mathfrak{f}^{-1}(\mathcal{J})_{\mathfrak{f}^{-1}(\mathcal{G})}$-regular space.

Proof: A similar proof to the proposition (3.7).

Corollary 3.13: *Let* $(\mathfrak{X}, \mathfrak{T}, \mathcal{J}, \mathcal{G})$*be* $\mathcal{J}_{\mathcal{G}} - \mathfrak{T}_2$*-Topological space such that* $\mathfrak{T} - \{\emptyset\} \subseteq \mathcal{G}$*.Then* $(\mathfrak{X}, \mathfrak{T})$*is* $\mathcal{J}_{\mathcal{G}}$*-compact iff* $(\mathfrak{X}, \mathfrak{T})$*is* H*-closed and* $\mathcal{J}_{\mathcal{G}}$*-regular.*

Proof: By Corollary (3.10) and Theorem (3.11)

Conclusion

1. The researchers have presented a new definition for $\mathcal{J}_{\mathcal{G}}$-regular space and studied the relationship between $\mathcal{J}_{\mathcal{G}}$-regular space and $\mathcal{J}$ -regular, $\mathcal{G} -$ regular, and $\mathcal{J}_{\mathcal{G}}$- compact space.
2. There are future works to enlarge the separation properties, which correspond to our new definition.

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